ENGINEERING TRIPOS PART IB

Monday 1 June 1998 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

(TURN OVER

1 (a) Show that von-Mises yield criterion, for a thin plate loaded with a uniaxial in-plane stress σ and a shear stress τ, with zero stress in the through-thickness direction, can be expressed as

$$\sigma^2 + 3\tau^2 = Y^2.$$ [6]

(b) A steel box-girder bridge of trapezoidal cross-section, as shown in Fig. 1(a), is being built by erecting a cantilever BC from a completed span AB, as shown in Fig. 1(b). The top flange has a thickness of 15 mm, the bottom flange has a thickness of 25 mm and the webs have a thickness of 10 mm. The centroid of this section is 1 m above the bottom surface of the beam, its cross-sectional area is 0.357 m^2 and its second moment of area is 0.319 m^4. The stress condition at the top of the webs, immediately below the top flange, and immediately to the right of support B, is to be investigated. (Do not calculate stresses anywhere else in the beam.)

(i) Determine the bending moment and shear force in the beam at the root of the cantilever, immediately to the right of B, due to the beam's own self-weight. [4]

(ii) Hence determine both the longitudinal-normal and shear stresses at the top of the web, immediately below the top flange. [4]

(c) The next stage of the construction process requires a load W to be lifted up to the beam from one of the corners at the extreme tip of the cantilever at C. Determine the additional longitudinal-normal and shear stresses (at the same place as in part (b)) due to W. Hence determine the maximum value of W that can be applied if the section is built from steel with a uniaxial yield stress of 450 N/mm^2. [6]

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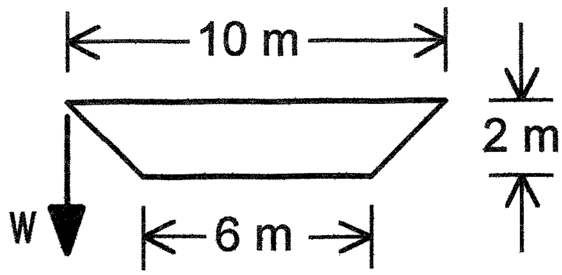

Fig. 1(a)

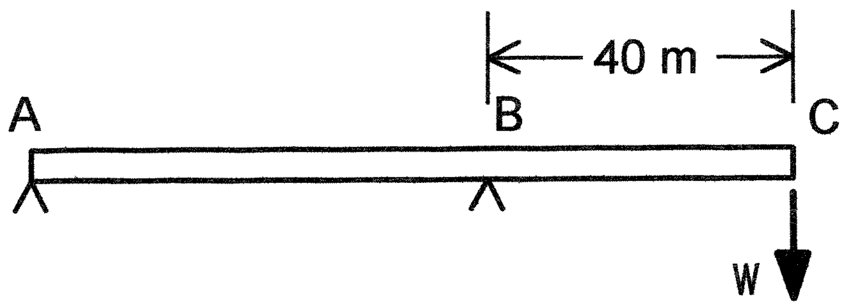

Fig. 1(b)

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2 An element of a steel plate is subjected to in-plane stresses

$\sigma_x = 130$ N/mm^2

$\sigma_y = -30$ N/mm^2

$\tau_{xy} = 60$ N/mm^2

(a) What strains would be recorded by strain gauges placed in the x and y directions? [6]

(b) Draw a Mohr's circle of stress, and determine the maximum and minimum principal stresses, and their orientation to the x direction. [6]

(c) Determine the strains that would be measured by strain gauges placed at $\pm 45°$ to the x direction. [6]

(d) If, instead of determining the strains from the stresses, you were asked to determine the stress state from measurements of direct strain, how many strain gauges would be needed? [2]

3 A beam of length L and fully-plastic moment capacity M_p , is simply supported at one end and is attached at the other end to a rigid wall by a bolting arrangement that can resist a moment of kM_p , where $k \leq 1$. The beam is loaded by a uniformly distributed load of intensity w .

(a) Sketch a likely collapse mechanism which has a single variable parameter. [3]

(b) For the collapse mechanism identified in (a), find an expression for the value of the collapse parameter, and show how the collapse load of the beam could be found as a function of k . [8]

(c) For a beam with $L = 10$ m and $k = 0.7$, which is designed to fail when $w = 20$ kN/m, determine the support reactions, and draw the corresponding bending moment diagram. Have you found the actual value of the collapse load? How do you know? [6]

(d) For the dimensions and loading from (c), choose a suitable Universal Beam section (Structures Data Book, page 10) if the steel has a yield stress of 400 N/mm^2. [3]

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4 (a) What are the conditions on a yield-line mechanism so that it satisfies the compatability constraints? [5]

(b) Figure 2 shows six slabs which are to be analysed using the yield line method. *A separate copy of these figures is provided; your answers should be drawn on this sheet which should be handed in as part of your answer book.* Slabs (i), (ii), (iii) and (iv) are subjected to distributed loads. Slabs (v) and (vi) are subjected to point loads at the position(s) marked X. All the slabs have the same fully plastic moment about any axis, in both hogging and sagging bending.

For each slab sketch a likely collapse mechanism, showing carefully the positions of the yield lines and the axes of rotation of each rigid region. Indicate on your drawings the number of variable dimensions needed for each yield-line pattern (if any) and identify suitable variables to be used in a yield-line analysis. *Do not attempt to carry out yield-line analyses for any of these mechanisms.* [10]

(c) Determine the collapse load of the slab shown in Fig. 3, if it is loaded by a uniformly distributed load of intensity w and has an ultimate moment capacity of $\pm m$ per unit length about any axis. [5]

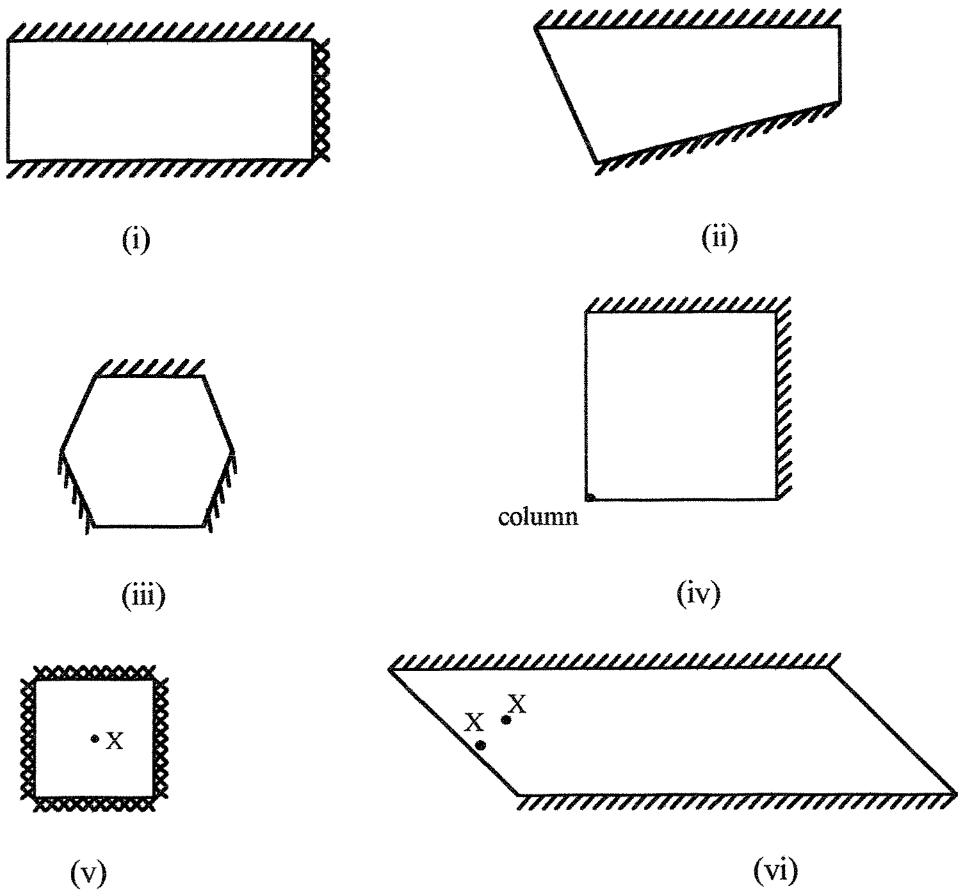


Fig. 2

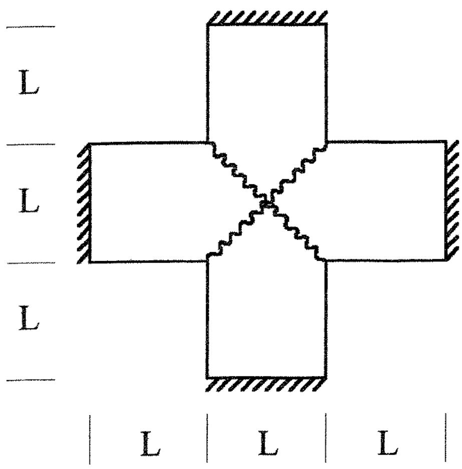
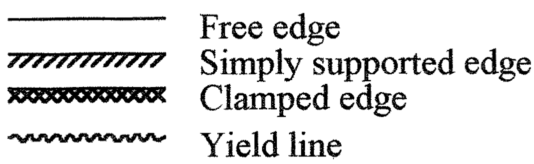


Fig. 3



Key to Figs 2 and 3

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5 Figure 4 shows a rail in a railway track which has a stiffness EI and is supported on sleepers which can be regarded as simple supports at spacing L. It is loaded by a wheel load W which can be idealised as a point load.

(a) By considering just one span L, determine bounds on the bending moment when the wheel is midway between two sleepers by assuming:

(i) the rail acts as a single, simply-supported span of length L; [1]

(ii) the rail acts as a completely clamped span of length L. [2]
(Hint. Use the Structures Data Book)

Show the two bending-moment diagrams on a single plot. [1]

(b) If there is a break in a rail, midway between two sleepers, the rail can be regarded as a propped cantilever, as illustrated in Fig. 5. Determine the maximum hogging and maximum sagging bending moments that now occur in the rail (i) with the wheel at the centre of the unbroken span, and (ii) at the tip of the cantilever. [8]

(c) On the basis of the assumptions made above, how does the maximum bending moment in the rail change when the break occurs? [3]

(d) Discuss the various assumptions that have been made in the above analyses, and consider qualitatively the effect they have had on the results you have obtained. [5]

(cont.

Fig. 4

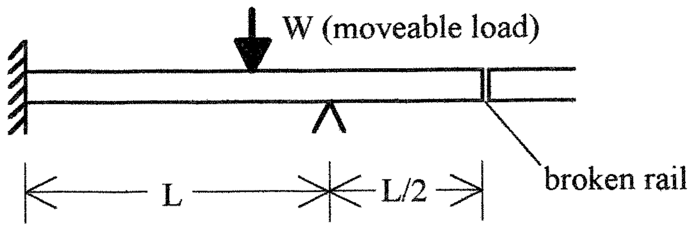

Fig. 5

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6 A pin-ended strut, with an initial imperfection in the form of a half sine wave whose magnitude at mid-span is v_0, is loaded by an axial force P.

(a) Show that the mid-span deflection v, measured from the line of action of P, can be expressed as:

$$v = \frac{v_0}{\left(1 - \frac{P}{P_{cr}}\right)}$$

as P tends towards P_{cr}, where P_{cr} is the buckling load of the perfect structure. [7]

(b) Hence show that a plot of δ / P versus δ will approximate to a straight line, if δ is the mid-span deflections measured from the unloaded position of the *imperfect* beam, and show how P_{cr} and v_0 can be determined from such a plot. [6]

(c) The table below gives a set of experimentally measured loads and mid-span deflections. Determine P_{cr} and v_0. [7]

P (kN)	0	200	320	360	380
δ (mm)	0	5	14	25	37

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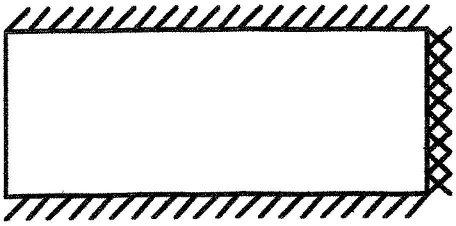
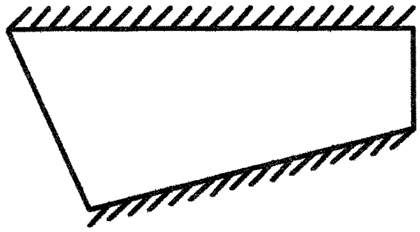
This sheet should be handed in with the answer to question 4.



(i)



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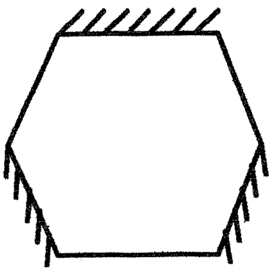

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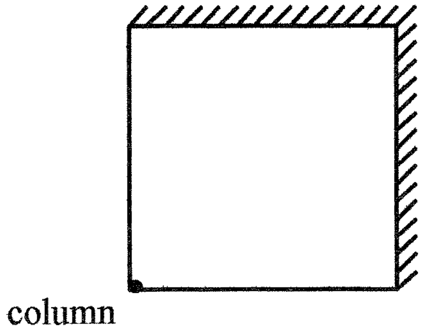

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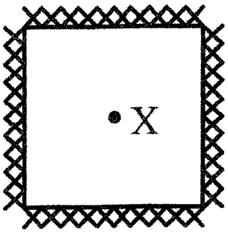
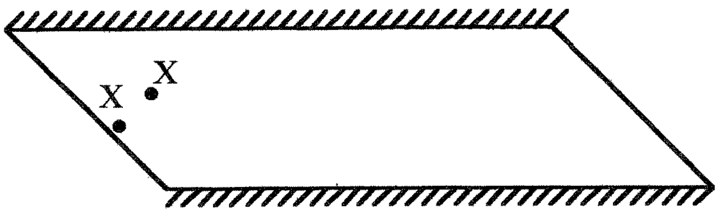


(v)



(vi)