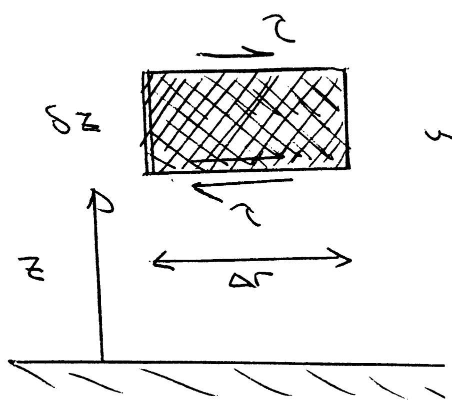
## Section A

### 1.

(a)




unit depth into page.

Net radial force: $F_r = \tau(z+\delta z)\Delta r - \tau(z)\Delta r$

(no pressure forces as $p = p_a$ at all points)

Net radial acceleration: $a_r = -\frac{u_\theta^2}{r} = -\Omega^2 r$

(ignore $u_r \frac{\partial u_r}{\partial r}$ since $u_r \ll u_\theta$)

Newton II : $$\underbrace{\left[\tau(z+\delta z) - \tau(z)\right]\Delta r}_{F_r} = \underbrace{(\rho\Delta r\Delta z)}_{m}\underbrace{(-\Omega^2 r)}_{a_r}$$

$$\Rightarrow \quad \underline{\underline{\tau(z+\delta z) - \tau(z) = -\rho\Delta z\,\Omega^2 r}}$$

(b) $$\tau = \rho\nu\frac{\partial u_r}{\partial z}, \quad \frac{\partial \tau}{\partial z} = -\rho\Omega^2 r$$

$$\Rightarrow \quad \cancel{\rho}\nu\frac{\partial^2 u_r}{\partial z^2} = -\cancel{\rho}\Omega^2 r$$

$$\Rightarrow \quad \frac{\partial u_r}{\partial z} = -\frac{\Omega^2 r}{\nu}(z + A)$$

Boundary condition at $z = h$ is $\tau = 0 \Rightarrow \left.\frac{\partial u_r}{\partial z}\right)_h = 0$

Thus $A = -h$ and,

$$\frac{\partial u_r}{\partial z} = \frac{\Omega^2 r}{\nu}(h - z)$$

Integrate again,

$$u_z = \frac{\Omega^2 r}{\nu}\left(hz - z^2/2\right)$$

(Constant of integration is zero since $u_z = 0$ on $z = 0$)

(c)

$$Q = 2\pi r \int_0^h u_z \, dz = \frac{2\pi \Omega^2 r^2}{\nu}\left(\frac{h^3}{2} - \frac{h^3}{6}\right)$$

$$\Rightarrow Q = \frac{2\pi \Omega^2 r^2}{3\nu} h^3$$

Continuity of mass $\Rightarrow$ $Q = $ const.

$$\Rightarrow h \sim r^{-2/3}$$

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2.

(a) $$\rho g h_1 + \tfrac{1}{2}\rho v_1^2 + \cancel{P_1}_{0} = \rho g h_2 + \tfrac{1}{2}\rho v_2^2 + \cancel{P_2}_{0}$$

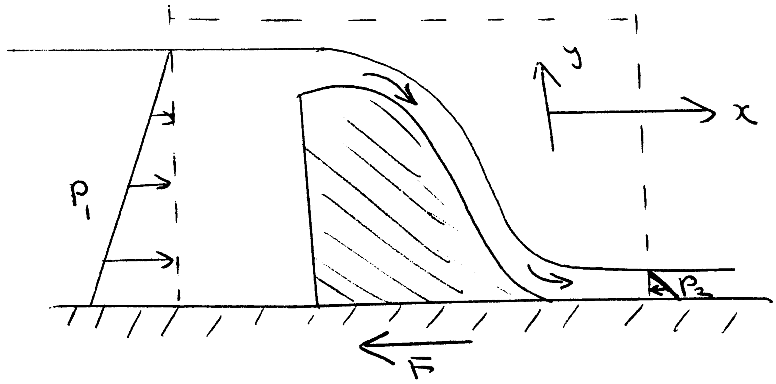
( Bernoulli)

$$\Rightarrow \tfrac{1}{2}\cancel{\rho}(V_2^2 - V_1^2) = \cancel{\rho}g(h_1 - h_2)$$

$$\Rightarrow V_2^2 \Big(\underbrace{1 - \left(\tfrac{h_2}{h_1}\right)^2}_{\text{from } v_1 h_1 = V_2 h_2}\Big) = 2g(h_1 - h_2)$$

$$\Rightarrow \begin{cases} V_2 = 9.28 \text{ m/s} \\ V_1 = 1.30 \text{ m/s} \end{cases}$$

(b) Streamlines are straight and ||$^{\text{el}}$ so no vertical acceleration in fluid ⇒ local force balance in z - direction is self-weight versus pressure gradient i.e. hydrostatic.




Apply force momentum in x - direction. Ignore shear on bed.

$$\underbrace{-F + \int_0^{h_1} P_1\,dy - \int_0^{h_1} P_2\,dy}_{\text{net external force.}} = \underbrace{(\rho h_1 V_1)}_{\dot{m}} V_2 - \underbrace{(\rho h_1 V_1)}_{\dot{m}} V_1$$

$$\Rightarrow \quad -F + \tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 = \rho h_1 V_1 (V_2 - V_1)$$

$$\Rightarrow \quad \underline{\underline{F = \tfrac{1}{2}\rho g (h_1^2 - h_2^2) - \rho h_1 V_1 (V_2 - V_1)}}$$

$$\Rightarrow \quad \underline{\underline{F = 68.44 \times 10^3\ N}}.$$

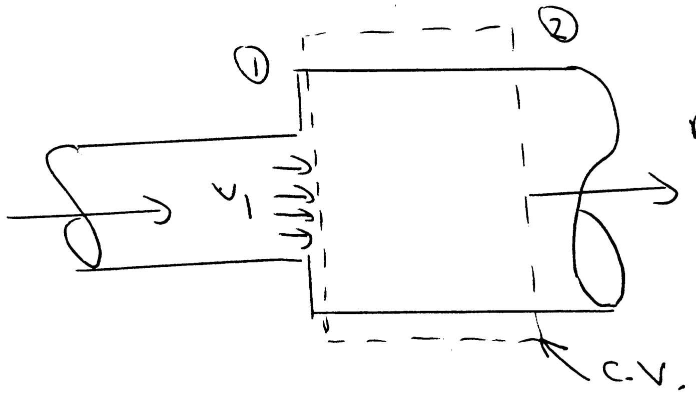
(c) Due to friction $V_2$ will be smaller than expected and so F will be larger.

3.

(a) $$\underbrace{\left(\tfrac{1}{2}\rho V_1^2 + \rho g y_1 + P_1\right)}_{C_1} - \underbrace{\left(\tfrac{1}{2}\rho V_2^2 + \rho g y_2 + P_2\right)}_{C_2}$$

$$= \underbrace{\sum \Delta C_{loss}}_{\substack{\text{energy losses} \\ \text{(per unit volume)}}} - \underbrace{\sum \Delta C_{pump}}_{\substack{\text{energy in from pumps} \\ \text{(per unit volume)}}}$$

(b)




Apply F-M-E in streamwise direction. Ignore shear on boundary.

$$\underbrace{P_1 A_2 - P_2 A_2}_{\text{net external force}} = \dot{m} V_2 - \dot{m} V_1 = \rho V_2 A_2 (V_2 - V_1)$$

$$\Rightarrow \quad \underline{\underline{P_1 - P_2 = \rho V_2 (V_2 - V_1)}}$$

(c) Extended Bernoulli:

$$\left(\tfrac{1}{2}\rho V_1^2 + P_1\right) - \left(\tfrac{1}{2}\rho V_2^2 + P_2\right) = \Delta C_{loss}$$

$$\Rightarrow \quad |\Delta C| = C_1 - C_2 = \tfrac{1}{2}\rho V_1^2 - \tfrac{1}{2}\rho V_2^2 + \rho V_2^2 - \rho V_2 V_1$$

$$= \tfrac{1}{2}\rho V_1^2 + \tfrac{1}{2}\rho V_2^2 + \rho V_1 V_2$$

$$= \tfrac{1}{2}\rho (V_1 - V_2)^2$$

(d)
$$
\begin{aligned}
\text{Power} &= \dot{Q}\,|\Delta C| \\
&= \dot{m}\,\tfrac{1}{2}(V_1 - V_2)^2 \\
&= \tfrac{1}{2}\,\dot{m}\,V_1^2\left(1 - \frac{A_1}{A_2}\right)^2 \\
&= \tfrac{1}{2} \times 10\ \text{kg/s} \times (10\ \text{m/s})^2 \times \left(1 - \tfrac{1}{2}\right)^2 \\
&= 125\ \text{Watts}.
\end{aligned}
$$

(e) Consider boundary layer - dissipation stays finite as $\nu \to 0$ because $\delta \to 0$ at a rate $\nu^{-1/2}$. Expect internal shear layers to form which become thinner as $\nu \to 0$ so that dissipation stays finite.

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4 (a) $$\Delta p = f(\rho, V, \mu, L, d)$$

Parameters $= 6$, Dimensions $= 3$ $(M, L, T)$

Groups $= 3$

Spot: $$\Pi_1 = \frac{\Delta p}{\frac{1}{2}\rho V^2}, \quad \Pi_2 = \frac{L}{d}, \quad \Pi_3 = \frac{\rho V d}{\mu}$$

$$\Rightarrow \quad \Pi_1 = f(\Pi_2, \Pi_3)$$

$$\Rightarrow \quad \Delta p = \tfrac{1}{2}\rho V^2 f\left(\frac{L}{d}, \frac{\rho V d}{\mu}\right)$$

But $\Delta p \propto L$ so $f$ must be such that,

$$\Delta p = \tfrac{1}{2}\rho V^2 \frac{L}{d} \underbrace{f\left(\frac{\rho V d}{\mu}\right)}_{\text{Darcy friction factor}}$$

Laminar flow, $\Delta p \neq f(\rho)$ so only possibility is $f(Re) = \frac{k}{Re}$ so $\rho$ cancels. Thus,

$$\Delta p = \tfrac{1}{2}\cancel{\rho} V^{\cancel{2}} \frac{L}{d} \frac{k\mu}{\cancel{\rho}\cancel{V}d}$$

$$= \frac{k}{2}\frac{\mu V L}{d^2}$$

(b) $$\Delta p = f(\rho, V, \mu, L, d, \epsilon)$$

Parameters $= 7$, Dimensions $= 3$

$\Rightarrow$ Groups $= 4$

$$\Pi_1 = \frac{\Delta p}{\frac{1}{2}\rho V^2}, \quad \Pi_2 = \frac{L}{d}, \quad \Pi_3 = \frac{\rho V d}{\mu}$$

(as before)

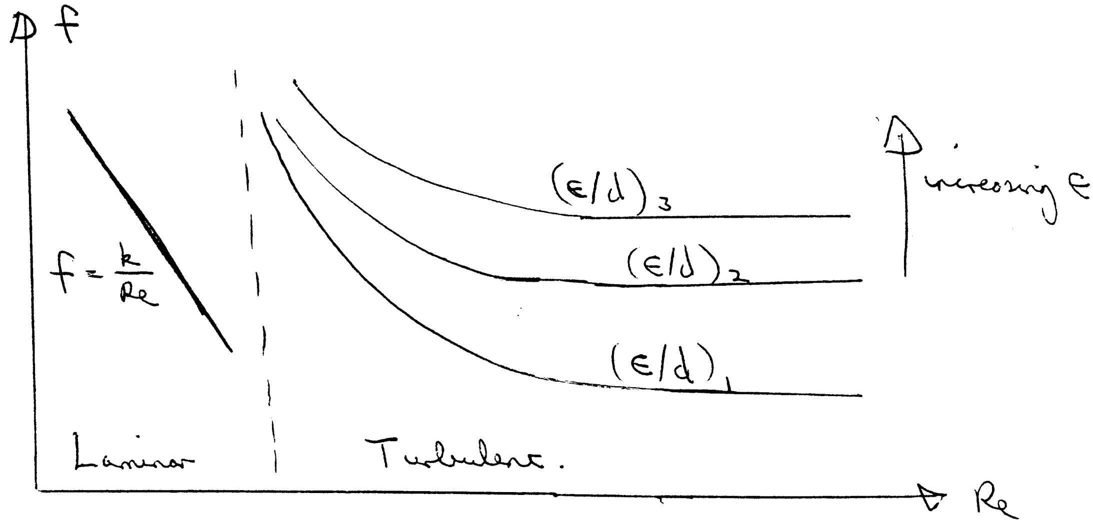
New group: $\Pi_4 = \frac{\epsilon}{d}$.

$$\Pi_1 = f(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$$

$$\Rightarrow \quad \Delta p = \frac{1}{2}\rho V^2 f\left(\frac{L}{d}, \frac{\rho V d}{\mu}, \frac{\epsilon}{d}\right)$$

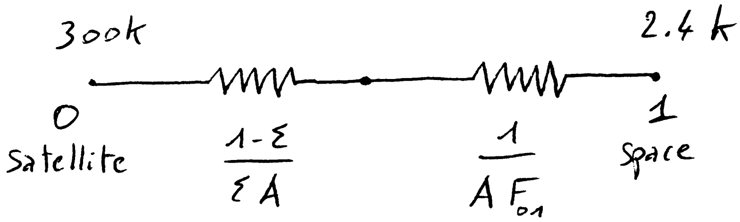
But $\Delta p \propto L$ so as before this becomes,

$$\Delta p = \frac{1}{2}\rho V^2 \frac{L}{d} f\left(Re, \frac{\epsilon}{d}\right)$$



# 5 Radiation

i) Equivalent circuit




$\frac{1-\varepsilon}{\varepsilon A} = \frac{9}{A}$ $\qquad$ $\frac{1}{A F_{01}} = \frac{1}{A}$

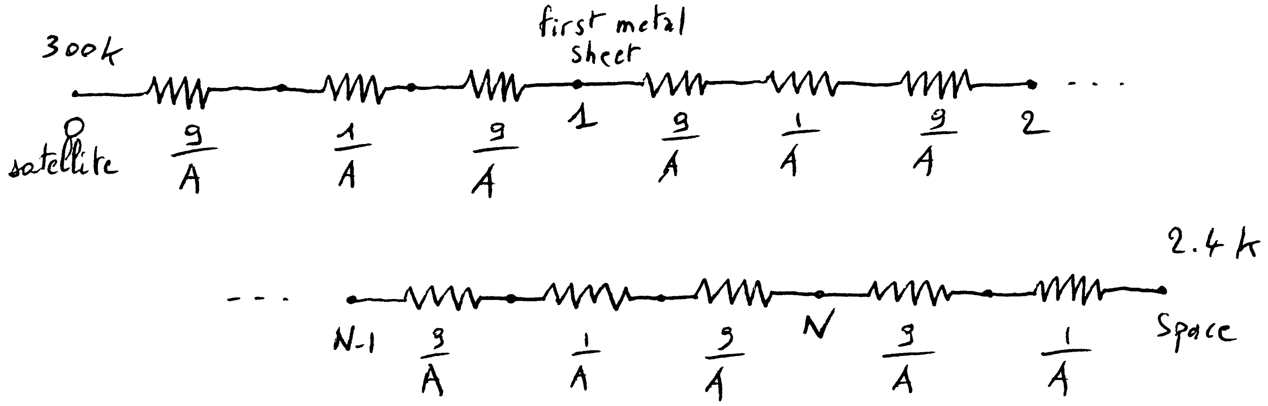
$F_{01} = 1$ every photon from the satellite goes towards space.

Rate of heat loss

$$\phi = \sigma\left(T^4 - T_{space}^4\right)\frac{A}{10} = 2\pi R L \frac{\sigma\left(T^4 - T_{space}^4\right)}{10}$$

$$\simeq 2.89 \text{ kW}$$

ii) Circuit with $N$ metal sheets




Resistance: $\frac{10}{A} + \sum_{i=1}^{N} \frac{19}{A} = \frac{10}{A} + \frac{19N}{A} = \frac{10 + 19N}{A}$

Ratio with the resistance in i): $\frac{10 + 19N}{10} = 1 + 1.9N$

This ratio is larger than 20 when $N \geqslant 10$

Heat flux by conduction along the metal per unit length along z:

$$q_c = - \lambda_m \, e \, \frac{dT}{dx}$$

Convection heat flux on both sides per unit length along z

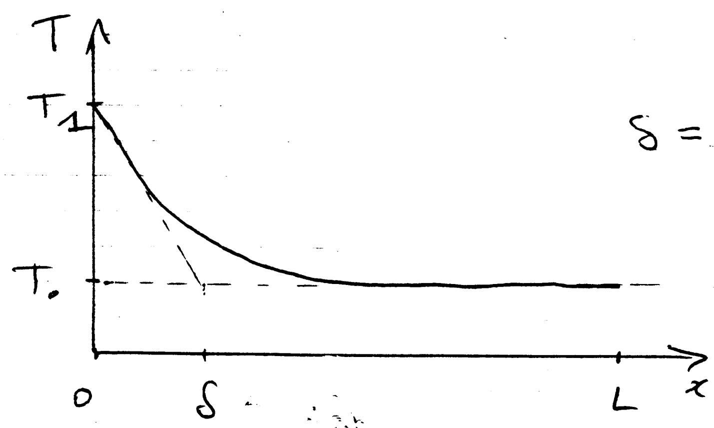
$$q_{out} = 2 \, h \, dx \, (T - T_0)$$

Heat conservation : $$\frac{d q_c}{dx} = - q_{out}$$

$$\therefore \quad \frac{d^2 T}{dx^2} = \frac{2 h}{\lambda_m e} (T - T_0)$$

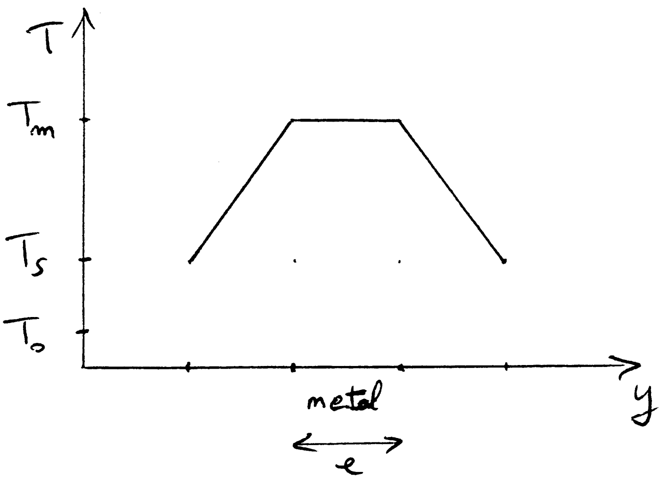
The solution is a linear combination of the two fundamental exponential solutions, but the growing exponential is set to zero.

$$T - T_0 = (T_1 - T_0) e^{-\sqrt{\frac{2h}{\lambda_m e}} x} = (T_1 - T_0) e^{-x/\delta} , \quad \delta = \sqrt{\frac{\lambda e}{2h}}$$




$\delta = 0{\cdot}14 \, m$

ii) With coating.




Heat conduction in the insulating material leads to a linear variation of temperature.

The heat transferred by conduction in the insulating coating must be transferred by convection in air

$$\lambda_i \frac{T_m - T_s}{e} = h\,(T_s - T_o)$$

$$\therefore \quad T_s = \frac{\frac{\lambda_i T_m}{e} + h T_o}{\frac{\lambda_i}{e} + h}$$

The convection heat on both sides becomes now:

$$q_{out} = 2h\,(T_s - T_o) = 2h \frac{T_m - T_o}{1 + \frac{he}{\lambda_i}}$$

$$\frac{he}{\lambda_i} = 10.$$

iii) The rest of the analysis is identical with $\frac{\lambda_m e}{2h}$ being replaced by $11 \frac{\lambda_m e}{2h}$. Now $\delta = \sqrt{\frac{11 \times 2}{2h}} = 0.47$ m.