ENGINEERING TRIPOS PART IB

Monday 1 June 1998 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

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1 A device comprises two uniform bars OA and AB shown in Fig. 1(a). The bars are each of length $2a$ and mass m and frictionless pivots at O and A constrain the bars to move in a horizontal plane (*i.e.* gravity does not influence the motion). The device is at rest when a hammer delivers an impulse I at A as shown in the figure.

(a) Show that the angular velocity of OA immediately after the impulse is $\frac{6I}{7ma}$ and find the corresponding angular velocity of AB . [7]

(b) For the device immediately after the impulse find expressions in terms of I, m and a for

(i) the moment of momentum about O [2]

and (ii) the kinetic energy. [5]

(c) During the subsequent motion it is observed that the device again passes through the fully-extended configuration as shown in Fig. 1(b). Use conservation laws and your answers from (b) to write down two equations which can be used to determine the angular velocities of the two bars. Do *not* solve these equations, but state how many solutions you expect to find. [6]

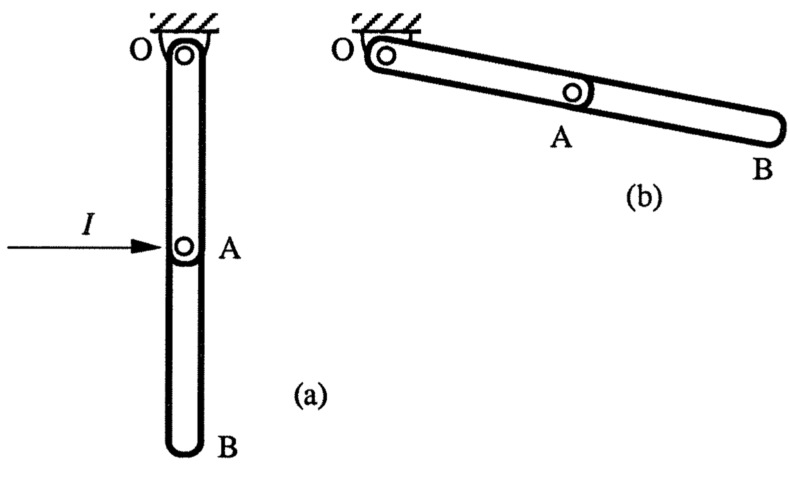

Fig. 1

2 (a) Derive from first principles:

(i) the perpendicular-axis theorem for a lamina;

(ii) the radius of gyration about a diameter of a thin uniform circular disc of mass m and radius a. [8]

(b) A lever OC is being operated by means of a hydraulic cylinder AB as shown in Fig. 2. Dimensions OA, OB and BC are each 1m. The cylinder is extending at a constant rate of 10 $\mathrm{mms^{-1}}$ at the instant shown (*i.e.* when the angle AOC is 90°). Find the velocity and acceleration of the point C. [12]

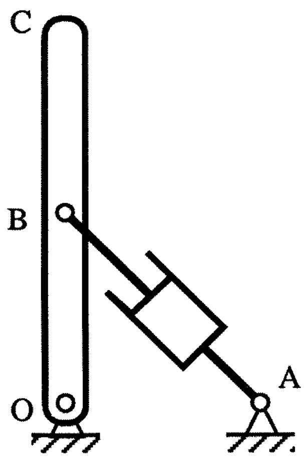

Fig. 2

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3 A thin-walled hollow sphere of mass m and radius a rolls on a horizontal surface. The coefficient of friction between the sphere and the surface is μ. An ideal impulse I is delivered to the sphere at a point P which is level with its centre. The sphere skids for some time before rolling without slip.

(a) Find the velocity of the sphere immediately after the impulse for the cases:

(i) the impulse acts horizontally through the centre of the sphere;

(ii) the impulse is contrived to act vertically downwards at P. [8]

(b) For each case find the final steady velocity of the sphere. [8]

(c) Show that skidding is possible only if $\mu < \frac{3}{5}$. [4]

[The moment of inertia for a hollow sphere is given in the Mechanics Data Book]

4 The centre C of the Earth has position vector $\boldsymbol{r}_e$, velocity $\dot{\boldsymbol{r}}_e$ and acceleration $\ddot{\boldsymbol{r}}_e$ measured with respect to some fixed origin O. The Earth has constant angular velocity vector $\boldsymbol{\omega}$. A train on the Earth has position vector $\boldsymbol{r}_t$, velocity $\dot{\boldsymbol{r}}_t$ and acceleration $\ddot{\boldsymbol{r}}_t$ measured with respect to an observer P at rest on the Earth's surface. The position vector from C to P is $\boldsymbol{r}_p$ in a reference frame fixed in the Earth.

(a) Write down the absolute position vector $\boldsymbol{r}$ for the train and hence derive from first principles vector expressions for:

(i) the absolute velocity $\dot{\boldsymbol{r}}$ of the train;

(ii) the absolute acceleration $\ddot{\boldsymbol{r}}$ of the train. [12]

(b) Under certain conditions it is possible to write $\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_t$. For a train not at the equator prove that this is *only* possible if it is travelling due west and find an expression for the required speed of the train when at a latitude λ. [8]

5 The planar mechanism shown in Fig. 3 is driven by a link AB rotating with constant angular velocity ω . Links AB and CD are light while BC is a uniform bar of mass m . All joints are frictionless.

(a) What are the angular velocities of BC and CD in the configuration shown? [4]

(b) What are the components of the acceleration of the midpoint of BC parallel and perpendicular to BC? [6]

(c) What is the angular acceleration of BC? [4]

(d) Use d'Alembert's principle to find the instantaneous magnitude of the force acting on each of joints B and C. [6]

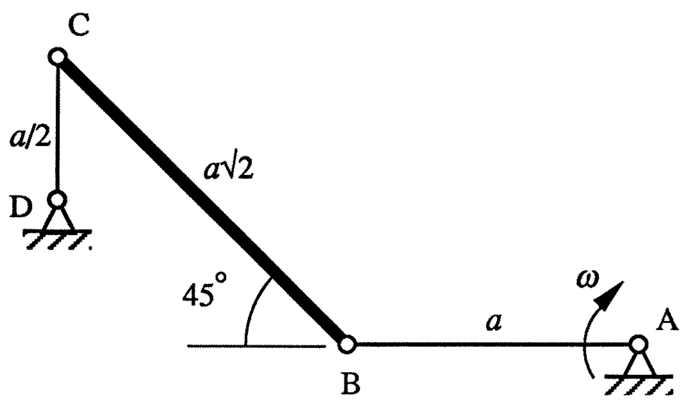

Fig. 3

6 A trolley of mass m is free to roll without friction on a horizontal rail. A uniform rigid arm of mass m and length $2a$ is attached to the trolley at a pivot as shown in Fig. 4. Initially ($t = 0$) the system is at rest with the arm hanging vertically downwards ($\theta = 0$) under the action of gravity. A motor at P then causes the arm to accelerate anticlockwise at a constant rate of $\ddot{\theta} = \frac{\pi}{2}$ rad s^{-2}.

(a) Find the initial acceleration of the trolley. [6]

(b) Find the corresponding motor torque required to drive the arm initially. [6]

(c) Find the angle θ after 2 seconds have elapsed and find the trolley acceleration and motor torque. [4]

(d) What is the amplitude of motion of the trolley? [4]

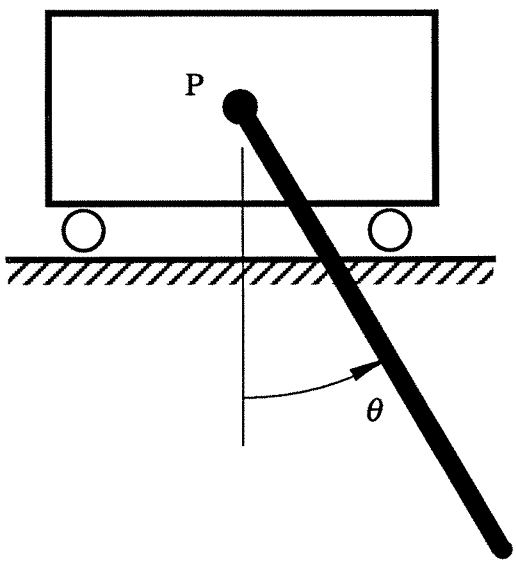

Fig. 4

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