IB 1999 PAPER 6 - INFORMATION ENGINEERING.

WORKED SOLUTIONS.

i

1. (a) The required condition is:

$$\int_0^\infty |g(t)|\,dt < \infty$$

(i) $$\int_0^\infty |\delta(t-1)|\,dt = 1$$

$\Rightarrow$ asympt. stable

(ii) $$\int_0^\infty \left|\frac{1}{1+t}\right| dt = \Big[\log(1+t)\Big]_0^\infty$$

$$= \infty$$

$\Rightarrow$ unstable.

1.(b)(i)

$$\int_0^\infty |g(t)|\,dt = \int_0^1 1.\,dt = 1$$

$\Rightarrow$ asympt. stable.

By convolution:

$$x(t) = \int_0^t g(\tau)\,u(t-\tau)\,d\tau$$

$$= \int_0^1 1 \,.\, \cos(\omega(t-\tau))\,d\tau$$

$$= -\frac{1}{\omega}\Big[\sin(\omega(t-\tau))\Big]_0^1$$

$$= -\frac{1}{\omega}\Big(\sin(\omega(t-1)) - \sin\omega t\Big)$$

(b)(ii)

$$\text{T.F.} = \mathcal{L}\{g(t)\} = G(s)$$

$$= \int_0^1 1 \cdot e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_0^1$$

$$= -\frac{1}{s}\left(e^{-s} - 1\right)$$

$$= \frac{1}{s} - \frac{e^{-s}}{s}$$

For a <u>stable</u> system the steady-state response to $\cos(\omega t)$ is given by:

$$x(t) = |G(j\omega)| \cos(\omega t + \angle G(j\omega))$$

where $G(j\omega)$ is the <u>frequency response</u>

i.e. $A = |G(j\omega)|$

$\phi = \angle G(j\omega)$

(3)

(b)(ii) contd.

$$G(j\omega) = \left.\frac{1 - e^{-s}}{s}\right|_{s=j\omega}$$

$$= \frac{j\left(e^{-j\omega} - 1\right)}{\omega}$$

$$\Rightarrow \quad A = |G(j\omega)|$$

$$= \frac{1}{\omega}\sqrt{(\cos\omega - 1)^2 + \sin^2\omega}$$

$$= \frac{1}{\omega}\sqrt{2 - 2\cos\omega}$$

$$= \underline{\underline{\frac{2}{\omega}\sin\frac{\omega}{2}}}$$

$$\phi = \arg\left(G(j\omega)\right)$$

$$= 90^\circ + \tan^{-1}\left\{\frac{-\sin\omega}{\cos\omega - 1}\right\}$$

$$= \tan^{-1}\left(\frac{\cos\omega - 1}{\sin\omega}\right)$$

✓

(b)(ii) contd.

Compare with (b)(i):

$$x(t) = -\frac{1}{\omega}\left(\sin\omega(t-1) - \sin\omega t\right)$$

$$= -\frac{1}{\omega}\left((\cos\omega - 1)\sin\omega t - \sin\omega\cos\omega t\right)$$

$$= A\cos(\omega t + \phi).$$

Solving for A & ϕ:

$$A^2 = \frac{1}{\omega^2}\left(\sin^2\omega + (1-\cos\omega)^2\right)$$

$$= \frac{1}{\omega^2}\left(2 - 2\cos\omega\right)$$

(same as (b)(ii)) ✓

$$\tan\phi = \frac{\cos\omega - 1}{\sin\omega}$$

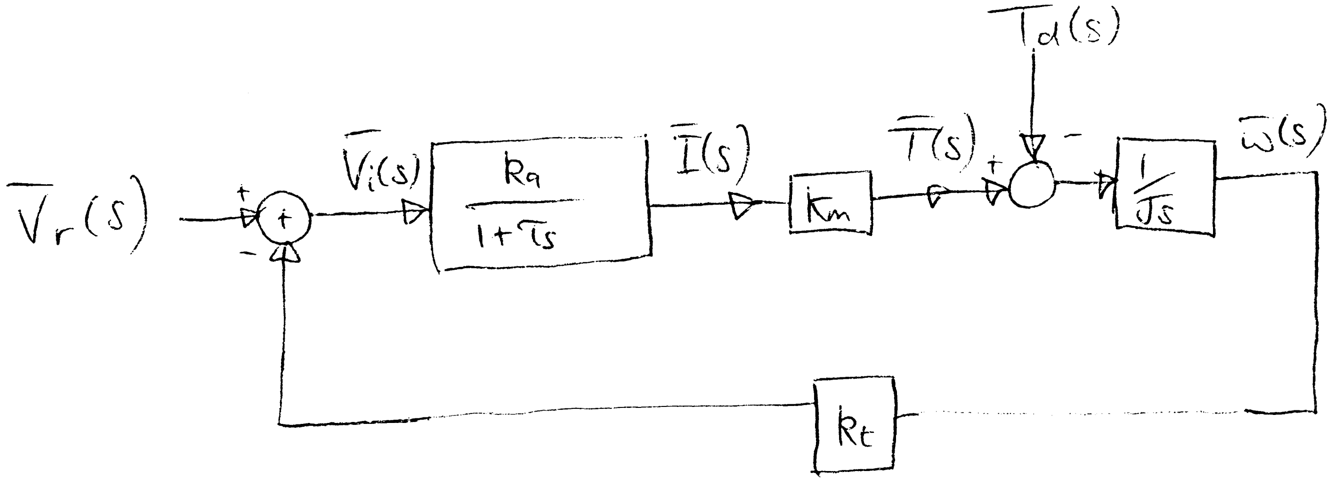
(same as (b)(ii)) ✓

Therefore agreement between (b)(i) & (b)(ii).

2.

(a) $\tau s \bar{I}(s) + \bar{I}(s) = k_a V_i(s)$ $\qquad T - T_d = J\ddot{\theta}$

$$\Rightarrow \bar{I}(s) = \frac{k_a V_i(s)}{1 + \tau s} \qquad \Rightarrow \bar{T}(s) - \bar{T}_d(s) = J s\, \bar{\omega}(s)$$



$$\text{OLTF} = \frac{k_a}{1+\tau s} \cdot K_m \cdot \frac{1}{Js} \cdot k_t$$

$$= \frac{k_a K_m k_t}{Js\,(1+\tau s)}$$

CLTF:

$$\bar{\omega}(s) = \frac{1}{Js}\left(\frac{K_m k_a}{1+\tau s}\left(\bar{V}_r(s) - k_t \bar{\omega}(s)\right) - \bar{T}_d(s) \right)$$

$$\bar{\omega}(s)\left(1 + \frac{k_t K_m k_a}{Js(1+\tau s)}\right) = \frac{K_m k_a}{Js(1+\tau s)} \bar{V}_r(s) - \frac{1}{Js}\bar{T}_d(s)$$

CLTF $\bar{V}_r \to \bar{\omega}$:

$$\frac{K_m k_a}{Js(1+\tau s)\left(1+\frac{k_t K_m k_a}{Js(1+\tau s)}\right)}$$

$$= \frac{K_m k_a}{\left(Js(1+\tau s)+k_t K_m k_a\right)} = \bar{H}_2(s)$$

$\bar{T}_d \to \bar{\omega}$:

$$\frac{-\frac{1}{Js}}{1+\frac{k_t K_m k_a}{Js(1+\tau s)}} = \frac{-(1+s\tau)}{Js(1+s\tau)+k_t K_m k_a}$$

$$= \bar{H}_1(s)$$

(b) Closed loop characteristic eqn :

$$Js(1 + s\tau) + k_t K_m k_a$$

$$= J\tau s^2 + Js + k_t K_m k_a$$

$$\propto s^2 + \frac{s}{\tau} + \frac{k_t K_m k_a}{J\tau}$$

Compare with 2nd order mechanical system $\longrightarrow$ $2c\omega_n s$ $\quad$ ω_n^2

$$\Rightarrow \omega_n^2 = \frac{k_t K_m k_a}{J\tau}$$

i.e. $\frac{1}{\tau} = 2c\omega_n$

and $c = \frac{1}{2\tau\omega_n}$

so : (subst. for ω_n^2) $\quad c^2 = \frac{1}{4\tau^{\cancel{2}}} \frac{J\cancel{\tau}}{k_t K_m k_a} = \frac{J}{0.8\, k_t K_m k_a}$

For $c \geqslant 0.5$:

$$\frac{J}{0.8\, k_t K_m k_a} \geqslant 0.5^2$$

$$0.8\, k_t K_m k_a \leqslant 4J$$

$$k_a \leqslant \frac{5J}{k_t K_m}$$

(c) Set $s = j0$ in CLTF $\bar{V}_r \to \bar{\omega}$:

$$\lim_{t \to \infty} \omega(t) = \alpha H_1(j0) = \frac{\alpha}{k_t}$$ when $T_d = 0$.

Now add in $1 \times H_2(j0)$:

response to $T_d = H(t)$

$$\lim_{t \to \infty} \omega(t) = \frac{\alpha}{k_t} + -\frac{1}{k_t K_m k_a}$$

$$= \frac{1}{k_t}\left(\alpha - \frac{1}{K_m k_a}\right)$$

3.
(a) Closed loop T.F. is obtained as:

$$\bar{y}(s) = G(s)\left(\bar{x}(s) - K(s)\,\bar{y}(s)\right)$$

$$\frac{\bar{y}(s)}{\bar{x}(s)} = \frac{G(s)}{1+G(s)K(s)} = \text{CLTF}$$

<u>CLCE is</u>

$$1 + G(s)K(s) = 0$$

<u>Closed - loop poles:</u>

Solutions of CLCE, i.e. values of s for which $1 + G(s)K(s) = 0$.

(b) $$1 + \frac{1}{s^2+1} \cdot k \cdot \frac{s+1}{s+a} = 0 \quad \leftarrow \text{CLCE}$$

$$\Rightarrow (s^2+1)(s+a) + k(s+1) = 0$$

Set: $s = -1 + 2j$:

$$(1 + 1 + -4j - 4)(a - 1 + 2j) + k.2j = 0$$

$$\Rightarrow (-2 + -4j)((a-1) + 2j) + k.2j = 0$$

ii

3.

Real Part:

$$-2(a-1) + 8 = 0$$

$$\Rightarrow a - 1 = 4$$

$$\underline{\underline{a = 5}}$$

Imag. Part:

$$-4 - 4(a-1) + 2k = 0$$

$$-4 - 4 \times 4 + 2k = 0$$

$$\Rightarrow 2k = 20$$

$$\underline{\underline{k = 10}}.$$

(c) Construction of Bode Plot as follows:

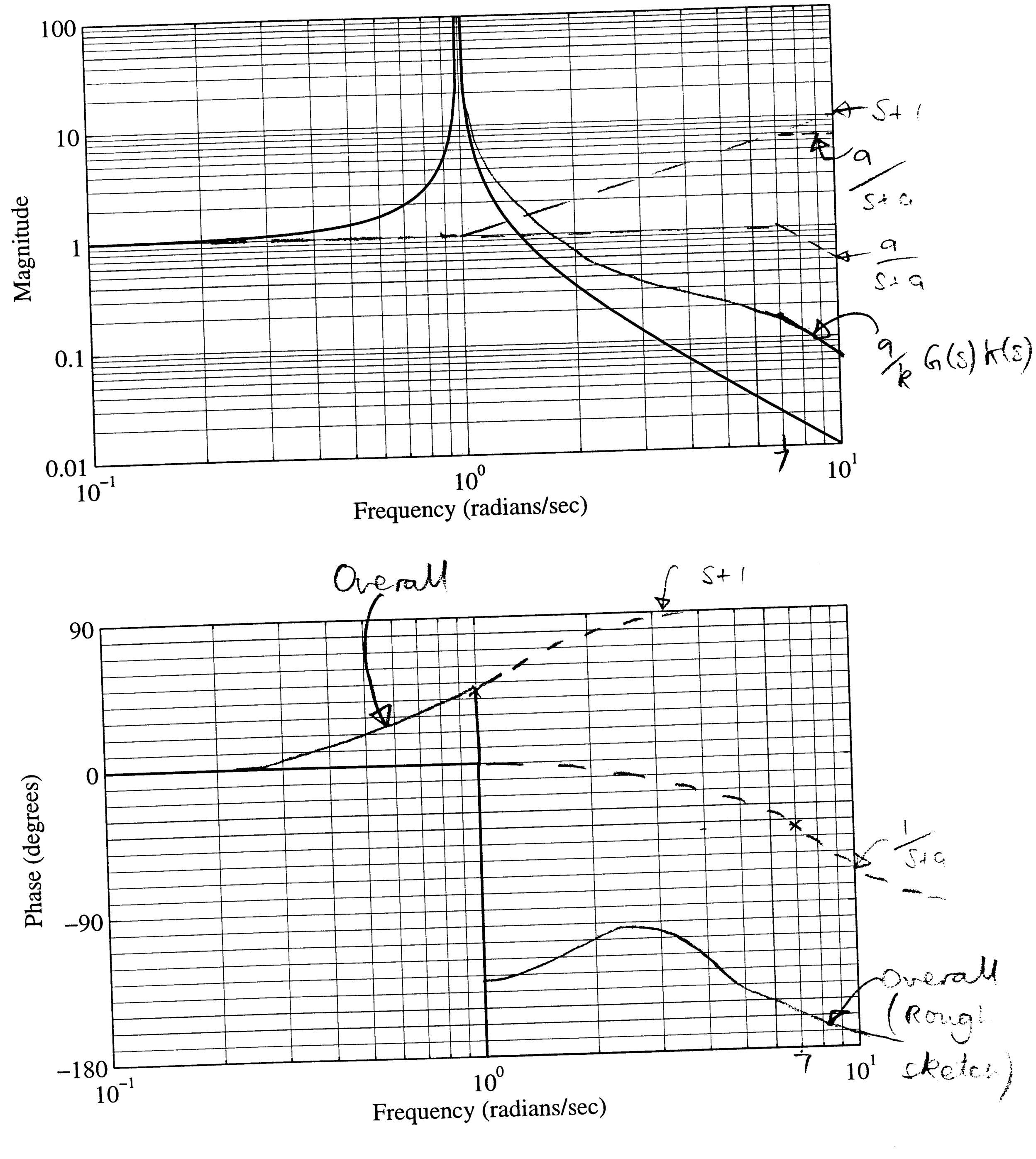


Fig. 3

(TURN OVER

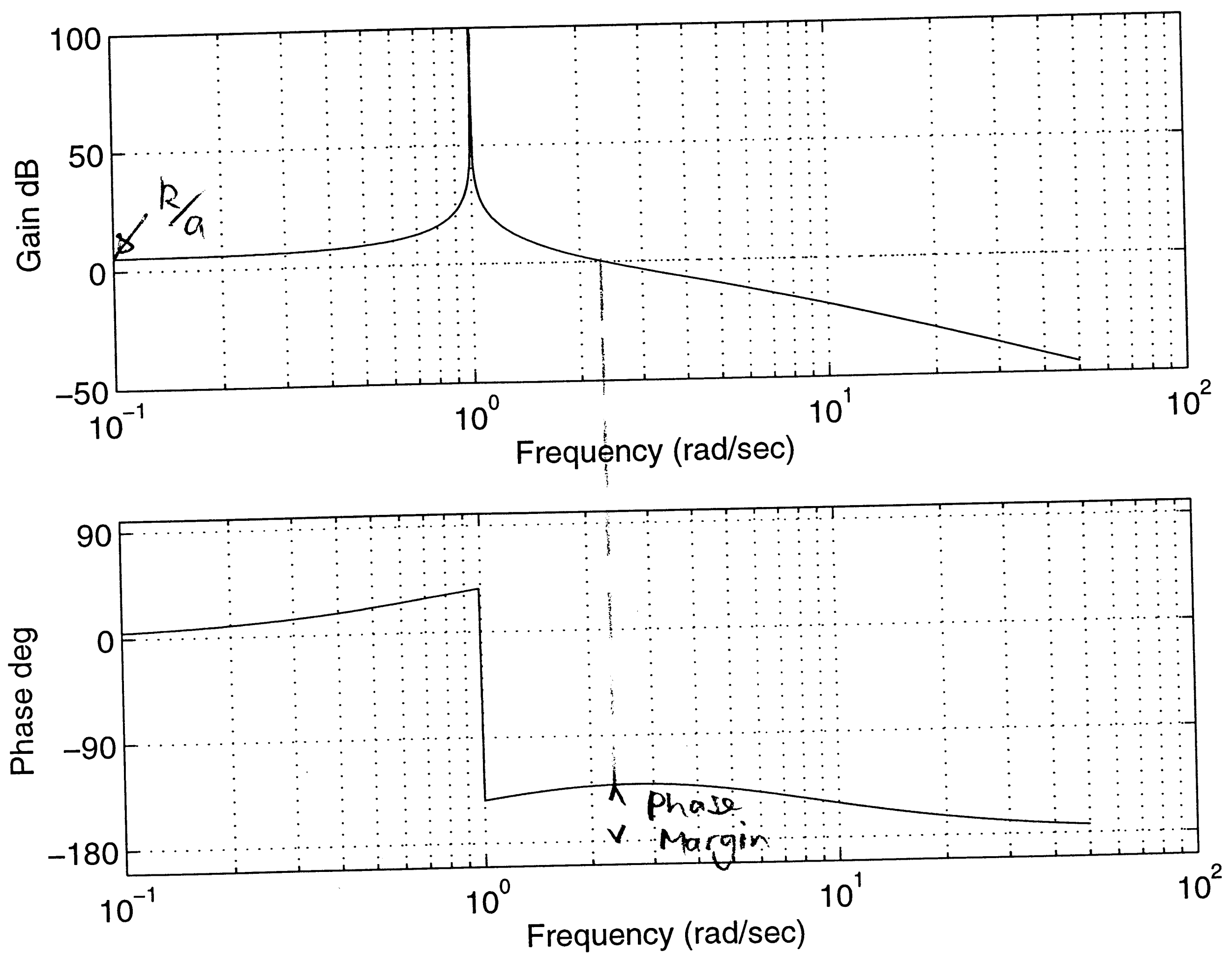
 iv

More accurate plot below:

Phase margin corresponds to 0dB gain point.

$\omega = 2.34$, gain $= 1.00$, phase $= -132^\circ$

Hence Phase margin $= 180 - 132 = \underline{\underline{48^\circ}}$

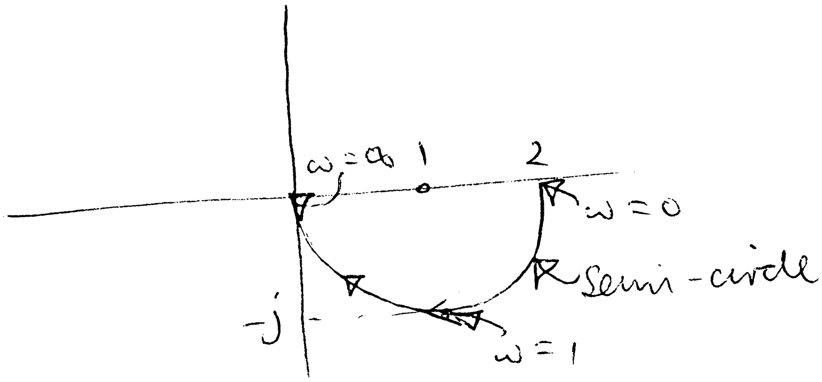


i

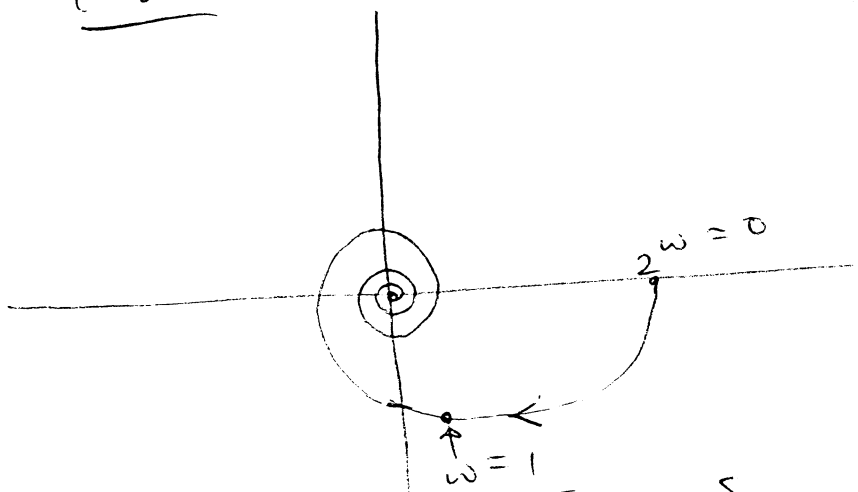
4. (a) The closed-loop system is stable if, and only if, the complete Nyquist diagram of $K(s)\,G(s)$ does not encircle the point '-1'.

(b) (i) $\tau = 0$

$$G(s)\,K(s) = \frac{2}{s+1}$$



$\tau = 0.5$



ii) Determine when $|G(j\omega)K(j\omega)| = 1$:
$2|G(j\omega)| = 1$

when $$\frac{4}{+\omega^2+1} = 1$$

$$\Rightarrow 1+\omega^2 = 4$$
$$\Rightarrow \omega = \sqrt{3}$$

$$\angle G(j\sqrt{3}) = -\tan^{-1}\sqrt{3} = -\pi/3$$

Therefore $e^{-j\omega T_{max}}$ has argument $-2\pi/3$:

$$\Rightarrow \sqrt{3}\,T_{max} = 2\pi/3$$

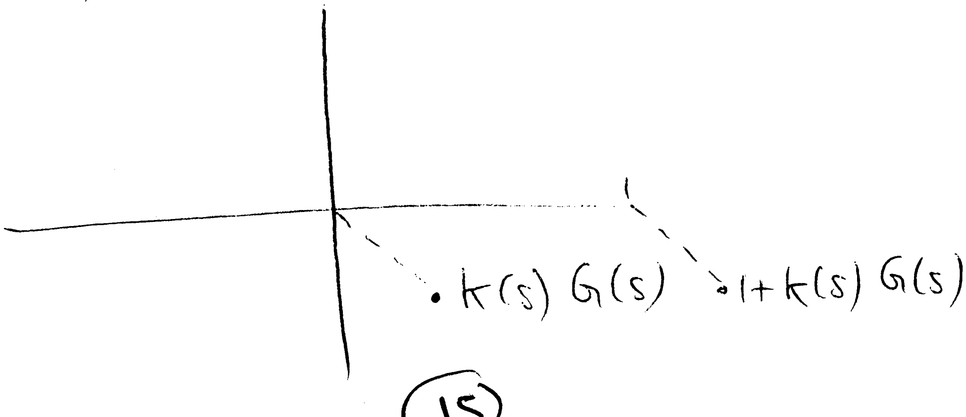
$$T_{max} = \frac{2\pi}{3\sqrt{3}} = \underline{\underline{1.209}}$$

iii) Closed loop gain is

$$\left|\frac{K(s)\,G(s)}{1+K(s)\,G(s)}\right| \qquad (s = j\omega)$$

Gain is 1 when

$$|K(s)\,G(s)| = |1+K(s)\,G(s)|$$



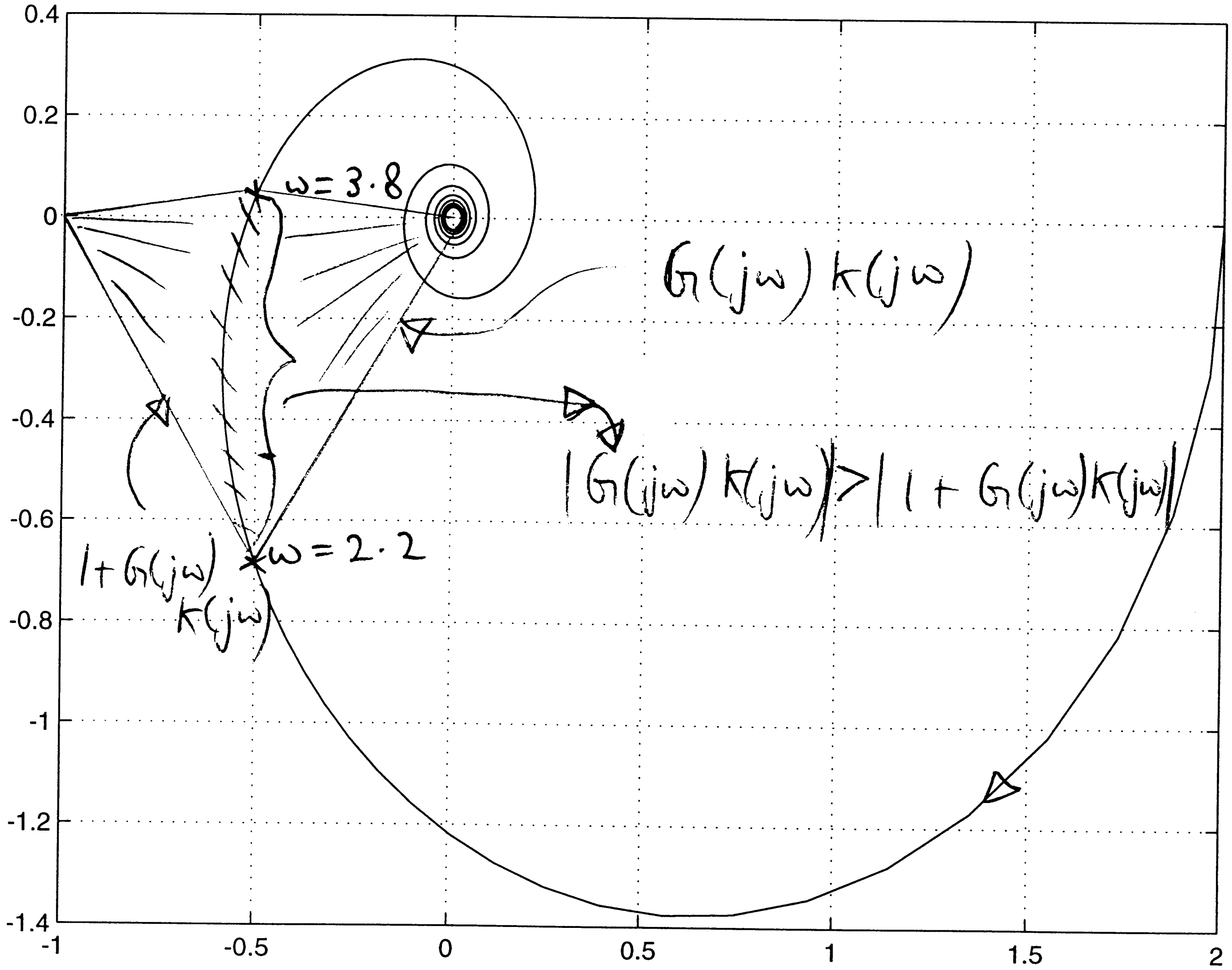
Lengths are equal only when $Re\left(K(j\omega)G(j\omega)\right) = -0.5$

By trial and error, this occurs when $\omega \approx 2.2$ and 3.8 rad s^{-1}.

Hence range of values is $\underline{\underline{2.2 < \omega < 3.8}}$

(See Nyquist diagram overleaf).

Nyquist plot for $\tau = 0.5$:



5.

(a) Image information is carried by scanning the image in a succession of horizontal scan lines.

lines/frame - the total number of lines in the frame. 625 is chosen to give satisfactory 'resolution' (fineness of detail) in the vertical direction. More would require higher bandwidth.

frames/s - 25 frames (complete sets of 625) lines are transmitted each second to give satisfactory perception of smooth motion (fewer would be 'jerky').

2:1 interlace - transmitting odd lines (in 1/50 s) followed by even lines (in 1/50 s) - this 'paints' screen once every 1/50 s, reducing flicker and tearing during motion.

aspect ratio - horizontal size/vertical size of image. Chosen for aesthetic reasons. (New HDTV standard is 16:9).

'line synch pulses' cause the receiver to start each line scan at a uniform leftmost position (off screen) in order to align the lines horizontally.

'frame synch pulses' cause the receiver to start vertical downward scanning of the odd lines and even lines in alternate interlaced scans correctly.

'blanking periods' are there between lines (and therefore containing the synch pulses) because at the end of each line [/frame] the circuitry must cease accurate linear scanning and 'flyback' ready to start the next line [/frame].

5.

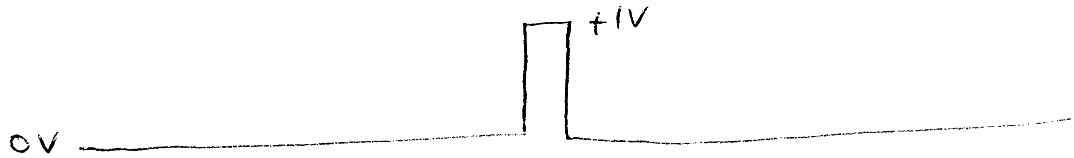
(b) Pixels / line $= 625 \times 4/3$

Pixels / frame $= 625 \times 4/3 \times 625$

Pixels / second $= 625 \times 4/3 \times 625 \times 25$

$= \underline{\underline{1.302 \times 10^7}}$

(c) Each line has the form:



which is periodically repeated every line.

Line period is $\dfrac{625 \times 4/3}{625 \times 4/3 \times 625 \times 25}$ s

$= \dfrac{1}{625 \times 25}$ s

From Data book, p. 24 EI:

$$g(t) = \frac{t_d}{T}\left[1 + 2\sum_{n=1}^{\infty} \frac{\sin(n\pi t_d/T)}{(n\pi t_d/T)}\cos(n\omega_0 t)\right]$$

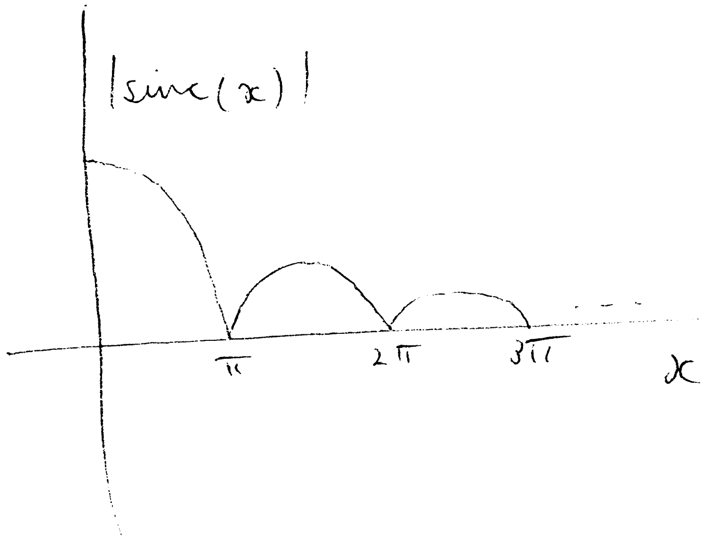
where $\quad 1/T = 625 \times 25$

$$\omega_0 = \frac{2\pi}{T} = 2\pi \times 625 \times 25 \text{ rad/s}$$

$$t_d/T = \frac{3}{625 \times 4}$$

i.e. harmonics of frequency 15.625 kHz.
with amplitudes proportional to $\operatorname{sinc}\left(n\pi \, t_d/T\right)$
for $n \geq 1$

(d)



Various bandwidth criteria possible, e.g. find '3dB' point of Fourier series components.

Now, $\operatorname{sinc}(0.45\pi)$ is just less than $\frac{1}{\sqrt{2}}$. Therefore, setting $0.45\pi \geq n\pi \, t_d/T$

gives $n = 375$.

$\Rightarrow$ Take $BW = 375 \times 15.625 \text{ kHz} = \underline{5.86 \text{ MHz}}$
('0.5 × pixel rate' rule is also acceptable.)

5. (e) Limiting the bandwidth will cause ir
'blurring' at vertical edges.

However, horizontal edges are transmitted at the line frequency, hence a much smaller bandwidth is required and no blurring is observed.

6. (a) (i) Speech is intelligible, but significant amounts of the low frequency content of male speech is lost. More importantly, high frequency information is removed which makes it harder to distinguish 's'/'f' and 'd'/'t'.

(ii) Music is substantially degraded because hifi music requires a passband from about 40 Hz → at least 10 kHz. Bass notes and high frequency notes and harmonics will be removed, 'dulling' the sound.

(b) $\omega_c = 200\pi \times 10^3$ rad./s

Restrictions are :

$$a_0 > \mathrm{Max}\left(\mathrm{abs}\left(x(t)\right)\right)$$

to avoid overmodulation, i.e. keeps modulation index $\left(\frac{\mathrm{Max}(\mathrm{abs}(x(t)))}{a_0}\right) < 1$.

a_0 not unnecessarily large to avoid power wastage.

(c) With $x(t) = \cos(\omega_M t)$:

$$s(t) = (a_0 + x(t))\cos(\omega_c t)$$

$$= (a_0 + \cos(\omega_m t))\cos(\omega_c t)$$

$$= \underbrace{a_0 \cos \omega_c t}_{\text{Carrier}} + \underbrace{0.5 \cos((\omega_c - \omega_M)t)}_{\text{Lower sideband}} + \underbrace{0.5 \cos((\omega_M + \omega_c)t)}_{\text{Upper sideband}}$$

(d) $\omega_M = 2\pi \times 10^3$ rad/s

$$\omega_c - \omega_M = 2\pi \times (10^5 - 10^3) \text{ rad/s}$$

$$\equiv \underline{\underline{99 \text{ kHz}}}$$

$$\omega_c + \omega_M = \underline{\underline{101 \text{ kHz}}}$$

Telephone BW = 3,400 Hz.

$\therefore$ Passband is 100 - 3.4 kHz → 100 + 3.4 kHz

i.e. 96.6 kHz → 103.4 kHz

Suitable carrier frequency spacing makes allowance for non-ideal bandlimiting filters (i.e. transition band). Assume a total bandwidth of 4 kHz (other reasonable values acceptable) to give a spacing of 2 × 4 kHz = <u>8 kHz</u>

(e) Demodulation :

Channel selection filter with passband 96-104 kHz.

Then use envelope detector (diode, R, C circuit).

AC coupling (DC / mean level removal) required at the output.