ENGINEERING TRIPOS PART IB

Thursday 4 June 1998 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 Figure 1 shows a feedback system with reference input zero, disturbance $d(t)$, output $y(t)$, and internal signals $e(t)$, $p(t)$ and $q(t)$.

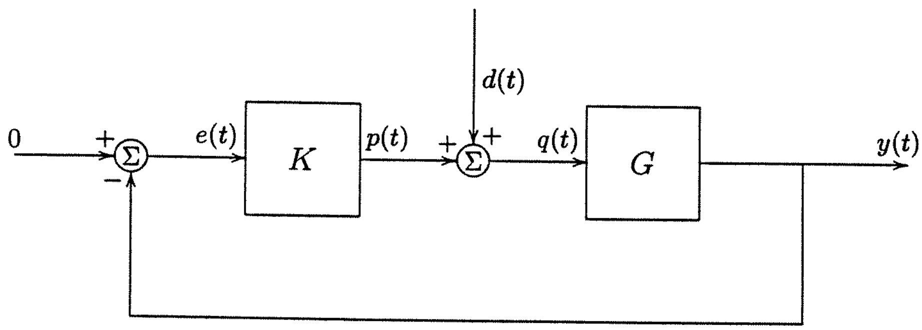

Fig. 1

The following equations are satisfied:

$$p(t) = 0.1\int_0^t e(\tau)d\tau + 0.2e(t)$$

$$\dot{y}(t) + y(t) = q(t-1)$$

(a) Determine the transfer functions of the blocks denoted K and G in Figure 1. [8]

(b) Calculate the closed loop transfer function relating $\bar{y}(s)$ to $\bar{d}(s)$. [6]

(c) Given that the system is closed loop stable and that $d(t) = 2H(t)$, where $H(t)$ is the unit step function, calculate the steady state value of $y(t)$. Comment on this result. [6]

2 (a) If a linear system has an impulse response $g(t)$, under what condition is the system *asymptotically* stable? [4]

(b) Show that the steady-state response of an asymptotically stable linear system to the input $u(t) = \cos(\omega t)$ is of the form $A\cos(\omega t + \phi)$, and derive expressions for A and ϕ. [8]

(c) Consider a linear system with impulse response

$$g(t) = \begin{cases} 0, & t \leq 0 \\ \sin(t), & 0 < t \leq \pi \\ 0, & t > \pi \end{cases}$$

Show that the system is asymptotically stable, and find its steady-state response to the input $u(t) = \cos(2t)$. [8]

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3 (a) Given a physical system describe how to determine experimentally the information needed to plot its Bode and Nyquist diagrams. Comment on any difficulties which might arise. [6]

(b) Figure 2 shows the Bode diagram for a marginally stable linear system with transfer function of the form

$$G(s) = \frac{a(1+bs)}{s(1+cs)\left(1+0.25s+(s/\omega_n)^2\right)}$$

Estimate the values of a, b, c and ω_n. [8]

(c) Explain what is meant by the gain margin and phase margin of a feedback system. The system with the Bode diagram given in Fig. 2 is controlled by a proportional negative feedback controller with a gain of 2. Graphically determine the gain and phase margins, and comment on the performance of the system given these values. [6]

Note: An extra copy of Fig. 2 is provided on a seperate sheet, and should be handed in with your answer if constructions are made on it.

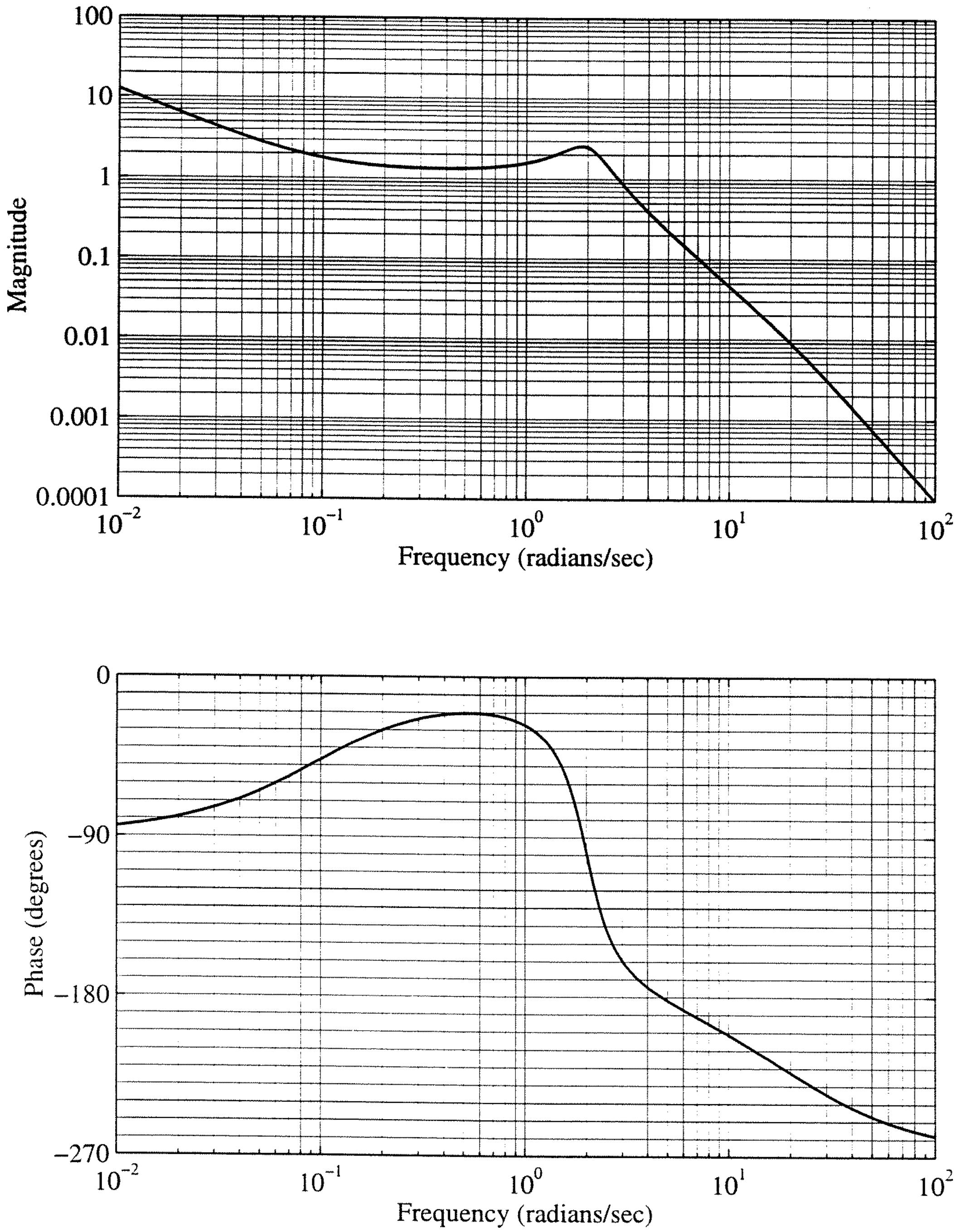

Fig. 2

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4 Figure 3 shows an electric motor driving a flywheel of inertia J. The motor has negligible inertia. The flywheel is fitted with a controller with transfer function $H(s)$ which generates the control voltage v in response to the error in shaft angle, $e = \theta_d - \theta$ where θ_d is the desired angle. The amplifier converts voltage to current according to $i = k_a v$ and the motor exerts a torque T equal to $k_m i$.

(a) Derive the equation of rotary motion of the flywheel about the axis of the shaft and hence determine the transfer function relating $\overline{\theta}(s)$ to $\overline{v}(s)$. Draw a block diagram of the closed-loop control system. [8]

(b) If $H(s) = K$ (constant), draw a Nyquist plot of the open-loop transfer function and sketch the time-response of the closed-loop system to a step input in θ_d. What is the effect of increasing K? [6]

(c) The controller is modified to have transfer function $H(s) = K(1+s)$ with $K = \frac{1}{\sqrt{2}} \cdot \frac{J}{k_m k_a}$. Sketch the Nyquist plot of the new open-loop transfer function. Determine the phase margin of the feedback system and explain whether you consider this to be a suitable value. [6]

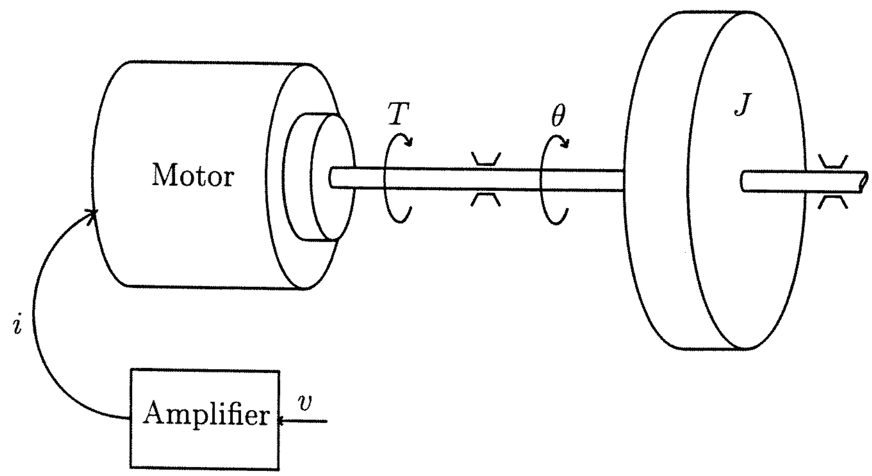

Fig. 3

SECTION B

5 A cosine carrier wave of frequency ω_c and amplitude A is amplitude modulated by a cosine signal of frequency ω_m. The modulation index is m_A.

(a) Obtain an expression for the modulated signal as a sum of three sinusoidal components. Explain the significance of these components in communication systems. [6]

(b) Figure 4 shows a demodulator for AM signals. The input is $b\cos\,\omega_c t$ where b is a constant. Use a standard Fourier series to obtain the frequency spectrum of the output from the rectifier. [6]

(c) Using the results from part (b), derive an expression for the output from the rectifier when the input is the amplitude modulated signal defined above. Show how the lowpass filter can recover the original modulating signal. What are the constraints on the filter bandwidth and the modulation index? [8]

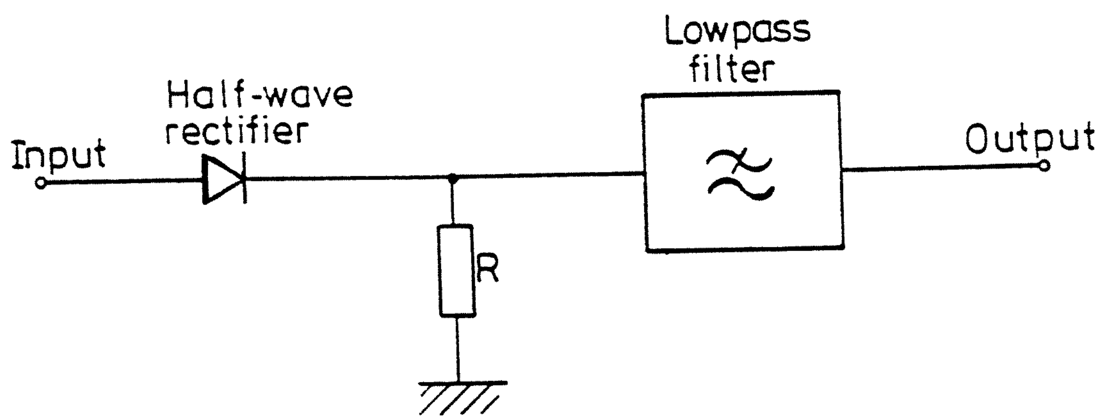

Fig. 4

6 (a) When a signal is digitised it must be sampled and quantised. Briefly explain the distortions introduced by these two processes and the steps which may be taken to minimise the distortions. [6]

(b) A compact disc (CD) outputs data in a digital form at a rate of approximately 1.5 Mbit/s. Estimate, giving reasons, how many bits per sample are available for a stereo (two channel) audio signal of 20 kHz bandwidth per channel. Hence determine the maximum signal-to-noise power ratio that can be achieved on each channel if the signal is sinusoidal, and express this ratio in decibels. [7]

(c) If the same CD is to contain digitised monochrome video signals with a signal-to-noise ratio of at least 37 dB and a frame rate of 8 frames per second, estimate the maximum image size (in lines and pixels) which could be accommodated. Compare this with a conventional television image and suggest possible ways in which the CD image could be improved. [7]

Note: the r.m.s. noise voltage from an ideal digitiser with a quantising step size δv is $\delta v / \sqrt{12}$.

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