Section A

1. (a)

$$\bar{p}(s) = \frac{0.1}{s}\bar{e}(s) + 0.2\,\bar{e}(s)$$

$$\therefore\ k(s) = \frac{\bar{p}(s)}{\bar{e}(s)} = \underline{\underline{\left(0.1/s + 0.2\right)}}$$

$$s\bar{y}(s) + \bar{y}(s) = e^{-s}\bar{q}(s)$$

$$\therefore\ G(s) = \frac{\bar{y}(s)}{\bar{q}(s)} = \underline{\underline{\frac{e^{-s}}{1+s}}}$$

(b)

$$\bar{y}(s) = G(s)\left(\bar{d}(s) + -k(s)\,\bar{y}(s)\right)$$

$$\bar{y}(s)\left(1 + G(s)\,k(s)\right) = G(s)\,\bar{d}(s)$$

$$H(s) = \frac{\bar{y}(s)}{\bar{d}(s)} = \frac{G(s)}{1 + G(s)k(s)} = \frac{e^{-s}}{1 + s + e^{-s}(0.1/s + 0.}$$

$$= \frac{s e^{-s}}{s^2 + (1 + 0.2e^{-s})s + 0.1e^{-s}}$$

$$= \frac{s e^{-s}}{s(s+1) + e^{-s}(0.1 + 0.2s}$$

1. contd.

(c) Steady state value $= 2 \times H(s)|_{s=0}$

$= \underline{\underline{0}}$

(Integral action reduces response to zero).

2. (a) Asymptotically stable if:

$$\int_0^\infty |g(t)|dt < \infty$$

(b) $$y(t) = \int_0^t g(\tau)\, u(t-\tau)\, d\tau$$ (Convolution)

$$= \int_0^t g(\tau) \cos(\omega(t-\tau))\, d\tau$$

$$= \int_0^t g(\tau)\, \mathrm{Re}\{ e^{j\omega(t-\tau)} \} d\tau$$

$$= \mathrm{Re}\int_0^t g(\tau)\, e^{j\omega t} e^{-j\omega\tau}\, d\tau$$

$$= \mathrm{Re}\left\{ \int_0^\infty g(\tau) e^{j\omega t} e^{-j\omega\tau} d\tau - \cancel{\int_t^\infty g(\tau) e^{j\omega t} e^{-j\omega\tau} d\tau} \right\}$$

$\to 0$ as $t \to \infty$ since $\int_0^\infty |g(t)|dt$ is finite

$$= \mathrm{Re}\; e^{j\omega t}\int_0^\infty g(\tau) e^{-j\omega\tau} d\tau$$

$$= \mathrm{Re}\left\{e^{j\omega t}\,\bar{g}(j\omega)\right\}$$

$$= \underbrace{\left|\bar{g}(j\omega)\right|}_{A}\cos\left(\omega t + \underbrace{\angle\bar{g}(j\omega)}_{\phi}\right)$$

2.(c) $$\int_0^\infty |g(t)|\,dt = \int_0^\pi \sin t\,dt$$

$$= \left[-\cos t\right]_0^\pi$$

$$= \underline{\underline{2}}$$

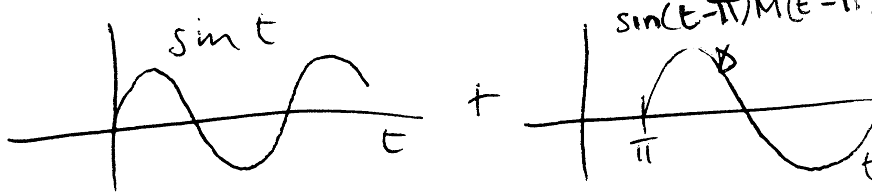
(Finite, ⇒ asympt. stable)

Frequency response at $\omega = 2$:

$$G(j2) = \int_0^\pi \sin t\, e^{-2jt}\,dt$$

Can integrate directly, but quicker to use:

$g(t) =$



$$\Rightarrow G(s) = \frac{1}{s^2+1} + \frac{e^{-s\pi}}{s^2+1}$$

$$= \frac{1+e^{-s\pi}}{s^2+1}$$

$$\Rightarrow G(j2) = \frac{1+e^{-2j\pi}}{-3} = \underline{\underline{-2/3}}$$

2.(c) contd.

$$\Rightarrow \quad \lim_{t \to \infty} \{ g(t) \} = -\tfrac{2}{3} \cos(2t)$$

$$= \underset{\substack{\uparrow \\ A}}{\tfrac{2}{3}} \cos\left(2t + \underset{\substack{\uparrow \\ \phi}}{\pi}\right)$$

3. (a)

1) Input a sinusoidal excitation at frequency ω.

2) Measure gain and phase shift from input → output, once any transients have decayed

3) Repeat over desired frequency range.

Difficulties:

- Integral action causes drift - system will 'hit the end-stops'.

- Unstable systems never reach steady state

- Noise, non-linearities, time variations, etc.

⋮ (other answers possible).

3 (b)

There are clearly corner frequencies at around 10^{-1}, 2 and 20 rad/s.

→ Draw in asymptotes on gain curve

Now, working L→R:

① Initial section up to 10^{-1} rad/s has gradient -20 dB/dec, corresponding to the $1/s$ term.

Gain at $\omega = 0.01$ is approx. $a/0.01$.

$$\Rightarrow \quad a/0.01 \approx 11 \quad \text{(from graph)}$$

$$\Rightarrow a \approx 0.1$$

(Can refine this figure once b, c, ω_n estimated)

② Gain curve flattens out to zero gradient at 10^{-1} rad/s, corresponding to $(1+bs)$ term.

$$\Rightarrow \quad 1/b \approx 10^{-1}$$

$$\underline{\underline{b = 10}}$$

3(b) contd.

(3) 'Overshoot' in gain around $\omega = 2$ corresponds to 2nd order term.

$$\Rightarrow \underline{\underline{\omega_n \approx 2}}$$

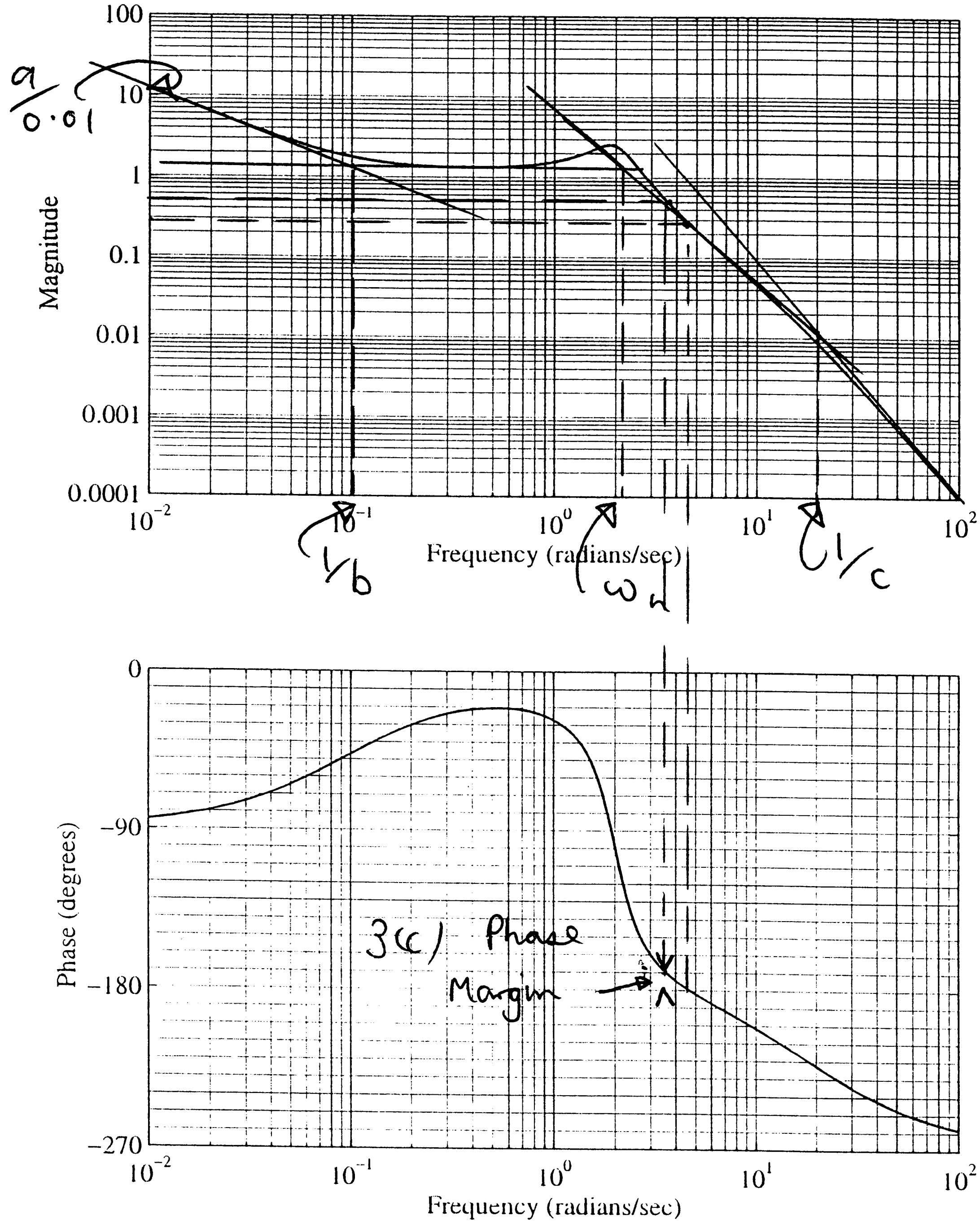
(4) Negative gradient increases by -20dB/dec. around $\omega = 20$.

$$\Rightarrow \frac{1}{c} \approx 20, \quad \underline{\underline{c \approx 0.05}}$$

3.(b) contd.

ENGINEERING TRIPOS PART IB

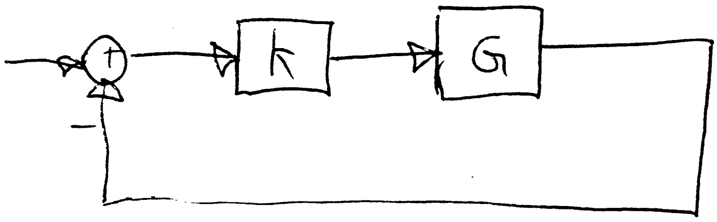
Thursday 4 June 1998, Paper 6, Question 3



Extra Copy of Fig. 2

3. (c)

Suppose the feedback system:



is stable for some k. The gain margin is the extra gain factor which can be applied before instability. The phase margin is the extra phase shift (in a negative sense) which renders the system unstable.

Feedback with $k=2$

Gain margin is obtained when phase $= -180^\circ$, i.e. ≈ 4.4 rad/s. From Bode plot, gain at 4.4 rad/s ≈ 0.28. $\therefore$ Open loop gain of closed-loop system is 2×0.28 and gain margin is $\frac{1}{2 \times 0.28} = \underline{\underline{1.8}}$.

Phase margin obtained when gain $=1$, i.e. when Bode plot gain $= 0.5$, i.e. $\approx 12^\circ$ (see diagram)

These are very poor figures which would lead to oscillatory response.

4. (a)

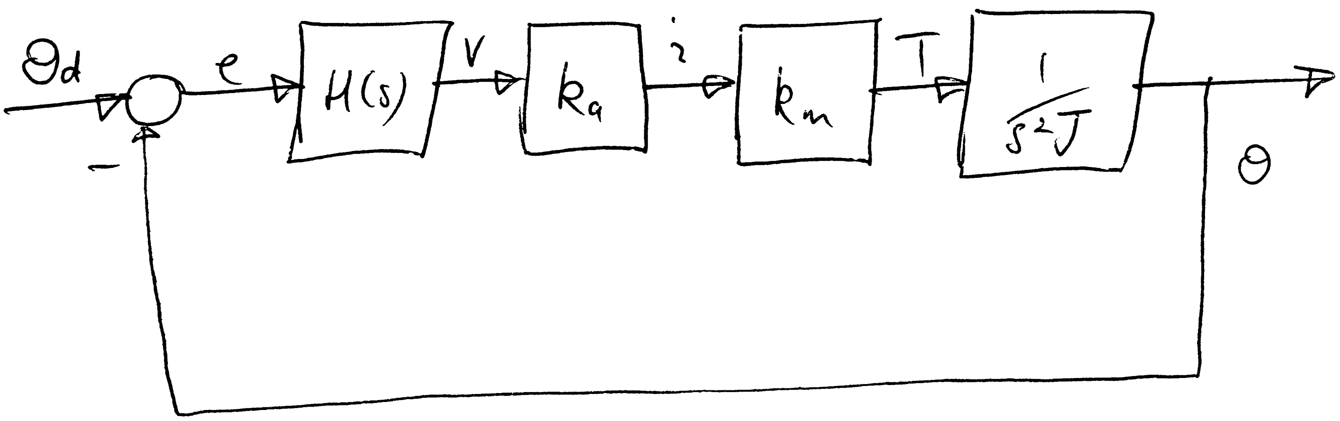
$$\bar{T}(s) = k_m \, k_a \, \bar{v}(s)$$

$$\bar{v}(s) = H(s)\left(\bar{\theta}_d(s) - \bar{\theta}(s)\right)$$

$$T = J\ddot{\theta}$$

$$\Rightarrow \bar{T}(s) = s^2 J \, \bar{\theta}(s)$$

$$\Rightarrow \quad \bar{\theta}(s) = \underbrace{\left\{\frac{k_m k_a}{s^2 J}\right\}}_{\text{T.F.}} \bar{v}(s)$$

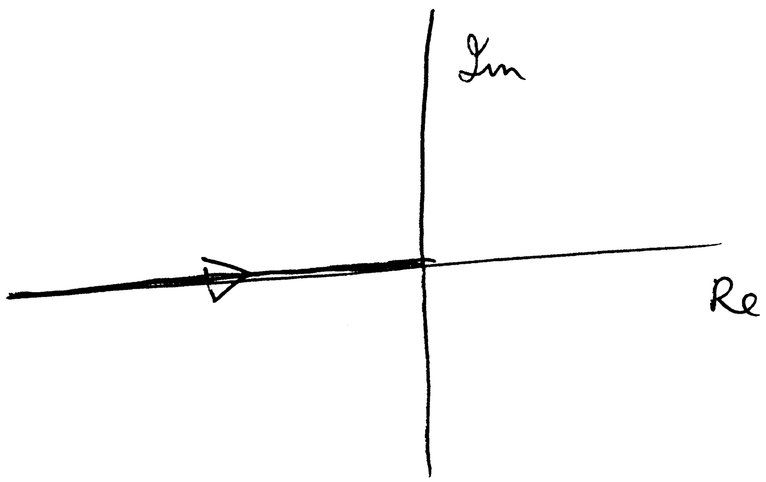


4.
(b)

Open loop system is

$$\frac{K\,K_m K_a}{s^2 J}$$

Nyquist diagram:



CLTF:

$$\frac{K'}{s^2+K'}, \quad \text{where } K' = \frac{K K_m K_a}{J}.$$

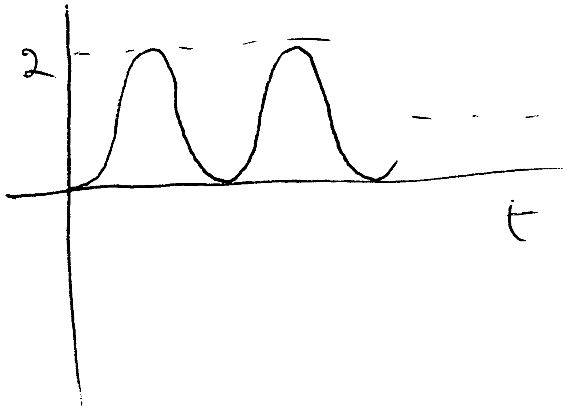
Step response is

$$\frac{K'}{s(s^2+K')} = \frac{1}{s} + \frac{-s}{s^2+K'}$$

$$\Rightarrow \theta(t) = 1 - \cos\left(\sqrt{K'}\,t\right)$$

4.(b) contd.

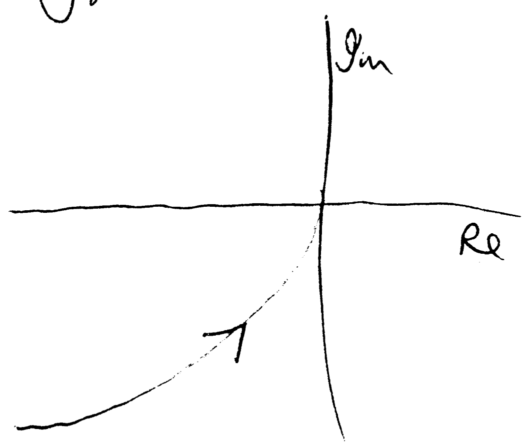
Sketch:



Increasing k just increases oscillation frequency.

(c) OLTF is now $\dfrac{(1+s)}{\sqrt{2}\ s^2}$

Nyquist:



4.(c)

<u>PM</u>

Solve for

$$\left| \frac{(1+s)}{\sqrt{2}s^2} \right| \Bigg|_{s=j\omega} = 1$$

$$\frac{|1+j\omega|}{\sqrt{2}\,\omega^2} = 1$$

$$\Rightarrow \quad (1+\omega^2) = 2\omega^4$$

$$2\omega^4 - \omega^2 - 1 = 0$$

$$\omega^2 = \frac{1 \pm \sqrt{1+8}}{4}$$

$$= 1 \text{ or } \cancel{-1/2}$$

$$\angle \left\{ \frac{1+j\omega}{\sqrt{2}\omega^2} \right\} \Bigg|_{\omega=1} = 45^\circ - 180^\circ$$

$$\Rightarrow \quad PM = \underline{\underline{45^\circ}}$$

Ample phase margin

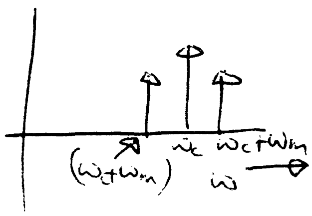
→ Good damping.

5. (a)

$$s(t) = a_0(1 + m_a \cos\omega_m t)\cos(\omega_c t + \phi)$$

$$= a_0 \cos(\omega_c t + \phi) + a_0 m_a \cos\omega_m t \cos(\omega_c t + \phi)$$

$$= \underbrace{a_0 \cos(\omega_c t + \phi)}_{\text{Carrier}} + \frac{a_0 m_a}{2}\Big(\underbrace{\cos((\omega_m + \omega_c)t + \phi)}_{\text{Upper sideband}} + \underbrace{\cos((\omega_c - \omega_m)t + \phi)}_{\text{Lower sideband}}\Big)$$

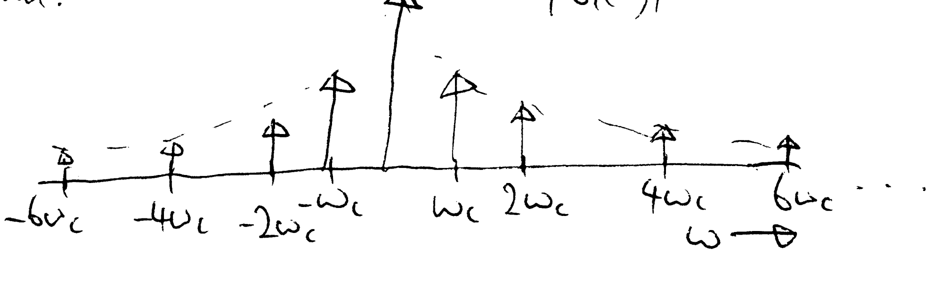


Hence can pass through bandlimited radio channel.

(b) p.24 of EIST D. Book:

$$g(t) = b\left(\frac{1}{\pi} + \frac{1}{2}\cos(\omega_c t) + \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos(2n\omega_c t)}{4n^2 - 1}\right)$$

Spectrum:



5. (c) Output from rectifier is:

Hence rectified O/P is:

Result from (b)

$$\propto \left(1 + m_a \cos \omega_m t\right)\left(\frac{1}{\pi} + \frac{1}{2}\cos \omega_c t + \frac{2}{\pi}\sum (-1)^{n+1}\frac{\cos(2n\omega_c t)}{4n^2-1}\right)$$

$$= \frac{1}{\pi} + \frac{1}{2}\cos\omega_c t + \frac{2}{\pi}\sum(-1)^{n+1}\frac{\cos 2n\omega_c t}{4n^2-1}$$

$$+ \frac{m_a \cos\omega_m t}{\pi} + m_a\frac{\cos\omega_m t \cos\omega_c t}{2}$$

$$+ \frac{2}{\pi} m_a \sum \frac{\cos \omega_m t \cos(2n\omega_c t)(-1)^{n+1}}{4n^2-1}$$

5. (c) Contd.

Terms of the form

$$\cos \omega_m t \cos n\omega_c t$$

are rewritten as:

$$\tfrac{1}{2}\left(\cos\left(\omega_m t + n\omega_c t\right)\right) + \tfrac{1}{2}\cos\left(n\omega_c t - \omega_m t\right)\Big)$$

Hence a low-pass filter with bandwidth less than $(\omega_c - \omega_m)$ will recover just the term '$\dfrac{ma \cos \omega_m t}{\pi}$',

i.e. the modulating signal.

Modulation index must be < 1 for analysis to work.

6. (a)

Aliasing is an irreversible distortion of the signal caused by sampling at a frequency f_s which does not satisfy the Nyquist criterion, i.e.

For no aliasing, $f_s > 2 f_B$, where f_B is the highest frequency component in the signal.

Solution - use anti-aliasing low-pass filter before sampling (bandwidth $= f_s/2$).

Quantisation - representation of continuous-valued signals by values chosen from a discrete set. To reduce, have more discrete levels in required amplitude range → more precise ADC.

6. (b)

With 20 kHz bandwidth, use anti-alias filters with cut-off $(20+\Delta)$ kHz to allow for filter roll-off. e.g. $\Delta = 2$ kHz, 10% of bandwidth gives a sampling rate of $2 \times (20+2) = 44$ kHz as for CD audio.

$\Rightarrow$ No. bits per sample

$$= \frac{1.5 \text{ MBit/s}}{\underset{\uparrow \atop \text{Stereo}}{2} \times 44\text{k}} = 17.04 \Rightarrow 17 \text{ bits/sample}$$

(16 bits used in real systems)

rms quantisation noise $= \dfrac{\delta v}{\sqrt{12}}$

If maximum input voltage range $= V$,

then $\delta v = \dfrac{V}{2^{17}}$

6.(b) contd.

Max. sine-wave signal has rms $\frac{V}{2\sqrt{2}}$

$$\Rightarrow SNR = 20 \log_{10} \left(\frac{V/2\sqrt{2}}{V/2^{17}\sqrt{12}} \right) = \underline{\underline{104\ dB}}.$$

(16-bit gives 98 dB – acceptable answer).

(c) Video: (assuming sinusoidal signal):

for $20 \log_{10} \left(\frac{\sqrt{12}}{\sqrt{8}} 2^n \right) > 37$

$n \geq 6$ bits/sample.

(square-wave gives same answer)

$$\frac{1.5M}{6 \times \underset{\uparrow\ \text{frames/s}}{8}} = 31250 \text{ samples/frame}$$

i.e. 176×177 (square)

or 153×204 (3:4 aspect ratio)

⋮

6. (c) contd.

This is poor compared with TV's

820x 625 @ 25

frames/s.

Improve by:

Interlacing

Data compression

Intra - frame coding (transmit only changing picture content).