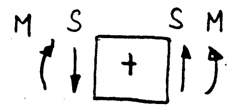
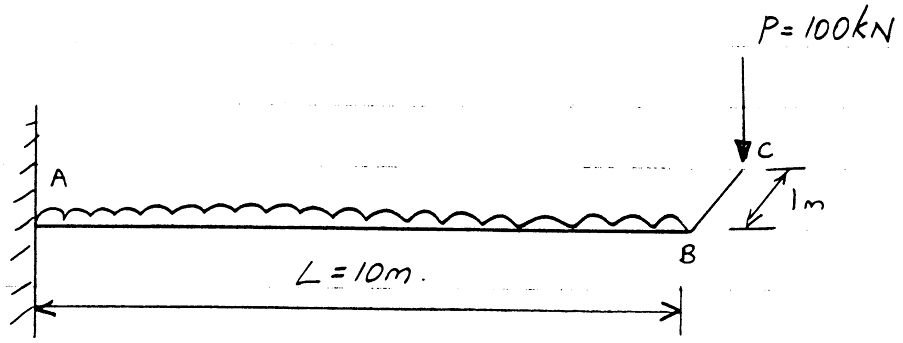
PAPER 2 IB STRUCTURES EXAM SOLUTION 1999 1



Q1



(a) Moment at A $M_A = \underbrace{\cancel{2} \times 10 \times \frac{10}{\cancel{2}}}_{UDL} + \underbrace{100 \times 10}_{Pt.\ load} = \underline{1100\ kNm}$

Shear at A $S_A = -(2 \times 10 + 100) = \underline{-120\ kN}$

Torque at A $T_A = 100 \times 1 = \underline{100\ kNm}$.

(b) Angle of rotation $T = G \cdot \dfrac{4 A_e^2}{\oint \frac{ds}{t}} \phi$ from Data Book

$G = 81\,GPa$ for steel (Data Book)

$A_e = 800 \times 1000 = 8 \times 10^5\ mm^2\ (0.8 m^2)$

$T = 100\ kNm$

$$\oint \frac{ds}{t} = \frac{800}{10} \times 2 + \frac{1000}{5} \times 2 = 160 + 400 = 560$$

$$\therefore \phi = \frac{100 \times 10^3 \times 560}{81 \times 10^9 \times 4 \times 0.8^2} = 270 \times 10^{-6}\ rads/m.$$

$\therefore$ Rotation $\theta_B = \phi . L = 270 \times 10^{-6} \times 10 = \underline{2.7 \times 10^{-3}\ rads}$ (0.16 degs)

(c) Longitudinal bending stress at A $\dfrac{\sigma}{y} = \dfrac{M}{I}$ from Data Book.

$$I = \frac{0.8}{12} \times 1^3 - \frac{0.79 \times 0.98^3}{12} = (66.7 - 62.0) \times 10^{-3} = 4.7 \times 10^{-3}\ m^4$$

$M = 1100\ kNm$ (from (a)) $y = 0.5\ m$

$$\sigma_A = \frac{1100 \times 10^3 \times 0.5}{4.7 \times 10^{-3}} = \underline{117 \times 10^6\ N/m^2}\ (117\ MPa)$$

PTO

Q1 (cont.)

(d) Shear stress at S

(i) Due to shear force alone $S_A = -120$ kN (from (a))

Longitudinal shear flow $q = \dfrac{S A_s \bar{y}}{I}$ (from Data Book)

where A_s = area of box above cut at position S.

$= 0.8 \times 0.01 = 8 \times 10^{-3}\ m^2$

$\bar{y} = 0.5 - 0.005 = 0.495$ m

$I = 4.7 \times 10^{-3}\ m^4$ (from (c))

$$\therefore q = \frac{120 \times 10^3 \times 8 \times 10^{-3} \times 0.495}{4.7 \times 10^3} = 101 \times 10^3 \text{ N/m}$$

At S where $t = 5$mm on each web, total cut thickness is 10mm.

$$\therefore \tau_S = \frac{q}{t_{cut}} = \frac{101 \times 10^3}{2 \times 0.005} = \underline{10.1 \times 10^6 \text{ N/m}^2} \quad (10.1 \text{ MPa})$$

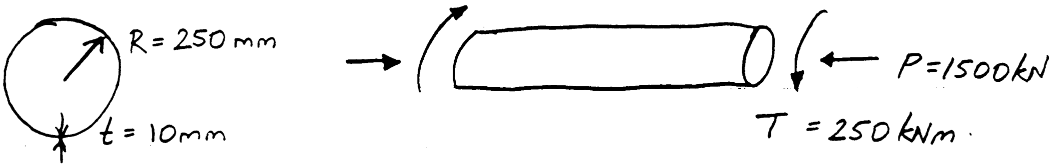
(ii) Due to torque alone $T_A = 100$ kNm (from (a))

$\tau_T = \dfrac{T}{2A_e t}$ from Data Book

$= \dfrac{100 \times 10^3}{2 \times 0.8 \times 0.005}$

$= \underline{12.5 \times 10^6 \text{ N/m}^2}$ (12.5 MPa)

Q2



(a) (i) Shear $T = 250\ \text{kNm}$

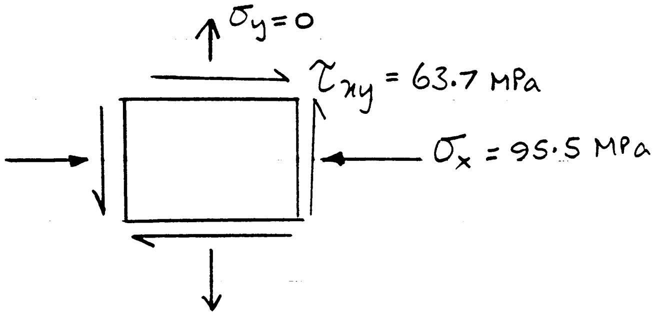
$$\tau = \frac{T}{2A_e t} \quad \text{(Data Book)}$$

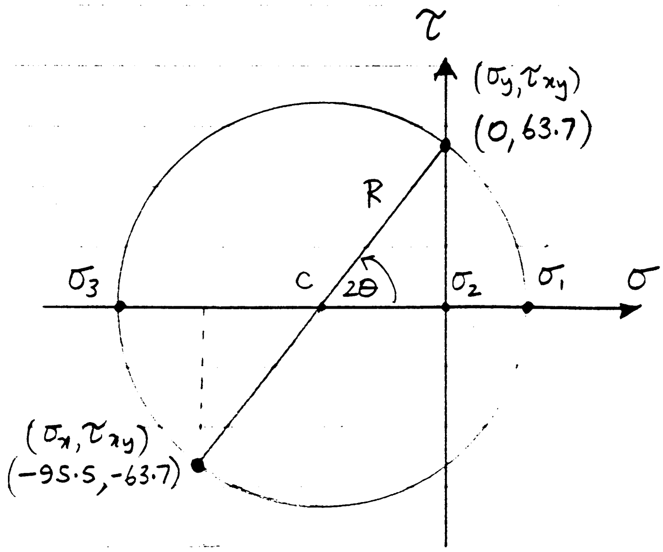
$$= \frac{250\times 10^3}{2\times\pi\, 0.25^2 \times 0.01} = 63.7\times 10^6\ \text{N/m}^2 \quad (64\ \text{MPa})$$

Longitudinal $\sigma = \dfrac{P}{A} = \dfrac{1500\times 10^3}{2\pi \times 0.25\times 0.01} = 95.5\times 10^6\ \text{N/m}^2$ (96 MPa)

(ii) Mohr's Circle for Stress (+ve ↻) Sub on surface $\Rightarrow \sigma_{hoop} = 0$

($\sigma_y = 0$)





(iii) Principal stresses $C = \dfrac{-95.5}{2} = -47.8\ \text{MPa}$

$$R = \sqrt{47.8^2 + 63.7^2} = 79.6\ \text{MPa}.$$

PTO

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Q2 cont.

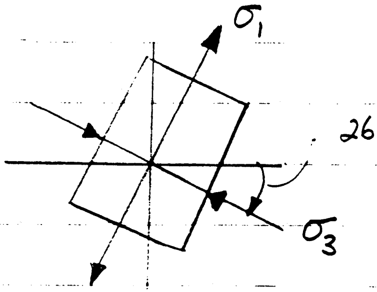
(a)(iii) cont. $\sigma_1 = -47.8 + 79.6 = \underline{31.8 \text{ MPa}}$

$\sigma_2 = \underline{0}$

$\sigma_3 = -47.8 - 79.6 = \underline{-127.4 \text{ MPa}}$

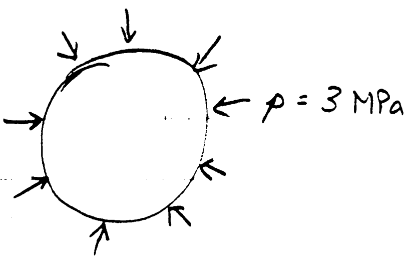
$\tan 2\theta = \dfrac{63.7}{47.8} = 1.333 \quad \Rightarrow \theta = 26.6°$

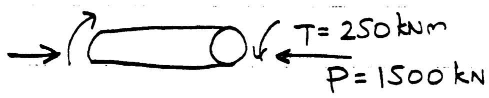
Principal stress σ_1 acts at 26.6° clockwise from y-axis.
σ_3 is at 26.6° clockwise from longitudinal axis of shaft.



(b) Depth $h = 300\text{m}$
Speed 10 knots $\Rightarrow P = 1500\text{kN}, T = 250\text{kNm}$.

Pressure $p = \rho g h = 10^3 \times 9.81 \times 300$
$= 2.94 \times 10^6 \text{ N/m}^2$
$\simeq 3 \text{ MPa}$.





$\sigma_h = -\dfrac{pR}{t} = \dfrac{-3 \times 10^6 \times 0.25}{0.01}$

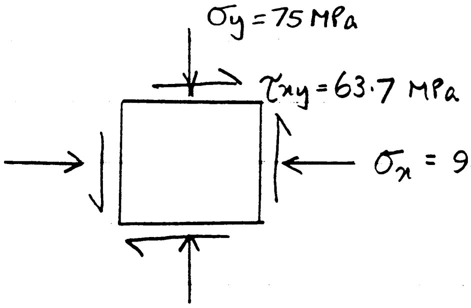
$= -75 \times 10^6 \text{ N/m}^2$

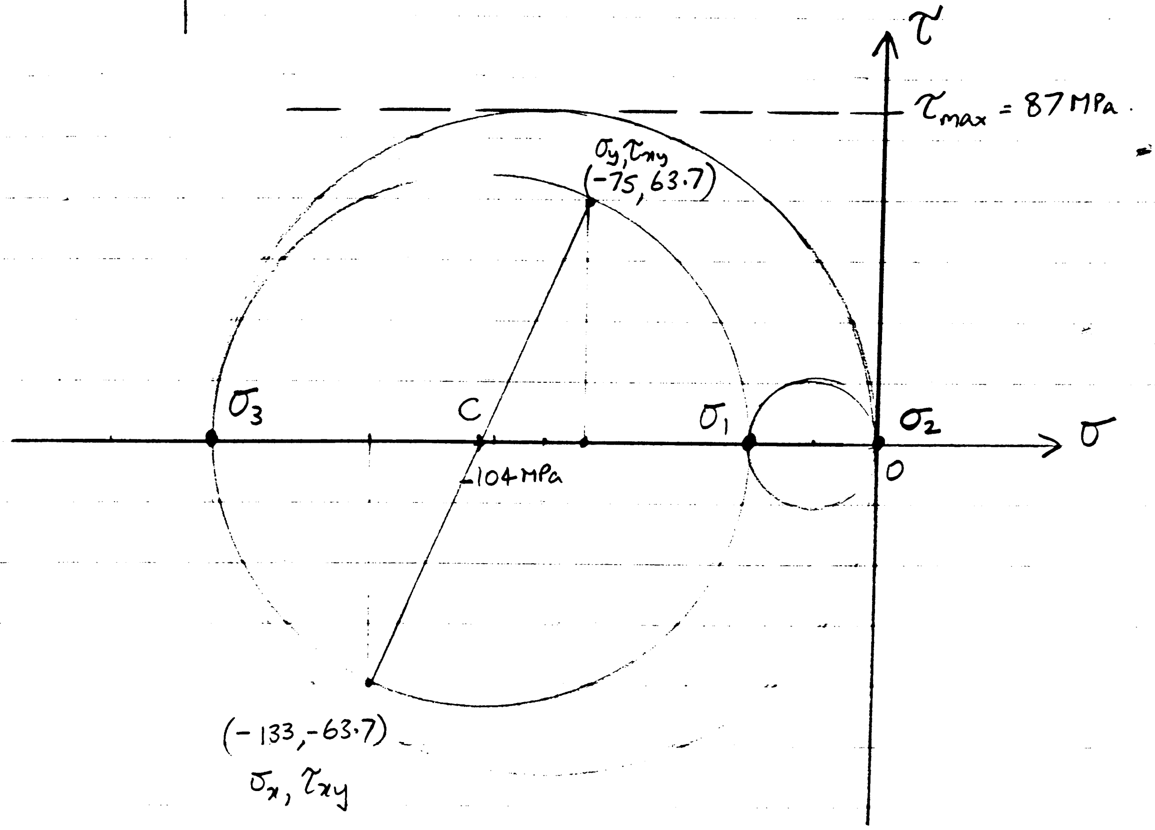
(-75 MPa)

$\sigma_\ell = -\dfrac{pR}{2t} = \dfrac{-75}{2} = -37.5 \text{ MPa}$

PTO

Q2 (b) cont.





$$C = \frac{-133-75}{2} = -104 \text{ MPa}$$

$$R = \sqrt{63.7^2 + (104-75)^2} = \sqrt{63.7^2 + 29^2} = 70 \text{ MPa}.$$

$\therefore \sigma_1 = -104 + 70 = -34$ MPa

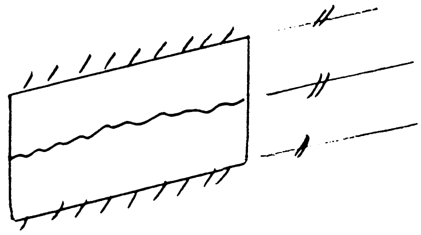
$\sigma_3 = -104 - 70 = -174$ MPa

$\sigma_2 = 0$

$\tau_{max} = \frac{174}{2} = \underline{87 \text{ MPa}}$ c.f. $\tau = 70$ MPa for 2θ (1-3)

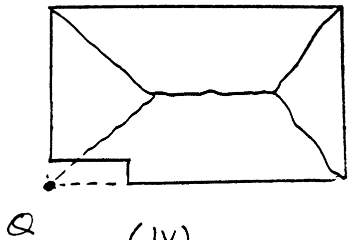
for 2θ (2-3)

(Q3) (a) (I) & (IV) NOT COMPATIBLE



Yieldline parallel with axes of rotation of adjacent rigid plates.

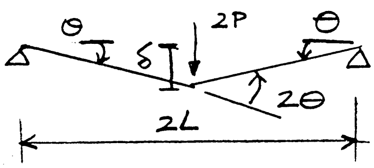
(I)

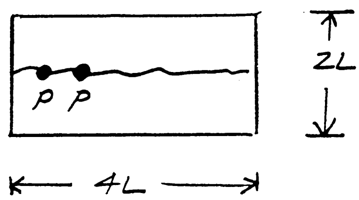


3 axes must meet at point Q. (ignore cut of section of slab.)

(IV)

(b) FULL-WIDTH MODE





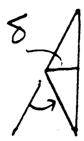
WORK DONE $\quad WD = 2P.\delta$

ENERGY DISSIPATED $\quad ED = m.4L.2\theta \quad$ where $\theta \simeq \tan\theta = \dfrac{\delta}{L}$

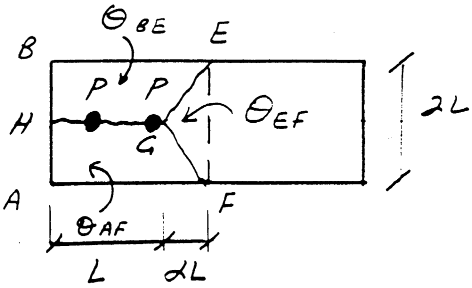
$WD = ED \quad\quad \cancel{2}P.\cancel{\delta} = m.4L.\dfrac{\cancel{2}\cancel{\delta}}{\cancel{L}}$

$$\underline{P = 4m}$$

(c) (i) PARTIAL-WIDTH MODE



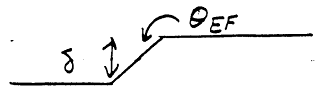
$\theta_{BE} + \theta_{AF} = 2\theta_{BE}$



WORK DONE $\quad WD = 2P.\delta$

ENERGY DISSIPATED $\quad ED = 2.mL(1+\alpha).\dfrac{\delta}{L}$

$$+ 2m.2L.\frac{\delta}{\alpha L}$$



Q3 (c)(i) cont. WD = ED

$$\cancel{2}P.\delta = 2m\cancel{L}(1+\alpha)\frac{\delta}{\cancel{L}} + 2m.2L\frac{\delta}{\alpha L}$$

$$P = m(1+\alpha) + m\frac{2}{\alpha}$$

$$\underline{P = m\left(1+\alpha+\frac{2}{\alpha}\right)}$$

(ii) Lowest P.

$$\frac{\partial P}{\partial \alpha} = m\left(1+(-1)2\alpha^{(-2)}\right) = m\left(1-\frac{2}{\alpha^2}\right) = 0 \quad \text{when } \frac{2}{\alpha^2} = 1$$

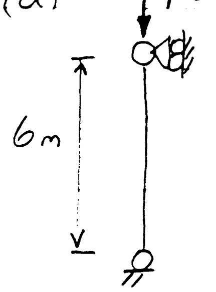
i.e. when $\alpha^2 = 2$ $\alpha = \sqrt{2} = 1.414$ $(< 3 \text{ OK})$

$$\therefore P = m\left(1+\sqrt{2}+\frac{2}{\sqrt{2}}\right) = m\left(1+2\sqrt{2}\right) = \underline{3.83m}$$

(iii) Partial width mode in Fig 3(c) governs since P= 3.83m is less than full-width mode in Fig. 3(b) with P= 4m.

(d) No. Yield-line theory requires ductile behaviour to allow the full failure mechanism to develop. Granite is brittle hence yield-line theory could not be used.

Q4 (a) $P = 1500\text{ kN}$



$\sigma_y = 350\text{ MPa}$

$E = 210\text{ GPa}$

Buckling $P_E = \dfrac{\pi^2 EI}{l_e^2}$ (from Data Book) where $l_e = L = 6\text{m}$.

$\therefore I_{required} \geq \dfrac{P_E \cdot l_e^2}{\pi^2 E} = \dfrac{1500\times10^3 \times 6^2}{\pi^2 \times 210\times10^9} = 26.1\times10^{-6}\text{ m}^4$ (2610 cm^4)

about weakest (Y-Y) axis

Yielding $\sigma_y = \dfrac{P}{A}$ $\therefore$ $A \geq \dfrac{P}{\sigma_y} = \dfrac{1500\times10^3}{350\times10^6} = 4.29\times10^{-3}\text{ m}^2$ (42.9 cm^2)

From UC tables For yield require $152\times152\times37$ or greater

For buckling require $203\times203\times86$ or greater

$\Rightarrow$ Select $\underline{203\times203\text{ UC}86}$ with $A = 110.1\text{ cm}^2$

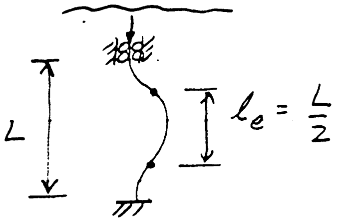
$I_{YY} = 3119\text{ cm}^4$

Check For this section $P_E = \dfrac{\pi^2 EI}{l_e^2} = \dfrac{\pi^2 \times 210\times10^6 \times 3119\times10^{-8}}{6^2} = 1796\text{ kN}$

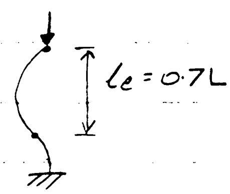
$P_y = \sigma_y . A = 350\times10^3 \times 110.1\times10^{-4} = 3854\text{ kN}$

(b) $P = 3750\text{ kN}$

(i) OPTION 1



OPTION 2



(ii) $P_E = \dfrac{\pi^2 EI}{(L/2)^2} = \dfrac{4\pi^2 EI}{L^2}$

$= 4\times1796$

$= \underline{7184\text{ kN}}$ OK

$P_E = \dfrac{\pi^2 EI}{(0.7L)^2} = \dfrac{\pi^2 EI}{0.49L^2}$

$= \dfrac{1796}{0.49} = \underline{3665\text{ kN}}$ BUCKLES

Q4 (b) Cont.

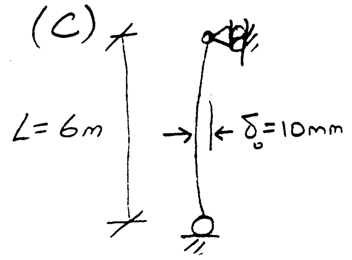
(III) OPTION 1 Failure governed by yield at $P = 3854$ kN

OPTION 2 Failure governed by buckling at $P = 3665$ kN

(IV) OPTION 1 OK

OPTION 2 INADEQUATE since required load is greater than buckling load.

(C)



(i) $$\delta = \frac{\delta_0}{1 - \frac{P}{P_E}}$$

$$\therefore \frac{P}{P_E} = 1 - \frac{\delta_0}{\delta}$$

$$\therefore P = P_E\left(1 - \frac{\delta_0}{\delta}\right) = 1796\left(1 - \frac{10}{20}\right) = 1796 \times 0.5$$

$\delta = 20$ mm at $P = 898$ kN

(ii) Maximum stress $\sigma = \frac{P}{A} + \frac{M}{Z} = \frac{P}{A} + \frac{Pe}{Z}$

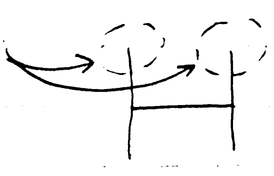
where $Z_{yy} = 298.7 \text{ cm}^3$ (from data book for 203×203 UC 86)

$$\therefore \sigma = \frac{898 \times 10^3}{110.1 \times 10^2} + \frac{898 \times 10^3 \times 20}{298.7 \times 10^3}$$

$$= 81.6 + 60.1$$

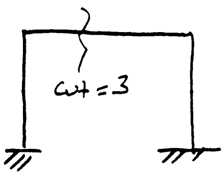
$$= 141.7 \text{ N/mm}^2 \quad (142 \text{ MPa}) < \sigma_y$$

This will occur at the extreme tip of 2 of the ends of the column flanges.



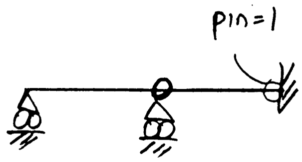
1B STRUCTURES SOL^{N} 1999. 10.

Q5 (a) (i)



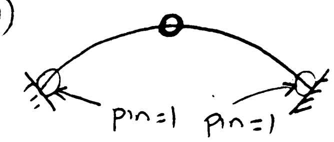
$n = 3$

(ii)



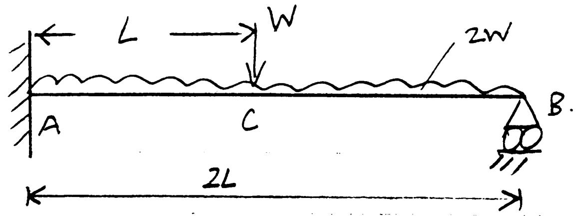
$n = 1$

(iii)



$n = 2$

(b)



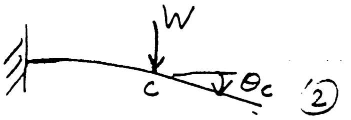
EI

Use Data Book cases.

$\equiv$

For ① $\delta_{B_1} = \dfrac{2W.(2L)^3}{8EI} = \dfrac{2WL^3}{EI}$

+



For ② $\delta_{C_2} = \dfrac{WL^3}{3EI}$; $\theta_{C_2} = \dfrac{WL^2}{2EI}$

$\therefore \delta_{B_2} = \dfrac{WL^3}{3EI} + \dfrac{WL^3}{2EI} = \dfrac{5WL^3}{6EI}$

+

For ③ $\delta_{B_3} = R_B.\dfrac{(2L)^3}{3EI} = \dfrac{8R_BL^3}{3EI}$

At B

Compatibility at B gives $\delta_{B_3} = \delta_{B_1} + \delta_{B_2}$

$$\therefore \frac{8R_BL^3}{3\cancel{EI}} = \frac{2WL^3}{\cancel{EI}} + \frac{5WL^3}{6\cancel{EI}}$$

$$R_B = \frac{3}{8}\left(2W + \frac{5W}{6}\right) = \frac{3}{8}\left(\frac{12W+5W}{6}\right) = \frac{3}{8}\cdot\frac{17W}{6} = \frac{17W}{16}$$

At A

$\Sigma V = 0 \qquad R_A + \dfrac{17}{16}W = 3W$

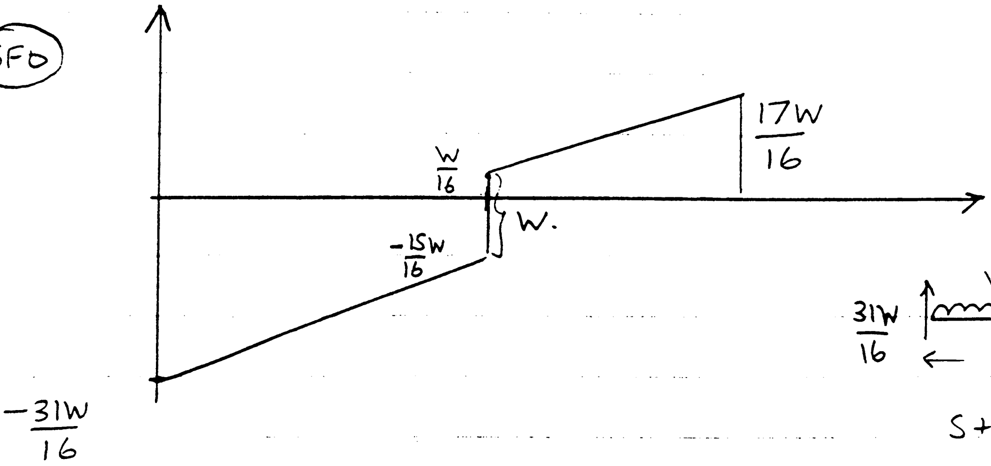
$$\therefore R_A = 3W - \frac{17W}{16} = \frac{48W - 17W}{16} = \frac{31W}{16}$$

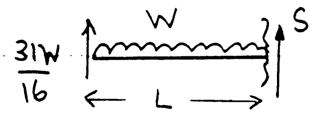
PTO

Q5 (b) cont.

$S \downarrow \boxed{+} \uparrow S$

SFD





$$S + \frac{31W}{16} - W = 0$$

$$S = W - \frac{31W}{16} = -\frac{15W}{16}$$

(c) $$\delta_{B\,con.} = \frac{17WL^3}{12EI}$$

With consolidation $\delta_{B_1} + \delta_{B_2} + \delta_{B_3} = \delta_{B\,con}$.

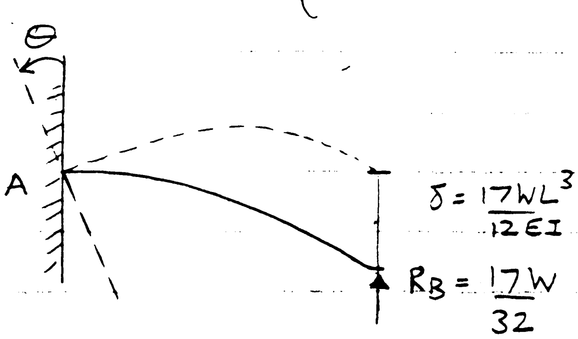
$$\therefore \frac{2WL^3}{EI} + \frac{5WL^3}{6EI} - \frac{8R_BL^3}{3EI} = \frac{17WL^3}{12EI}$$

$$R_B = \frac{3}{8}\left(2W + \frac{5W}{6} - \frac{17W}{12}\right) = \frac{3W}{8}\left(\frac{24+10-17}{\cancel{12}_{\,4}}\right) = \frac{17W}{32}$$

Reduction in $R_B = \frac{17W}{32}$ c.f. $R_B = \frac{17W}{16}$ for no consolidation.

(i.e. Consolidation results in R_B being halved)

(d)



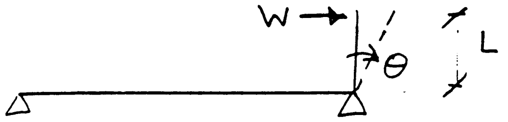
For $R_B = 0$, need to rotate wall anticlockwise about A by θ such that displacement at B due to θ is $\delta_{B_\theta} = \frac{17WL^3}{12EI}$

$$\therefore \delta_{B_\theta} = 2L.\theta = \frac{17WL^3}{12EI}$$

$$\therefore \theta = \frac{17WL^2}{24EI} \text{ anti-clockwise.}$$

Q6 (a)

(i)



$WD = W.L\theta$

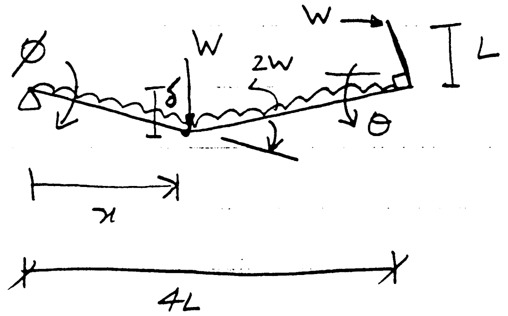
$ED = M_p.\theta$

$WD = ED$

$W.L\cancel{\theta} = M_p.\cancel{\theta}$

$\therefore W = \dfrac{M_p}{L}$

(ii)



$\delta = x\phi = (4L-x)\theta$

$\phi = \left(\dfrac{4L-x}{x}\right)\theta$

$\underline{WD} = 2W.\dfrac{\delta}{2} + W.\delta - W.L\theta = 2W(4L-x)\theta - WL\theta$

$= W\theta(7L-2x)$

$\underline{ED} = M_p(\phi+\theta)$

$= M_p\left[\left(\dfrac{4L-x}{x}\right)\theta + \theta\right]$

$\underline{WD = ED}$ $\quad W\cancel{\theta}(7L-2x) = M_p.\theta\left(\dfrac{4L-x}{x}+1\right) = M_p\cancel{\theta}\dfrac{4L}{x}$

$\therefore W = M_p\dfrac{4L}{(7L-2x)x}$ (1) $\qquad \dfrac{1}{W} = \dfrac{(7L-2x)x}{4M_p.L}$

$\dfrac{d\left(\frac{1}{W}\right)}{dx} = \dfrac{1}{4M_p.L}\dfrac{d}{dx}\left[x(7L-2x)\right] = \dfrac{1}{4M_pL}\left[(7L-2x)-2x\right] = \dfrac{1}{4M_pL}(7L-4x)$

$= 0$ when $(7L-4x) = 0$ i.e. when $7L = 4x$

$x = \dfrac{7L}{4} = 1.75L$

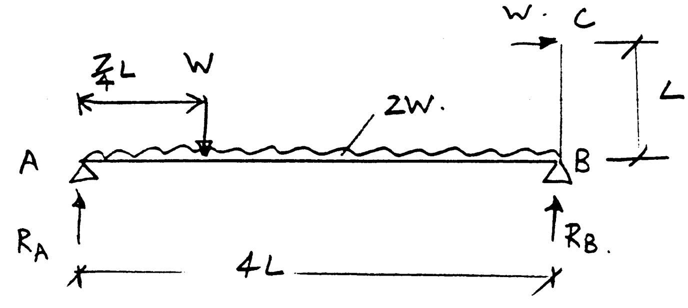
Substitute for x in (1) above.

At $x = \dfrac{7L}{4}$ $\quad W = \dfrac{M_p.4L}{\left(7L - 2.\frac{7L}{4}\right)\left(\frac{7L}{4}\right)} = \dfrac{4M_p}{\left(\frac{28}{4}-\frac{14}{4}\right)\frac{7L}{4}} = \dfrac{4M_p}{\frac{14}{4}\cdot\frac{7L}{4}} = \dfrac{4M_p}{\frac{49L}{8}} = \dfrac{32M_p}{49L}$

Q6 (a)(ii) Cont.

$$\therefore W = \frac{32 M_p}{49 L} \quad \text{at} \quad x = \frac{7L}{4} = 1.75L.$$

(b) Mode 2 with hinge at D is critical failure mechanism with $W = \frac{32 M_p}{49 L}$ c.f. Mode 1 with $W = \frac{M_p}{L}$.



For Beam AB

Moments about B. ($\Sigma M = 0$)

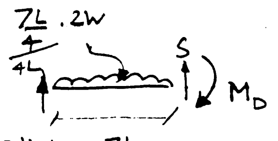
$$R_A . 4\cancel{L} - W . \frac{9\cancel{L}}{4} - 2W . 2\cancel{L} + W\cancel{L} = 0$$

$$R_A = \frac{4W - W + \frac{9}{4}W}{4} = \frac{21W}{16} \quad (1.31W)$$

$\Sigma V = 0$ $\quad R_A + R_B = 3W$

$$\therefore R_B = 3W - \frac{21}{16}W = \left(\frac{48-21}{16}\right)W = \frac{27}{16}W \quad (1.69W)$$

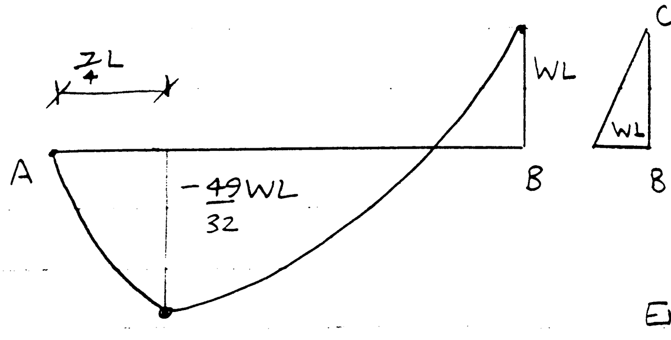
At D $\quad x = \frac{7}{4}L$.



$$M_D = -\frac{21W}{16} . \frac{7L}{4} + \frac{7}{16} . \cancel{2}W . \frac{7L}{4} . \frac{1}{\cancel{2}}$$

$$= WL\left(\frac{49 - 147}{64}\right) = -\frac{98}{64} WL$$

$$= -\frac{49}{32} WL \quad (-1.53WL)$$



END OF SOLUTIONS

C.R. MIDDLETON
APRIL 1999