IB Paper 5 June 1998 - Crib.

Section A

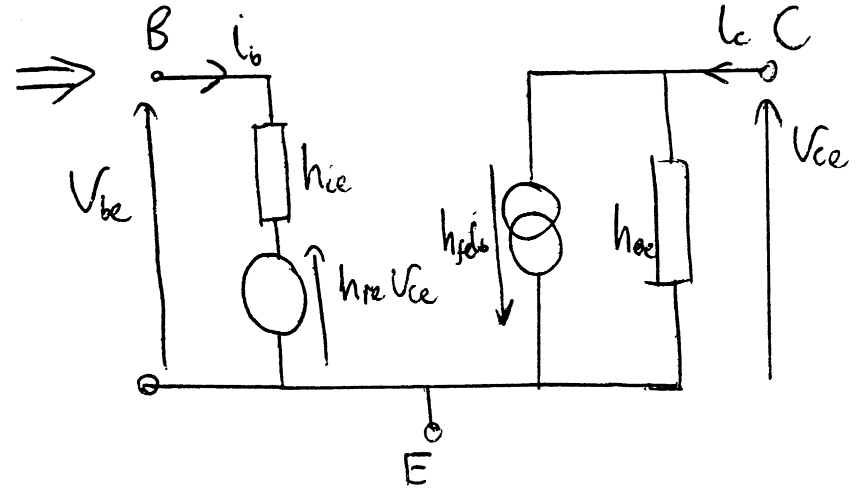
1. $\delta I_c \to i_c$, $\delta I_B \to i_b$, $\delta V_{CE} \to v_{ce}$, $\delta V_{BE} \to v_{be}$

$$\frac{\partial I_c}{\partial I_B} = h_{fe}, \quad \frac{\partial I_c}{\partial V_{CE}} = h_{oe}, \quad \frac{\partial V_{BE}}{\partial I_B} = h_{ie}, \quad \frac{\partial V_{BE}}{\partial V_{CE}} = h_{re}$$

$$\Rightarrow \quad i_c = h_{fe} i_b + h_{oe} v_{ce} \qquad (1)$$

$$v_{be} = h_{ie} i_b + h_{re} v_{ce} \qquad (2)$$

$\Rightarrow$



It is seen that the circuit linking C to E represents (1), that linking B to E represents (2)

a) Voltage across R_c is $20 - V_{CE} = 10V$ (R_L is ignored in bias analysis because of C_2)

$\therefore \; I_c = \frac{10}{R_c} = 25mA$

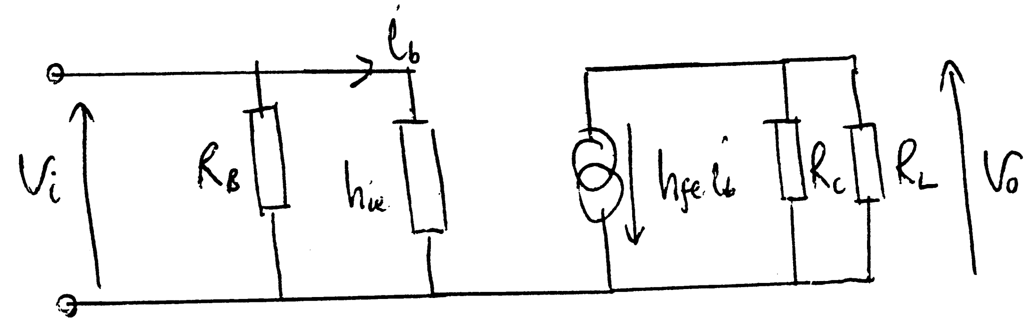
$\Rightarrow \; \underline{\underline{R_c = 400\Omega}}$

$$h_{FE} = 200 = \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{200} = 125\,\mu A$$

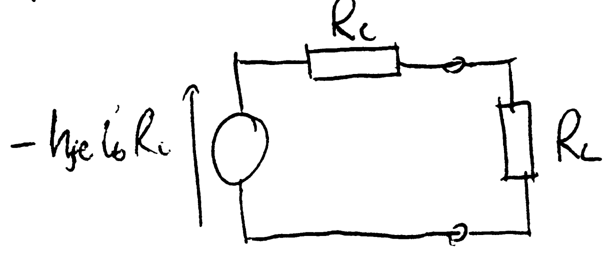
Quiescent voltage across $R_B = 20 - V_{BE} = 19.3\,V$

$$\therefore \frac{19.3}{R_B} = 125\,\mu A \Rightarrow \underline{\underline{R_B = 154.4\,k\Omega}}$$

b)



Replace $h_{fe} i_b \,//\, R_C$ with Thevenin equivalent



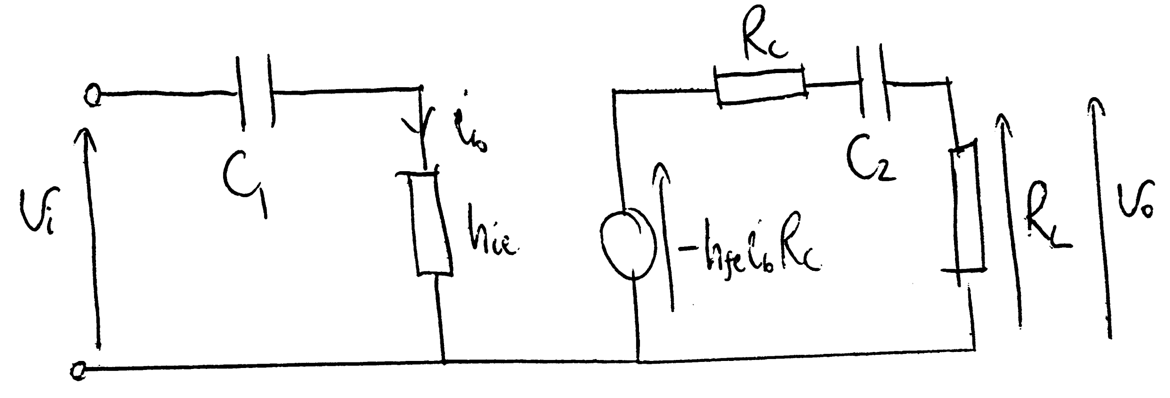
By maximum power transfer theorem $R_L = R_C$

$$\Rightarrow \underline{\underline{R_L = 400\,\Omega.}}$$

$$i_b = \frac{V_i}{h_{ie}}, \quad V_o = -h_{fe} i_b\, R_C // R_L$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-h_{fe}\, R_C//R_L}{h_{ie}} = -\frac{200}{800} \times 200 = \underline{\underline{-50}}$$

c) At low frequencies the reactances of C_1 and C_2 increase. C_1 will reduce the base current, and C_2 is effectively in series with R_L. Both of these will cause the gain to be reduced.



As before, replace $h_{fe}i_b // R_c$ with $-h_{fe}i_bR_c$ in series with R_c

$$i_b = \frac{V_i}{h_{ie} + 1/j\omega C_1} \qquad V_o = \frac{R_L}{R_L + R_c + 1/j\omega C_2} \times (-h_{fe} i_b R_c)$$

$$\frac{V_o}{V_i} = \frac{-h_{fe} R_c R_L}{(h_{ie} + 1/j\omega C_1)(R_L + R_c + 1/j\omega C_2)}$$

<u>Check</u> $\omega \to \infty$, $\frac{V_o}{V_i} \to -\frac{h_{fe}}{h_{ie}} \frac{R_c R_L}{R_c + R_L} = -\frac{h_{fe}}{h_{ie}} R_c // R_L$

ie tends towards mid-band gain.

d) Notice that $h_{ie} = R_L + R_c$, and $C_1 = C_2$. For half-power,

$$\left(h_{ie}^2 + \left(\frac{1}{\omega C_1}\right)^2\right)^{1/2} \left((R_L+R_c)^2 + \left(\frac{1}{\omega C_2}\right)^2\right)^{1/2} = \sqrt{2}\, h_{ie}(R_L + R_c)$$

$$\Rightarrow \quad h_{ie}^2 + \left(1/\omega C_1\right)^2 = \sqrt{2}\, h_{ie}^2$$

$$\frac{1}{\omega C_1} = h_{ie}(\sqrt{2} - 1)^{1/2} \qquad \omega = 97.1 \qquad f = 15.5\ Hz.$$

2/ Two signals V_1 and V_2 can always be broken down as:-

$$V_1 = V_c + \frac{V_d}{2} \quad , \quad V_2 = V_c - \frac{V_d}{2}$$

where V_c is the average, or common-mode component of the signals

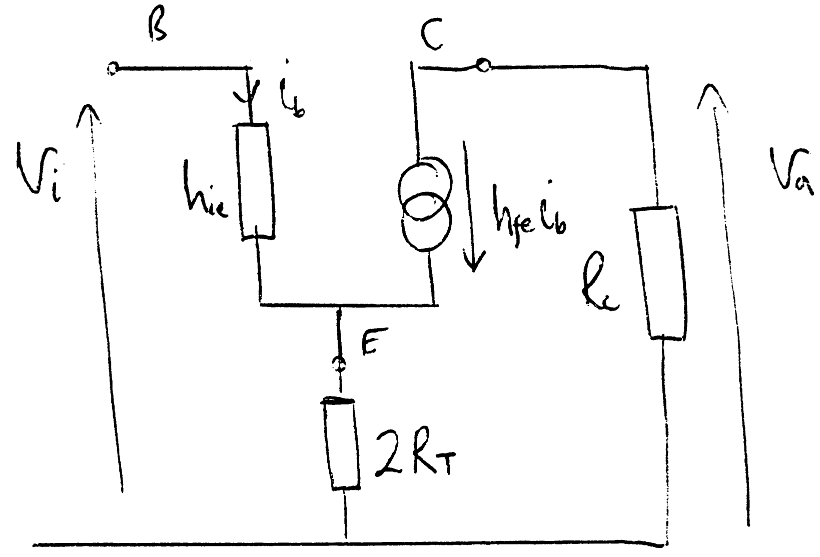
V_d is the difference between, or the differential components of the signals.

∴ The common-mode signal is of the form $V_1 = V_2$

The differential signal is of the form $V_1 = -V_2$.

In this application, the thermocouple signal is a differential signal of one order magnitude less than the mains interference, which is a common-mode signal. A differential amplifier is designed to have a much higher gain for differential than for common-mode signals, as required here.

b) $V_1 = V_2 \Rightarrow i_{e_1} = i_{e_2} \Rightarrow$ need to include R_T. To use half-circuit principles, replace R_T with two parallel resistors of value $2R_T$.



$$V_i = h_{ie} i_b + 2R_T (h_{fe}+1) i_b$$

$$V_a = -h_{fe} i_b R_c$$

$$\frac{V_a}{V_i} = \frac{-h_{fe} R_c}{h_{ie} + 2R_T (h_{fe}+1)}$$

Differential gain $\Rightarrow V_1 = -V_2 \Rightarrow i_{e_1} = -i_{e_2}$ so no small signal current in R_T, so emitters of transistors are at small-signal earth. $\therefore$ Circuit for differential signal as above, but with $2R_T = 0$.

$\therefore$ Find differential gain from common-mode gain, but with $R_T = 0$

$$\frac{V_a - V_b}{V_1 - V_2} = \frac{-h_{fe} R_c}{h_{ie}}$$

$$CMRR = \frac{A_{diff}}{A_{common\text{-}mode}} = \frac{-h_{fe} R_c / h_{ie}}{-h_{fe} R_c / (h_{ie} + 2R_T (h_{fe}+1))} = \frac{h_{ie} + 2R_T (h_{fe}+}{h_{ie}}$$

$$A_{diff} = 1000 = \frac{h_{fe} R_c}{h_{ie}} = \frac{200 R_c}{10^3}$$

$$\Rightarrow \underline{\underline{R_c = 5 k\Omega}}$$

$$CMRR = 2000 = \frac{h_{ie} + 2(h_{fe}+1) R_T}{h_{ie}} = \frac{10^3 + 2(201) R_T}{10^3}$$

$$\Rightarrow \underline{\underline{R_T = 4973 \Omega}}$$

c) The new small-signal circuit valid for differential signals is the same as that for common-mode signals, except with $2R_T$ replaced by R_E.

$$\therefore \underline{\underline{A_{diff} = \frac{-h_{fe} R_c}{h_{ie} + R_E(h_{fe}+1)}}}$$

$$\text{Gain reduction factor} = \frac{-h_{fe} R_c / h_{ie}}{-h_{fe} R_c / (h_{ie} + R_E(h_{fe}+1))}$$

$$= \frac{h_{ie} + R_E(h_{fe}+1)}{h_{ie}}$$

$$= 1 + \frac{R_E(h_{fe}+1)}{h_{ie}}$$

d) Applying idea from theory of negative feedback, open-loop gain is reduced by factor "1+AB" $\Rightarrow$ input resistance is increased by the same factor

$$\therefore 1 + \frac{R_E(h_{fe}+1)}{h_{ie}} = 10 \qquad \underline{R_E = 44.8 \Omega.}$$

4/ Generate as a.c. because transformers can be used to step up the voltage for transmission, enabling less losses (high voltage ⇒ low current ⇒ reduced I^2R losses in lines)

Three-phase is chosen to increase power output for given size of generator, but without excessive conductors needed to transmit the power.

i) Δ load: $V_{ph} = V_{line} = 415V$

$$P_\Delta = \frac{3V_{ph}^2}{R} = \frac{3\times 415^2}{10} = \underline{51.7 \text{ kW}}$$

$$Q_\Delta = \frac{3V_{ph}^2}{X_L} = \frac{3\times 415^2}{10} = 51.7 \text{ kVAR}$$

$\curlywedge$ load: $V_{ph} = V_{line}/\sqrt{3} = 240V$

$$I_{ph} = \frac{V_{ph}}{|Z|} = \frac{240}{(10^2+5^2)^{1/2}} = 21.43 \text{ A}$$

$$P_\curlywedge = 3I_{ph}^2 R = \underline{13.8 \text{ kW}}$$

$$Q_\curlywedge = -3I_{ph}^2 X_c = -6.9 \text{ kW}$$

ii) Total power $= P_\Delta + P_\curlywedge = 65.5\text{kW}$

Total reactive power $= Q_\Delta + Q_\curlywedge = 44.8 \text{ kVAR}$

∴ Total apparent power $= (P^2+Q^2)^{1/2} = 79.4 \text{ kVA}$

$$S = \sqrt{3}\, V_L I_L \Rightarrow I_L = \frac{79.4\times 10^3}{\sqrt{3}\times 415} = \underline{110.4 \text{ A}}$$

iii) $\cos\phi = \frac{P}{S} = \frac{65.5}{79.4} = \underline{\underline{0.825 \text{ lagging}}}$

Star-connected capacitors $\Rightarrow V_{cap} = V_L/\sqrt{3} = 240\,V$

$$Q_{cap} = 3\frac{V_{ph}^2}{X_c} = \frac{3\times(415/\sqrt{3})^2}{1/\omega C} = \omega C \times 415^2$$

$$= 100\pi \times 415^2 \times C = 54.1\times10^6\,C.$$

For unity power factor, $Q_{cap} = Q_{load} = 44.8\times10^3$

$\therefore$ $\underline{\underline{C = 828\,\mu F.}}$

Now, $S = P$ since $Q = 0$ ie $S = 65.5\,kVA$

$S = \sqrt{3}\,V_L I_L \Rightarrow I_L = 91.1\,A.$

$\%$ reduction $= \frac{110.4 - 91.1}{110.4} \times 100 = \underline{\underline{17.5\%}}$

For 0.95 lagging power factor, new total reactive power $= P\tan\phi$

where $\cos\phi = 0.95 \Rightarrow Q_{total} = 21.5\,kVAR$

$\therefore$ $Q_{cap} = 44.8\,kVAR - 21.5\,kVAR = 23.3\,kVAR$

$= 54.1\times10^6\,C$

$\underline{\underline{C = 430\,\mu F}}$

$$S = (P^2 + Q^2)^{1/2} = (65.5^2 + 21.5^2)^{1/2} = 68.9\,\text{kVA}$$

$$\sqrt{3}\, V_L I_L = 68.9 \times 10^3 \Rightarrow I_L = 95.9\,\text{A}.$$

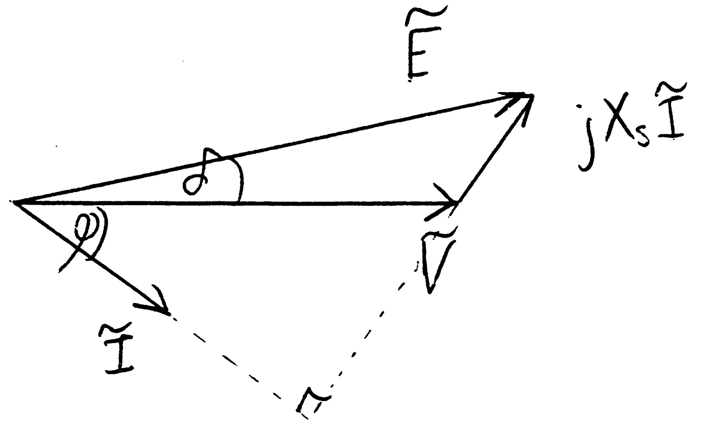
$$\%\ \text{reduction} = \frac{110.4 - 95.9}{110.4} \times 100\% = 13.1\%$$

Correcting p.f. from $0.825 \rightarrow 0.95$ reduces losses in transmission by $\left(\frac{95.9}{110.4}\right)^2 = 75.5\%$

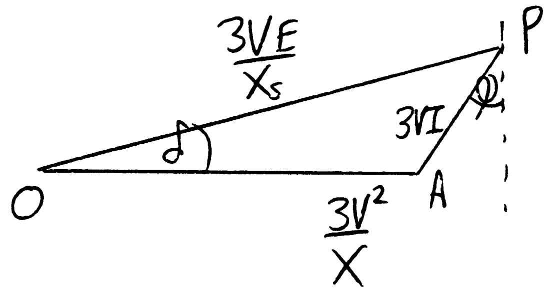
Correcting p.f. from $0.825 \rightarrow 1$ reduces losses in transmission by $\left(\frac{91.1}{110.4}\right)^2 = 68.1\%$, but requires almost twice the size of capacitance.

$\therefore$ For ~~an~~ extra reduction in losses of 7.4%, the power factor correction capacitors will cost twice as much, and the 'pay-back' period therefore may not justify correction to unity.

4 a)



Phasor diagram for a typical operating point when generating has its sides scaled by $3V/X_s$:



Now the limits on P_{max}, VA, excitation and stability can all be marked on to give the operating chart.

b) $V = 60$ kV line $\Rightarrow V_{ph} = \frac{60\,kV}{\sqrt{3}} = 34.6\,kV$

OA is fixed at $\frac{3V^2}{X_s} = \frac{3 \times (34.6\times10^3)^2}{5} = 720 \times 10^6$

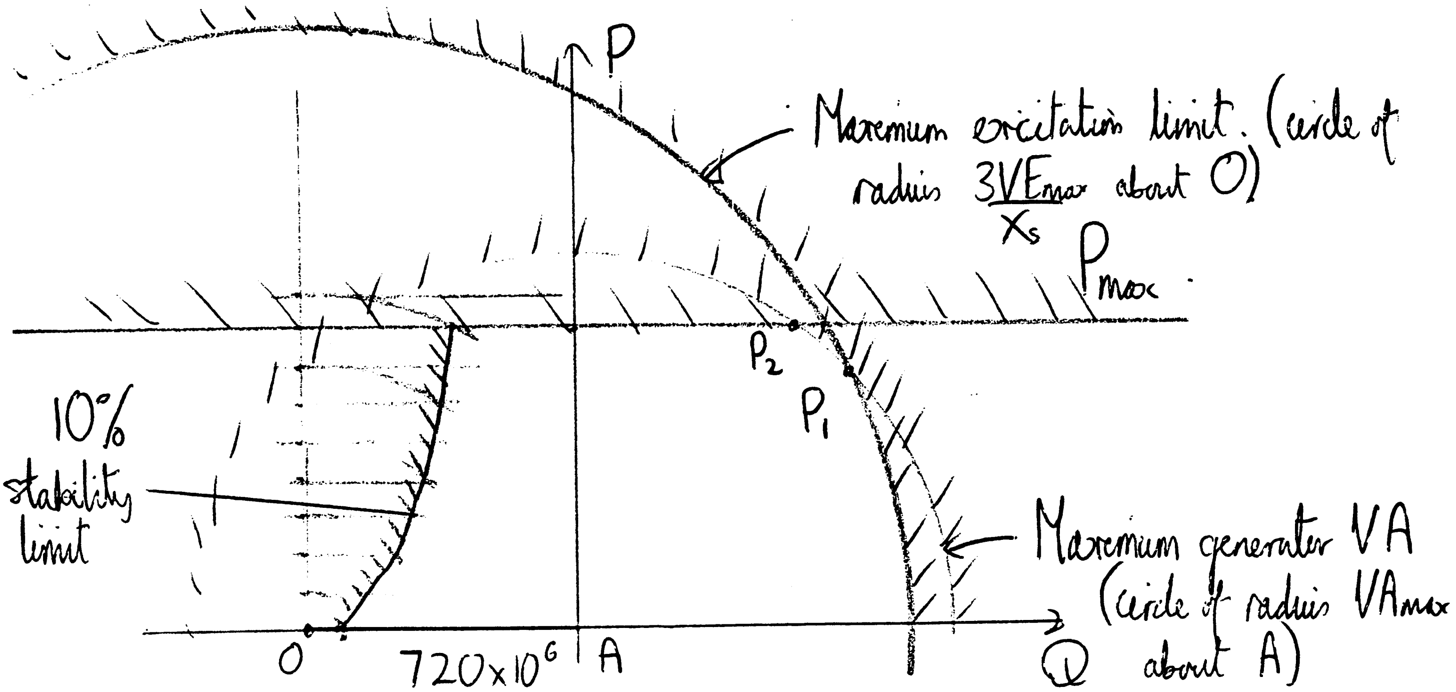
Maximum possible length $AP_{max} = 1000 \times 10^6$, since $AP = 3VI$, and $3VI$ has a maximum value equal to the generator VA rating.

$E_{max} = 135\,kV$ line $= \dfrac{135}{\sqrt{3}}\,kV$ phase $= 77.9\,kV$

$\therefore$ Maximum length of $OP = \dfrac{3VE_{max}}{X_s} = \dfrac{3 \times 34.6 \times 10^3 \times 77.9 \times 10^3}{5}$

$= 1620 \times 10^6.$

Choose scale of $1\,cm \equiv 200 \times 10^6$



$P_{max} = 800\,MW \Rightarrow$ horizontal line at 800×10^6 since $P = 3VI\cos\varphi$

which is the vertical projection of the point P.

10% stability limit $\Rightarrow \Delta P = 10\% \times$ rated VA $= 100\,MW$ without

losing synchronism.

ii) Circle of radius VA_{max} about A – limit is due to the maximum value of current in the stator windings which can occur before the winding overheats due to I^2R losses

Circle of radius $3VE_{max}/X_s$ about O – limit due to maximum excitation voltage; which in turn is limited by the maximum field current which can flow in the rotor winding before it overheats due to I^2R losses.

Horizontal line at P_{max} is a limit on the maximum real power which the generator can produce, which in turn is limited by the prime-mover.

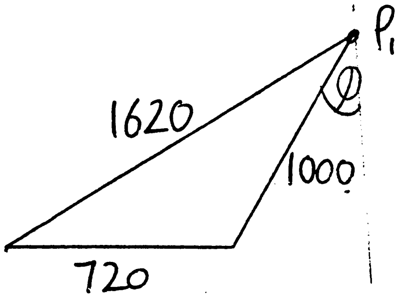
Stability margin – this prevents the generator losing synchronism when a step change in demanded power occurs, by allowing the load angle δ to swing to $90°$.

iii) Rated MVA can be delivered over power factors ranging from the point P_1 to the point P_2.

P_2 corresponds to the intersection of $(3VI)_{max}$ and P_{max}

$$\Rightarrow S = 1000 \text{ MVA},\ P = 800 \text{ MW} \therefore \cos\phi = \frac{P}{S} = \underline{\underline{0.8 \text{ lagging}}}$$

P_1 is the intersection of $(3VI)_{max}$ with $\left(\frac{3VE}{X_s}\right)_{max}$

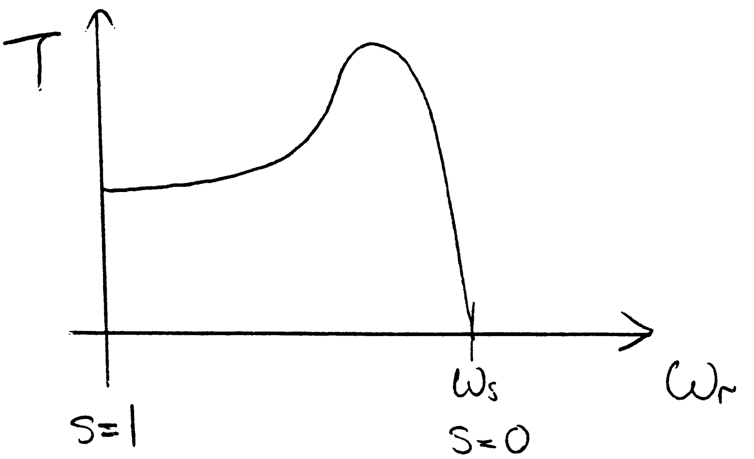


Pythagoras:

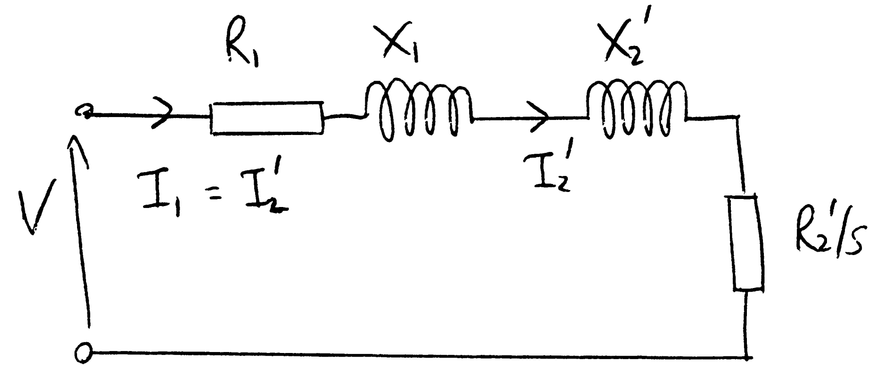
$$1620^2 = (1000\cos\phi)^2 + (720 + 1000\sin\phi)^2$$

$$\Rightarrow \cancel{\cos\phi} \sin\phi = 0.768 \quad \underline{\underline{\cos\phi = 0.64 \text{ lagging.}}}$$

5/ a)



b)



$$P_{in} = 3 I_2'^2 (R_1 + R_2'/s)$$

$$P_{loss} = 3 I_2'^2 (R_1 + R_2')$$

$$\therefore P_{out} = P_{in} - P_{loss} = 3 I_2'^2 R_2' \frac{(1-s)}{s}$$

This output power is the power which is converted to mechanical power

$$\Rightarrow T\omega_r = T(1-s)\omega_s = 3 I_2'^2 R_2' \frac{(1-s)}{s}$$

$$\Rightarrow T = \frac{3}{\omega_s} I_2'^2 \frac{R_2'}{s}$$

$$I_2'^2 = \frac{V^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \quad \Rightarrow T = \frac{3V^2}{\omega_s \left((R_1 + R_2'/s)^2 + (X_1 + X_2')^2 \right)} \times$$

c) Maximum torque occurs when maximum power is 'consumed' in R_2'/s.

By maximum power transfer theorem, source impedance = load resistance

$$\Rightarrow \left(R_1^2+(X_1+X_2')^2\right)^{1/2} = \frac{R_2'}{S_{max}}$$

$$S_{max} = \frac{R_2'}{\left(R_1^2+(X_1+X_2')^2\right)^{1/2}}$$

d) $$S_{max} = \frac{1.4}{\left(1^2+(2+2.4)^2\right)^{1/2}} = 0.31$$

4 pole motor $\Rightarrow$ synchronous speed $= \frac{60f}{p} = \frac{60\times 50}{2} = 1500$ rpm

$$\therefore N = (1-s)N_s = (1-0.31)\times 1500 = \underline{\underline{1035 \text{ rpm.}}}$$

Maximum torque is the torque evaluated at $s = S_{max}$. $V = V_{ph} = \frac{415}{\sqrt{3}}$

$= 239.6$ V (star-connected)

$$\therefore T_{max} = \frac{3\times(415/\sqrt{3})^2}{(1+1.4/0.31)^2+(2+2.4)^2}\times\frac{1.4}{0.31}\times\frac{1}{1500\times 2\pi/60}$$

$$= \underline{\underline{99.5 \text{ Nm.}}}$$

e) Need S_{max} to occur at 1, since $s=1$ at starting

$$\Rightarrow 1 = \frac{R_2'+R_{extra}'}{\left(R_1^2+(X_1+X_2')^2\right)^{1/2}} = \frac{1.4+R_{extra}'}{4.51}$$

$R'_{extra} = 3.11\ \Omega$

This is the extra resistance referred to the stator.

$N_S : N_R = 2 : 1$

$$\Rightarrow R_{extra} = \left(\frac{N_R}{N_S}\right)^2 R'_{extra} = \left(\frac{1}{2}\right)^2 \times 3.11 = \underline{\underline{0.778\ \Omega}}$$

A cage induction motor does not have slip-rings and brushes, which enable extra resistance to be added in series. To increase starting resistance, cage motors use specially shaped slots which exploit skin effect to give a high starting resistance (since rotor currents are at supply frequency when $S=1$), or alternatively, a double cage, with a high resistance starting cage, and a low resistance running cage.

6/ a)

$$v = \frac{1}{\sqrt{LC}} \qquad 2.5\times10^{8} = \frac{1}{\sqrt{80\times10^{-12}\times L}}$$

$$\underline{L = 0.2\ \mu H/m.}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.2\times10^{-6}}{80\times10^{-12}}} = \underline{50\Omega}$$

$$v = f\lambda \qquad 2.5\times10^{8} = 100\times10^{6}\,\lambda$$

$$\underline{\lambda = 2.5\,m}$$

b) These equations mean that the transmission line supports forwards- and backwards-travelling voltage and current waves, of angular frequency ω, and wavelength $2\pi/\beta$.

At the load, $x = 0$

$$\bar{V}(0,t) = (\bar{V}_f + \bar{V}_r)e^{j\omega t} \quad (1)$$

$$\bar{I}(0,t) = (\bar{I}_f + \bar{I}_r)e^{j\omega t} \quad (2)$$

$$\bar{I}_f = \frac{\bar{V}_f}{Z_0}, \quad \bar{I}_r = -\frac{\bar{V}_r}{Z_0}$$

Substitute for $\bar{V}_f$ and $\bar{V}_r$ in (1)

$$\bar{V}(0,t) = (Z_0\bar{I}_f - Z_0\bar{I}_r)e^{j\omega t}$$

Ohms law for the load:

$$\bar{I}(0,t) = \frac{\bar{V}(0,t)}{\bar{Z}_L} \Rightarrow (\bar{I}_f + \bar{I}_r)e^{j\omega t} = \frac{(Z_0\bar{I}_f - Z_0\bar{I}_r)e^{j\omega t}}{\bar{Z}_L}$$

$$\bar{Z}_L \bar{I}_f + \bar{Z}_L \bar{I}_r = Z_0 \bar{I}_f - Z_0 \bar{I}_r$$

$$\frac{\bar{I}_r}{\bar{I}_f} = \bar{\rho}_I = \underline{\underline{\frac{Z_0 - \bar{Z}_L}{Z_0 + \bar{Z}_L}}}$$

$$\bar{\rho}_V = \frac{\bar{Z}_L - Z_0}{\bar{Z}_L + Z_0} \qquad \text{(Data-book)}$$

c) $$\bar{V}(x,t) = \bar{V}_f e^{j(\omega t - \beta x)} + \bar{\rho}_V \bar{V}_f e^{j(\omega t + \beta x)}$$

$$\bar{I}(x,t) = \bar{I}_f e^{j(\omega t - \beta x)} + \bar{\rho}_I \bar{I}_f e^{j(\omega t + \beta x)}$$

$$\frac{\bar{V}(x,t)}{\bar{I}(x,t)} = \bar{Z}(x) = \frac{\bar{V}_f}{\bar{I}_f} \frac{(e^{-j\beta x} + \bar{\rho}_V e^{j\beta x})\, e^{j\omega t}}{(e^{-j\beta x} + \bar{\rho}_I e^{j\beta x})\, e^{j\omega t}}$$

$$= \underline{\underline{Z_0 \frac{e^{-j\beta x} + \bar{\rho}_V e^{j\beta x}}{e^{-j\beta x} + \bar{\rho}_I e^{j\beta x}}}}$$

d) ~~To~~ Find $\bar{Z}(-2.5)$ $\qquad \beta = \frac{2\pi}{\lambda}$, and $\lambda = 2.5m$

$\therefore \beta x = -2\pi$

$$\therefore \bar{Z}(-2.5) = Z_0 \frac{e^{j2\pi} + \bar{\rho}_V e^{-j2\pi}}{e^{j2\pi} + \bar{\rho}_I e^{-j2\pi}}$$

$$= 50 \frac{(1 + \bar{\rho}_V)}{(1 + \bar{\rho}_I)}$$

$$= 50 \; \frac{\left(1 + \frac{\bar{Z}_L - Z_0}{\bar{Z}_L + Z_0}\right)}{1 + \frac{Z_0 - \bar{Z}_L}{Z_0 + \bar{Z}_L}}$$

$$= 50 \cdot \frac{2\bar{Z}_L}{2Z_0} = \bar{Z}_L = (30 + j40)\,\Omega.$$

i.e. If the length of the transmission line is equal to a wavelength (or multiple of a wavelength), the impedance presented by the line plus load at the start of the line is equal to the load impedance.

7/ $\underset{\sim}{E} = \underset{\sim}{u}_x E_o e^{j(\omega t - \beta z)}$

$$\frac{\partial \underset{\sim}{E}}{\partial z} = (-j\beta)\, u_x \underset{\sim}{E} \ , \quad \frac{\partial^2 \underset{\sim}{E}}{\partial z^2} = -\beta^2 \underset{\sim}{E}$$

$$\frac{\partial \underset{\sim}{E}}{\partial t} = j\omega \underset{\sim}{E} \ , \quad \frac{\partial^2 \underset{\sim}{E}}{\partial t^2} = -\omega^2 \underset{\sim}{E}$$

$\therefore$ For $\underset{\sim}{E}$ to be a solution :-

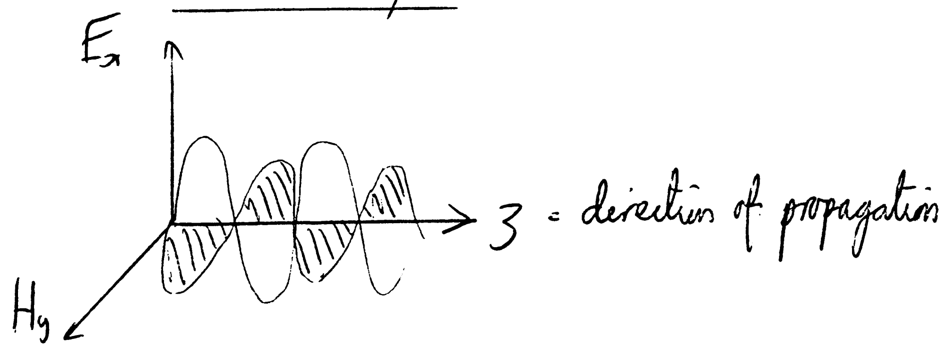
$$-\beta^2 E - \epsilon_o \mu_o (-\omega^2 E) = 0$$

$$\omega^2 \epsilon_o \mu_o = \beta^2$$

$$\underline{\omega/\beta = 1/\sqrt{\epsilon_o \mu_o}}$$

But, ω/β is the speed of propagation of the wave

$$\Rightarrow \quad \underline{v = 1/\sqrt{\epsilon_o \mu_o}}$$



$$\underline{\eta_o = \frac{|\underset{\sim}{E}|}{|\underset{\sim}{H}|} = \sqrt{\frac{\mu_o}{\epsilon_o}}}$$

$$\therefore \quad \underline{\underset{\sim}{H} = \underset{\sim}{u}_y \frac{E_o}{\eta_o} e^{j(\omega t - \beta z)}}$$

$$\text{Power per area} = \left| Re\left\{ \frac{1}{2} \underset{\sim}{E} \times \underset{\sim}{H}^* \right\} \right|$$

$$= \left| Re\left\{ \frac{1}{2}\, \underset{\sim}{u}_x E_0 e^{j(\omega t - \beta z)} \times \underset{\sim}{u}_y \frac{E_0}{\eta_0} e^{-j(\omega t - \beta z)} \right\} \right|$$

$$= \left| \underset{\sim}{u}_z \frac{E_0^2}{2\eta_0} \right| = \underline{\underline{\frac{E_0^2}{2\eta_0}}}$$

$$\text{Power p.u. area} = \frac{10^{-3}}{\pi \times (0.5\times 10^{-3})^2} = 1273\ Wm^{-2} = \frac{E_0^2}{2\eta_0}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi\times 10^{-7}}{8.85\times 10^{-12}}} = 376.8\,\Omega$$

$$\therefore\ 1273 = \frac{E_0^2}{2\times 376.8} \Rightarrow E_0 = \underline{980\ Vm^{-1}}$$

$$H = \frac{E_0}{\eta_0} = \frac{980}{376.8} = \underline{2.6\ Am^{-1}}$$

$$\text{Power reflection coefficient} \equiv \rho_V^2 = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)^2 \equiv \left(\frac{\eta_L - \eta_0}{\eta_L + \eta_0}\right)^2$$

$$= \left(\frac{\sqrt{\frac{\mu_0}{3\epsilon_0}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{3\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon_0}}} \right)^2 = \left(\frac{1/\sqrt{3} - 1}{1/\sqrt{3} + 1} \right)^2 = 0.072$$

$\therefore$ Reflected power $= 0.072$ mW, transmitted power $= 0.928$ mW.

$$\therefore\ \text{Transmitted power per area} = \frac{0.928\times 10^{-3}}{\pi \times (0.5\times 10^{-3})^2} = \underline{1182\ Wm^{-2}}$$

Power p.u. area = $\dfrac{E^2}{2\eta_L}$ $\qquad \eta_L = \sqrt{\dfrac{\mu_0}{3\epsilon_0}} = \sqrt{\dfrac{4\pi \times 10^{-7}}{3 \times 8.85 \times 10^{-12}}}$

$= 217.6\,\Omega$

$\therefore\ 1182 = \dfrac{E^2}{2 \times 217.6} \Rightarrow \underline{E = 717\ Vm^{-1}}$

$H = \dfrac{E}{\eta_L} = \dfrac{717}{217.6} = \underline{3.30\ Am^{-1}}$

T Black

June 1998

1998 Part 1B - PAPER 5 - ANSWERS

1. a) $i_c = h_{fe}i_b + h_{oe}v_{ce}$ $\quad v_{be} = h_{ie}i_b + h_{re}v_{ce}$ $\quad$ b) $R_C = 400\ \Omega$; $R_B = 154\ \text{k}\Omega$

c) $R_L = 400\ \Omega$; Mid-band gain $\dfrac{v_o}{v_i} = -\dfrac{h_{fe}R_C R_L}{h_{ie} + R_C + R_L} = -50$

d) $\dfrac{v_o}{v_i} = -\dfrac{h_{fe}R_C R_L}{\left(h_{ie} + 1/j\omega C_1\right)\left(R_L + R_C + 1/j\omega C_2\right)}$ $\quad$ e) $f_{3dB} = 15.5$ Hz

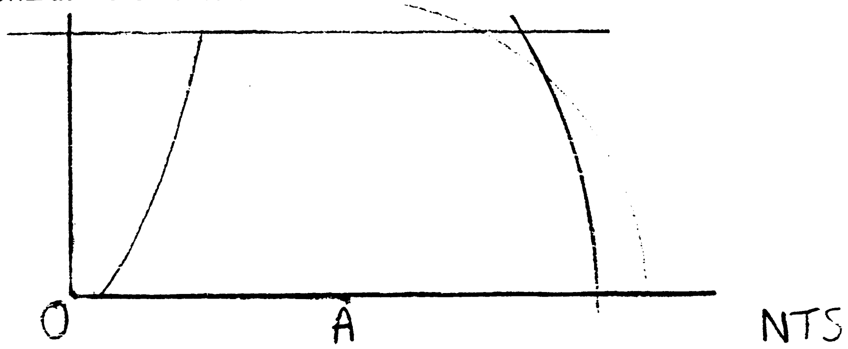
2. b) Common mode gain $= -\dfrac{h_{fe}R_C}{h_{ie} + 2R_T\left(h_{fe} + 1\right)}$; Differential gain $= -\dfrac{h_{fe}R_C}{h_{ie}}$;

$R_C = 5\ \text{k}\Omega$; $R_T = 4.97\ \text{k}\Omega$ c) Differential gain $= -\dfrac{h_{fe}R_C}{h_{ie} + R_E\left(h_{fe} + 1\right)}$;

Gain reduction factor $= 1 + \dfrac{R_E\left(h_{fe} + 1\right)}{h_{ie}}$ d) $R_E = 44.8\ \Omega$

3. b) (i) $P_{delta} = 51.7$ kW; $P_{star} = 13.8$ kW (ii) $I_{line} = 110.4$ A
(iii) Power factor = 0.825 lagging c) (i) 828 μF; (ii) 430 μF; 17.5 % and 13.1 % reduction in line current respectively.

4. b) i) Prime-mover limit - horizontal line at 800×10^6; Stator heating limit - circle of radius 1000×10^6, centre A; Rotor heating limit - circle of radius 1620×10^6, centre O; Length OA = 720×10^6; Stability limit - intersection of circles and horizontal lines drawn at increments of 10 % of rated MVA = 100×10^6.



(iii) Power factor between 0.64 and 0.8 lagging.

5. d) $T_{max} = 99.5$ Nm; N = 1035 rpm e) Extra rotor resistance = 0.778 Ω

6. a) L = 0.2 μHm^{-1}; $Z_0 = 50\ \Omega$; Wavelength = 2.5 m d) $Z_{in} = Z_L = (30 + j40)\ \Omega$ - this is because the transmission line is exactly one wavelength long.

7. a) $\omega^2\varepsilon_0\mu_0 = \beta^2$ b) ***E***, ***H*** and direction of propagation mutually orthogonal, such that ***E***×***H*** gives direction of propagation; $\eta_0 = \dfrac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$; $\overline{\mathbf{H}} = \mathbf{u}_y \dfrac{E_0}{\eta_0} \exp j(\omega t - \beta z)$

c) E = 980 Vm^{-1}; H = 2.6 Am^{-1} d) Power pu area = 1182 Wm^{-2}; E = 717 Vm^{-1}; H = 3.30 Am^{-1}