IB 1999 Answers

1999 Part IB Paper 4

Answers

Fluid Mechanics and Heat Transfer

1.c $Q = \frac{2\pi\Omega^2 r^2 h^3}{3\nu}$

2.a $V_1 = 1.30 m/s \quad , \quad V_2 = 9.28 m/s$

b $F = \frac{1}{2}\rho g\left(h_1^2 - h_2^2\right) + \rho h_1 V_1 (V_1 - V_2)$

$= 68.4 kN$

c F will be larger.

3.b $P_1 - P_2 = \rho V_2 (V_2 - V_1)$

d 125 Watts

4. -

5.a $2.89 kW$

b Resistance = (10 + 19N)/A

N > 10

6. $T - T_0 = (T_1 - T_0)\exp(-x/\delta)$

$\delta = (\lambda e / 2h)^{1/2}$

$q = 2h(T - T_0)/(1 + he/\lambda_i)$

Engineering Tripos IB 1999 - Answers Paper 1 Mechanics

Q1 POA = 90^0: $\omega_{AP} = 0$; $a_P = 17.7\omega^2$ ⇑; Q+50F = - $441.9\omega^2$.

OAP = 90^0: $\omega_{AP} = \omega / 9$; ; Q+52.7 F = + $188.6\,\omega^2$.

Q2. $M_{max} = \frac{2}{27} mg\ell \sin\theta$.

Loses contact when $\cos\theta = \frac{2}{3}\cos\alpha \rightarrow M_{max} = \frac{2}{27} mg\ell\sqrt{1 - \frac{4}{9}\cos^2\alpha}$

Q3. Force on ring $-2m\Omega\dot{\theta}\cos\theta\,\mathbf{e}^*$. $\dot{\theta}^2 = \Omega^2(\sin^2\theta - \sin^2\alpha) + \frac{2g}{R}(\cos\theta - \cos\alpha)$

$\cos\theta = \cos\alpha$ or $\cos\theta = \frac{2g}{\Omega^2 R} - \cos\alpha$

Q4 (a) $\omega_2 = -\frac{1}{2}\omega_1$: (b) $\omega_2 = \frac{13}{7}\omega_1$, $\omega_2 = -\frac{11}{7}\omega_1$

Q5 $\frac{m}{7}(v_0 - a\omega_0)^2$ where $v_0 = \frac{P}{m}(\cos\theta - \mu\sin\theta)$, $a\omega_0 = -\frac{5P}{2m}(1-\mu)\sin\theta$

Q6(i) AOC = 118^0, COB = 152^0, BOA = 90^0. Q=350.9 Nm.

Reduce to 272Nm by putting C in middle: to zero by balancing:

eg ±11 Kg mm on A, C at 43^0 to OA.

Q6(ii) $\frac{dh}{dt} = \Omega \times J\omega$:

Precession at $\Omega/2$: spin $\omega = \Omega R / 2r$: $Q = 0.1 m R r \Omega^2$ pointing backwards along track.

1999 Part IB Paper 3

Materials

Answers

1. (c) 2K for heterogeneous and 40K for homogeneous undercooling.

2. (b) $\sigma_0(P_s = 37\%) = 336$ MPa, $m = 9.3$, $\sigma_{mean}(5V) = 269$ MPa

   (c) $P_s = 0.995$

   (d) $P_s(V) = \exp\left\{-\left(\frac{1}{V_0\,\sigma_0^m}[(2\sigma)^m V_1 + \sigma^m(2V_2)]\right)\right\}$, invalid due to neglection of stress concentration at notches.

3. (a) 23% Fe_3C

   (b) A: Austenite, B: Ferrite, C: Pearlite, D: Bainite, E: Martensite
   (i) 100% martensite (ii) 100% bainite
   (iii) 75% bainite and 25% martensite
   (iv) $\approx$ 10% Ferrite (α) and 90% fine pearlite

4. (b) $\frac{d\,p(x)}{d\,x} = -\frac{\tau(x)}{h}$, assuming Tresca yield criterion $[p(x) - \sigma_x = \sigma_Y]$

   (c) $p_{av} = 64$ MPa, $F = 384$ kN

   (d) Analysis most relevant at elevated temperature (i.e. hot forging).

5. (a) $M1 = \frac{\sigma_y^{2/3}}{\rho}$ (Al, Ti, GFRP)

   (c) $M2 = \frac{G^{1/2}}{\rho}$ (GFRP, Al, Ti). Need L, T and ϕ to evaluate m.

6. (a) Brittle fracture, cold drawing, crazing, shear banding, viscous flow

   (b) $\dot{\varepsilon}_{total} = \dot{\varepsilon}_{dashpot} + \dot{\varepsilon}_{spring} = \frac{\sigma(t)}{\eta} + \frac{\dot{\sigma}(t)}{E}$, applicable in visco-elastic regime

   (c) $\sigma(0) = 5.3$ MPa, $E = 0.2$ GPa, $t_0 = 6$s. LDPE.

Engineering Tripos IB 1999 – Answers Paper 2 Structures

1 a) 1100 kNm, -120 kN, 100 kNm

b) 2.7×10^{-3} rads

c) 117 N/mm^2

d) (i)10.1 N/mm^2
(ii) 12.5 N/mm^2

2 a) (i) 64 N/mm^2 , 96 N/mm^2
(iii) 31.8 N/mm^2, 0 , -127.4 N/mm^2, 26.6°

b) 87 N/mm^2

3 a) (i) & (iv)

b) $4m$

c) (ii) $(1+2\sqrt{2})m$
(iii) $(1+2\sqrt{2})m$

d) No

4 a) 203 × 203 UC 86

b) Option 1 (i) $L/2$ (ii) 7184 kN (iii) Yield (iv) Yes
Option 2 (i) $0.7L$ (ii) 3665 kN (iii) Buckling (iv) No

c) (i) 898 kN
(ii) 142 N/mm^2

5 a) (i) 3 (ii) 1 (iii) 2
b) $31W/16$, $17W/16$
c) $17W/32$
d) $17WL^2/24EI$

6 a) (i) $W = M_p/L$
(ii) $W = 32\ M_p/49L$

b) $M_D = -49WL/32$

Engineering Tripos IB 1999 - Answers Paper 5 Electrical Engineering

1 (a) 0.3526 V; 12.243 V.
(b) 0.3224 V; 12.978 V.
(d) -1650; 64dB.

2 (a) $1/(2\pi T)$ Hz.
(b) 100 kHz; 1.001 kHz.
(c) 45 mV; 28 mV.

3 (b) 21.06 kW; 0.936 leading; 31.25 A.
(c) 443 V.

4 (b) 33.8 kV; 15.6°.
(c) 16.3°; 6776 A; 0.9296.

5 (a) 119.4 Nm.
(b) 820 rpm.
(c) 630 rpm.

6 (a) 2.11×10^8 m/s; 63.2 Ω; 2.02.
(b) 175.8 mm; 351.7 mm.
(c) 0.2255; 1.582.
(d) 37.5 W.

7 (b) $\frac{(\pi a)^2}{2\lambda} E_o$ or equivalent.
(c) 49.6 km.
(d) 60°.

ANSWERS IB Paper 6 1999

1. (a) (i) Stable

(ii) Unstable

(b) (i) $-\frac{1}{\omega}$ (sin (ω (t – 1)) sin ω t)

(ii) $G(s) = \frac{1-e^{-s}}{s}$

$A = \frac{2}{\omega} \sin \frac{\omega}{2}$

$\varphi = \tan^{-1}\left(\frac{\cos\omega - 1}{\sin\omega}\right)$

2. (a)

(b) $k_a \leq \frac{5J}{k_t k_m}$

(c) $\frac{\alpha}{k_t}$; $\frac{1}{k_t}\left(\alpha - \frac{1}{k_m k_a}\right)$

3. (b) a = 5 , k = 10

(c) 48°

4. (b) (ii) 1·209

(iii) 2·2 < ω < 3·8 (approx)

5. (b) 1·302 x 10^7 Pixel/sec.

(d) 5·86 MHz

6. (b) $\omega_c = 200\,\pi \times 10^3$ rads^{-1}

(c) 99/101 kHz

(d) Passband 96·6 → 103·4 kHz . Spacing 8 kHz (approx).

Part IB 1999

Paper 7 Mathematical Methods

Answers to Questions

1. (a) $\phi = x^n f(y/x)$

(b) $I_1 = \frac{\pi}{4}\left(1 - e^{-a^2}\right) \qquad I_2 = \frac{\sqrt{\pi}}{2}$

2. (i) $x^2 + y^2 + z = 4$ (a paraboloid)

(ii) 16π

(iii) 16π

3. (i) 6

(ii) $\phi = x^2 yz + C$

(iii) You will need to prove $\nabla f(\theta) = (df/d\theta)\nabla\theta$

4. (i) Local truncation errors $O(h^2)$ (Euler) $O(h^3)$ (P-C)

(ii) $y = e^{-t}$ 0.368 (Exact) 0.250 (Euler) 0.391 (P-C)

(iii) $\Delta t \approx 0.0005$ (Euler) $\Delta t \approx 0.04$ (P-C) (Very approximate)

(Need to consider global truncation errors)

5. (iii) $\ell n(k_1) = 1.79 \qquad k_2 = 0.51$

7. (ii) $\mathrm{var}(f_n) = \sigma^2/N$ for all n More than 100 samples

8. (i) $n \to \infty$, $p \to 0$ such that $\lambda = np$ is fixed.

(ii) (a) £5.12 (b) £8.98

(iii) Pack 101 components per box – but why?

Engineering Tripos IB 1999 – Paper 8

Question 1

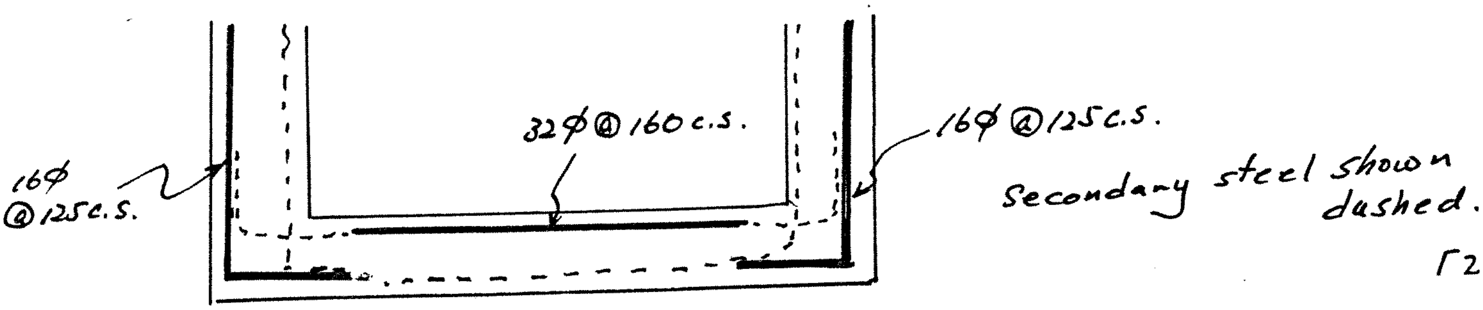
(a) $w = 40\,\text{kN/m}^3$

(b) 372 kNm/m

(c) $d = 249$ mm (say 250 mm)

(d) At the centre of the slab: 32 mm bar at 160 mm centre to centre spacing
At the corners: 16 mm bat at 25 centre to centre spacing

(e)



Question 2

(a) 2.1

(b) (i) 1.61
(ii) 1.50
(iii) 0.76

Question 5

(c) $R_1 = 1 \cdot 1\ \text{M}\Omega$
$R_2 = 10 \cdot 1\ \text{M}\Omega$
$C_1 = 0 \cdot 1\ \mu\text{F}$
$C_2 = 0 \cdot 1\ \mu\text{F}$

(d) 300 Hz

(e) 108 Mbytes

Question 6

(f) A = spanning pipe – most displacement
B = exposed pipe
C = buried pipe – least displacement – most stiffness

(g) Response is data-book case (a)
$c \approx 0 \cdot 3$ $f_n \approx 5$ Hz

(h) $L \approx 26$ m

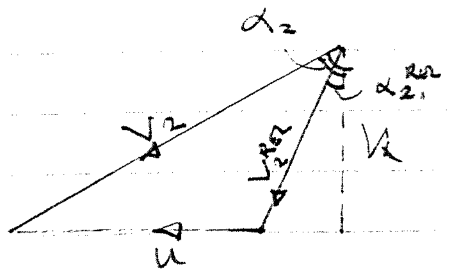
<u>Question 7</u>

(a) In the Stator, the stagnation P & T are essentially constant, the velocity increases and so the pressure falls.

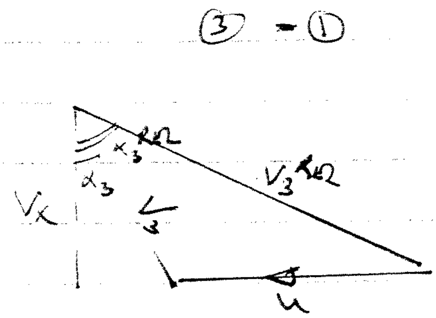
In the rotor, if the radius is constant the <u>relative</u> stagnation P & T are essentially instant, the <u>relative</u> velocity increases and to the pressure falls.

(b)

<u>Stator Exit/Rotor Inlet</u>



<u>Rotor Exit/Stator Inlet</u>



(c) 155·21 m/s

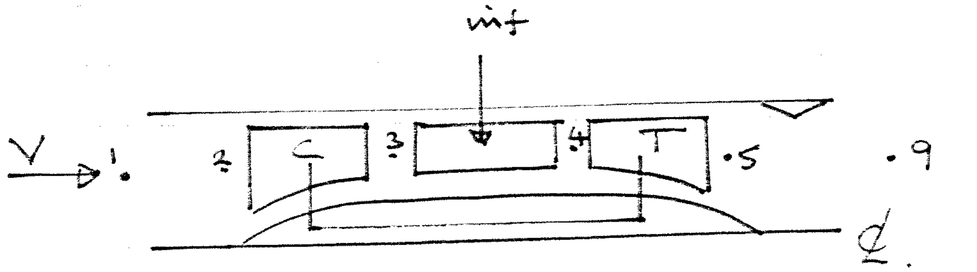
(d) 60.6 KW/kgs^{-1}, 59·96 K

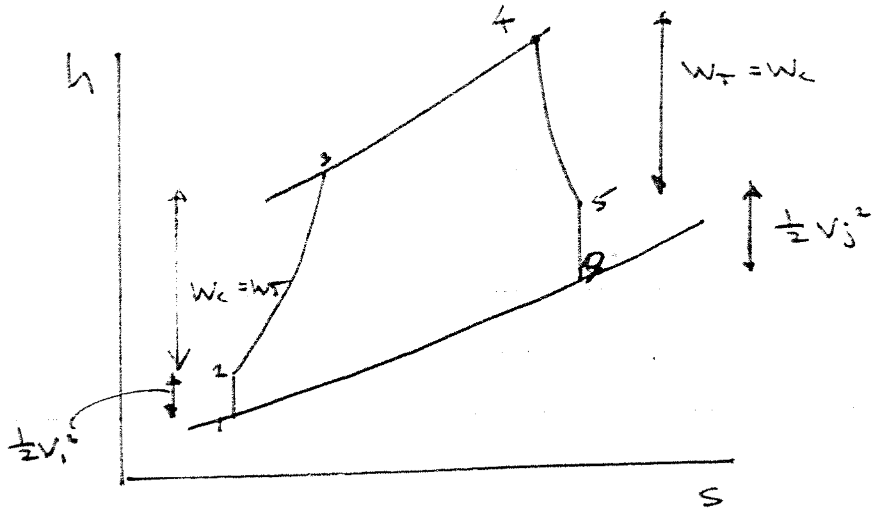
(e) 6 stages, $0 \cdot 156 = \frac{1}{6 \cdot 4}$

(f) (i) the blade speed of the fan is much higher due to its larger radus and most importantly (ii) the fan has a much higher mass flow rate.

Question 8

(a)





(b) (i) 1003 K
(ii) 1·92 kg/s

(c) 536 k

(d) 775K, 89·3 kPa

(e) 1·34 kN

(f) 0·86 kN

Question 9

(a) $$\eta_p = \frac{\text{Power to aircraft}}{\Delta(K.E)\text{of jet}}$$

(b) Tyres and the length of the runway

(c) 1·60

(d) Increased speed $\Rightarrow$ lower density for same lift

(e) X_n/Engine = 84 kN

(g) 432 m/s, 0·755

Question 10

(a) 0·7 ohm^{-1} m^{-1}

(b) 0·0071 eV

(c) 0·176 μm $f_T = 4{\cdot}5.10^{10}$ Hz

Question 11

(a) 2.10^{-7} m

(b) $6{\cdot}25.10^{22}$m^{-3} $<< N_{max}$

(c) 3 ps

(d) 3V and 6.10^{-4}A

Question 12

2.36 hours