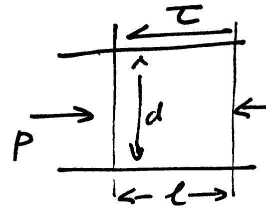
ENGINEERING TRIPOS IB 1998

PAPER 4. FLUID MECHANICS AND HEAT TRANSFER.

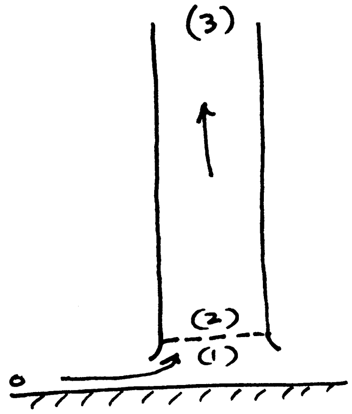
Q.1. (a)



$$\Delta p \frac{\pi d^2}{4} = \tau \pi d l$$

$$\therefore \Delta p = 4\tau \frac{l}{d} = 4C_f \tfrac{1}{2}\rho V^2 \frac{l}{d}$$

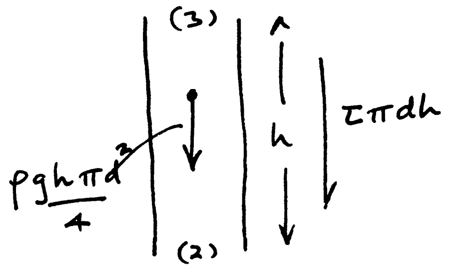
(b)



Continuity 1—2, $\rho_1 V_1 = \rho_2 V_2$

$$\therefore \rho_1 V_1 = 0.8\rho_1 V_2$$

$$\therefore V_2 = V_1/0.8$$



No change in momentum

$$\therefore p_3 = p_2 - \left[4\tau \frac{h}{d} + \rho_2 g h\right]$$

$$= p_2 - \left[4C_f \tfrac{1}{2}\rho_2 V_2^2 \frac{h}{d} + \rho_2 g h\right]$$

also $p_2 = p_1 - 0.1\rho_1 V_1^2$ and Bernoulli 0—1, $p_1 = p_0 - \tfrac{1}{2}\rho_1 V_1^2$

and pressure p_3 = atmospheric pressure at this height

$$= p_0 - \rho_1 g h$$

$$\therefore p_0 - \tfrac{1}{2}\rho_1 V_1^2 - 0.1\rho_1 V_1^2 - \left[4C_f \tfrac{1}{2} 0.8\rho_1 \frac{V_1^2}{0.8^2}\frac{h}{d} + 0.8\rho_1 g h\right]$$

$$= p_0 - \rho_1 g h$$

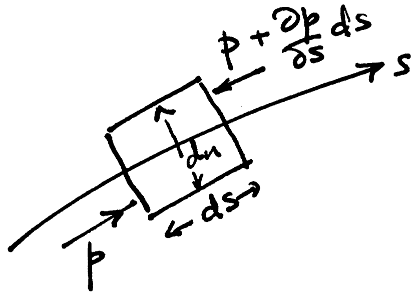
1 (cont.) $\therefore \; \rho_1 V_1^2 \left[0.6 + 4 \times 0.007 \times \frac{20}{2} \times \frac{1}{0.8}\right] = 0.2\, \rho_1 \times 9.81 \times 20$

$$V_1^2 (0.95) = 39.24$$

$$V_1 = 6.43$$

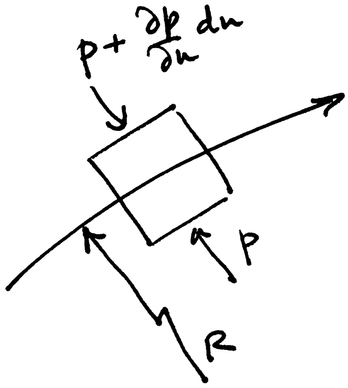
$$\underline{\underline{V_2 = 8.034 \text{ m/s}}}$$

Q2.



$$- \frac{\partial p}{\partial s} ds\, dn = \rho V \frac{\partial V}{\partial s} ds\, dn$$

$$\therefore \; \underline{\underline{\frac{\partial p}{\partial s} + \rho V \frac{\partial V}{\partial s} = 0}}$$



$$\frac{\partial p}{\partial n} dn\, ds = \frac{\rho V^2}{R} dn\, ds$$

$$\therefore \; \underline{\underline{\frac{\partial p}{\partial n} = \frac{\rho V^2}{R}}}$$

For circular streamlines, no variation with s

(a) $\frac{dp}{dr} = \frac{\rho V^2}{r} = \rho k_1^2 r$

$$\therefore \; \underline{p = \rho \frac{k_1^2 r^2}{2} + C_1}$$

(b) $\frac{dp}{dr} = \frac{\rho k_2^2}{r^3} \quad \therefore \; \underline{p = - \frac{\rho k_2^2}{2 r^2} + C_2}$

For the combined flow, at R the velocity and pressure are continuous

2 (cont.) $\therefore \quad k_1 R = \dfrac{k_2}{R}$

and $$\rho \frac{k_1^2 R^2}{2} + C_1 = -\rho \frac{k_2^2}{2R^2} + C_2$$

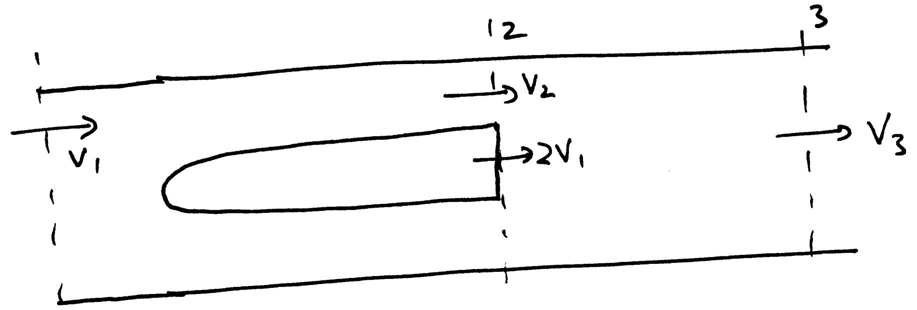
but $C_1 = p_0 \;, \quad C_2 = p_\infty$

$$\therefore \quad \rho \frac{k_2^2}{2R^2} + p_0 = -\rho \frac{k_2^2}{2R^2} + p_\infty$$

$$\therefore \quad \rho \frac{k_2^2}{R^2} = p_\infty - p_0$$

$$\left.\begin{aligned} k_2^2 &= \frac{R^2 (p_\infty - p_0)}{\rho} \\ k_1^2 &= \frac{(p_\infty - p_0)}{\rho R^2} \end{aligned}\right\}$$

Q 3.



No losses 1 – 2 $\therefore$ Bernoulli

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

Continuity $$\pi V_1 \frac{(1.5)^2}{4} = \frac{\pi V_2}{4}(1.5^2 - 0.5^2)$$

$$\therefore \quad V_2 = 1.125\, V_1$$

$$\therefore \quad \frac{p_2 - p_1}{\tfrac{1}{2}\rho V_1^2} = 1 - \left(\frac{V_2}{V_1}\right)^2 = \underline{\underline{-\,0.266}}$$

3(cont)

Continuity 2-3

$$V_1(1.5)^2 + 2V_1(0.5)^2 = V_3(1.5)^2$$

$$\therefore\ V_3 = 1.22\,V_1$$

Mixing 2-3 hence no Bernoulli but momentum (no force on sides)

$$\therefore\ (p_2 - p_3)\frac{\pi(1.5)^2}{4} = \pi\rho V_3^2\frac{(1.5)^2}{4} - \rho V_2^2(1.5^2 - 0.5^2)\frac{\pi}{4}$$

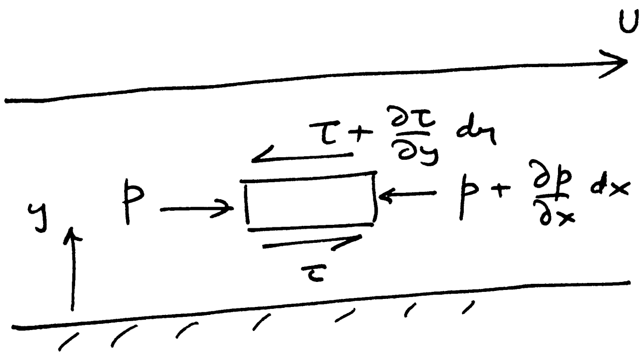
$$- \rho(2V_1)^2 0.5^2\pi/4$$

$$= \rho\pi\left[(1.22)^2 V_1\frac{(1.5)^2}{4} - \frac{(1.125)^2 V_1^2\, 2\times 1}{4} - V_1^2\frac{0.5^2\times 4}{4}\right]$$

$$\therefore\ \frac{p_2 - p_3}{\frac{1}{2}\rho V_1^2} = \frac{2}{1.5^2}\left[(1.22)^2(1.5)^2 - 2(1.125)^2 - 4\times 0.5^2\right]$$

$$= -0.162$$

Q4.



$$\frac{\partial\tau}{\partial y}dy\,dx = \frac{\partial p}{\partial x}dx\,dy \quad \therefore \quad \frac{\partial\tau}{\partial y} = \frac{\partial p}{\partial x}$$

$$\tau = \mu\frac{\partial u}{\partial y} \qquad \therefore\ \mu\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}\ \text{(const)}$$

$$\therefore\ \mu u = \frac{dp}{dx}\frac{y^2}{2} + K_1 y + K_2$$

4 (cont.) Boundary conditions $u=0, y=0$ and $u=U, y=h$

$$\therefore K_2 = 0 \qquad \mu U = \frac{dp}{dx}\frac{h^2}{2} + K_1 h$$

$$\therefore K_1 = \frac{1}{h}\left[\mu U - \frac{dp}{dx}\frac{h^2}{2}\right]$$

$$\therefore u = \frac{1}{2\mu}\frac{dp}{dx}\left[y^2 - yh\right] + \frac{Uy}{h}$$

$$\text{Flow rate} = \int_0^h u\,dy = \frac{1}{\mu}\int_0^h \left[\frac{dp}{dx}\left(\frac{y^2}{2} - \frac{yh}{2}\right) + \mu\frac{Uy}{h}\right]dy$$

$$= \frac{1}{\mu}\left[\frac{dp}{dx}\left(\frac{y^3}{6} - \frac{y^2 h}{4}\right) + \frac{\mu U y^2}{2h}\right]_0^h$$

$$= \frac{1}{\mu}\left[\frac{dp}{dx}\left(\frac{h^3}{6} - \frac{h^3}{4}\right) + \frac{\mu U h}{2}\right] = 0$$

$$\text{when} \quad \underline{\underline{\frac{dp}{dx}\frac{h^2}{\mu U} = 6}}$$

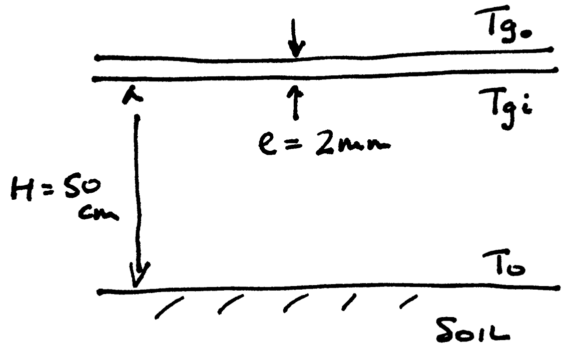
Dimensional Analysis for zero flow

$$\frac{dp}{dx} = \text{fn}\left[U, \mu, h\right] \text{ only}$$

$$\therefore \underline{\frac{dp}{dx}\frac{h^2}{\mu U} = \text{const}}$$

 T_∞

Q5.



Equilibrium of dark surface (soil)

$$Q_0 + \sigma T_{gi}^4 = \sigma T_0^4 + \frac{N_u \lambda_a}{H}(T_0 - T_{gi}) \quad \text{———} (1)$$

solar | radiation from glass | radiation from surface | convection inside greenhouse

Equilibrium of glass

$$Q_0 = \lambda_g \frac{T_{gi} - T_{go}}{e} \quad \text{———} (2)$$

downwards flux | upwards flux

Equilibrium of air outside

$$Q_0 = \sigma T_{go}^4 + h(T_{go} - T_\infty) \quad \text{———} (3)$$

solar | radiation to atmosphere | convection

5 (cont) Eq. (3) $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

$Q_o = 600\ W m^{-2}$

$$\therefore \quad 600 = 5.67 \times 10^{-8}\ T_{g_o}^4 + 10(T_{g_o} - 278)$$

$$\underline{T_{g_o} = 22^\circ C} \quad \text{checks}$$

(2) $$T_{g_i} = \frac{Q_o e}{\lambda_g} + T_{g_o}$$

$$= \frac{600 \times 2 \times 10^{-3}}{1} + 295 = 296.2\,K$$

$$= \underline{23.2\ ^\circ C}$$

(1) $$600 + 5.67 \times 10^{-8} \times (296.2)^4 = 5.67 \times 10^{-8} (T_o)^4 + \frac{500 \times 2.5 \times 10^{-2}}{50 \times 10^{-2}} (T_o - 296.2)$$

Iteration $T_o = 315.3\,K = \underline{42.3^\circ C}$

Q. 6

$$CO + (1+x)\tfrac{1}{2}\left(O_2 + \tfrac{79}{21} N_2^{+}\right) \longrightarrow CO_2 + \left(\frac{1+x}{2}\right) \frac{79}{21} N_2^{+} + \frac{x}{2} O_2$$

SFEE $$Q = H_{P_2} - H_{R_1} = (H_{P_2} - H_{P_0}) + (H_{P_0} - H_{R_0}) + \underbrace{(H_{R_0} - H_{R_1})}_{=0 \text{ both at } 25^\circ C}$$

6 (cont.)

$$\therefore \quad H_{P_2} - H_{P_0} = Q - \overline{CV}$$

$$-CV = \underset{\substack{\text{table}\\1}}{28} \times \underset{\substack{\text{table}\\2}}{10.1} = 282.8 \text{ MJ/Kmol co}$$

$$\frac{Q}{\dot{m}} = -10^3(2000 - 298) = -1.702 \text{ MJ/kg}$$

$$= -\frac{28 \times 1.702}{\dot{m}} = -\frac{47.7}{10} = -4.77 \text{ MJ/Kmol Co}$$

$$H_{P_2} - H_{P_0} = 282.8 - 4.77 = 278 \text{ MJ/Kmol co}$$

$$\therefore \quad H_{P_2} - H_{P_0} = \underset{1}{n_{CO_2}}\left(h^{2000}_{CO_2} - h^{298}_{CO_2}\right) + \underset{\frac{(1+x)}{2}\frac{79}{21}}{n_{N_2}}\left(h^{2000}_{N_2} - h^{298}_{N_2}\right)$$

$$+ \underset{\frac{x}{2}}{n_{O_2}}\left(h^{2000}_{O_2} - h^{298}_{O_2}\right)$$

$$= (100.97 - 9.37) + (1+x)\frac{79}{42}(64.84 - 8.67)$$

$$+ \frac{x}{2}(67.86 - 8.66)$$

$$\therefore \quad x = \frac{278 - 91.6 - \frac{79}{42}56.17}{\frac{79}{42}56.17 + \frac{59.2}{2}} = \underline{\underline{59.7\%}}$$