ENGINEERING TRIPOS PART IB

Monday 31 May 1999 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right hand margin.*

(TURN OVER

1 A piston P drives a crank OA of length 50 mm through a connecting rod PA of length 150 mm as shown in Fig. 1. If the crank rotates at a constant angular velocity ω, find expressions for the acceleration of the piston (a) when the angle POA is $90°$; (b) when the angle OAP is $90°$. [12]

If the connecting rod and crank may be treated as uniform bars, each of mass m, and the mass of the piston may be neglected, find the relation between the force acting on the piston and the torque exerted by the crank in each of the above two configurations. [8]

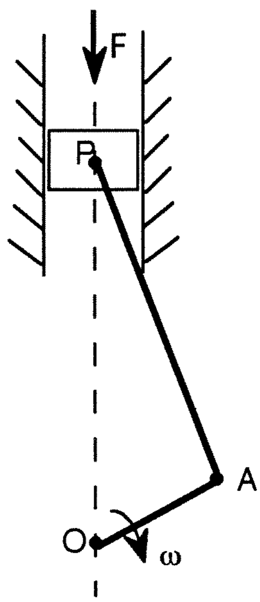

Fig. 1

2 A column of mass m and length 2ℓ is being slowly hauled into position by pulling it against a stop as shown in Fig. 2a, but when the column is inclined at an angle α to the vertical the supporting rope breaks and the column falls to the ground.

Treating the column as a uniform rod, and assuming that the 'fall' consists of rotation about the stop, show that when during its fall the column is inclined at θ to the vertical, its angular velocity is given by

$$\dot{\theta}^2 = \frac{3}{2}\frac{g}{\ell}(\cos\alpha - \cos\theta).$$ [4]

Show that the maximum bending moment in the column is always at a distance of $(2\ell / 3)$ from the stationary end, and find its value. [10]

Show that if there is no friction between the column and the ground, the column will lose contact with the stop before hitting the ground, and hence find the greatest bending moment occurring while the above analysis is valid. [6]

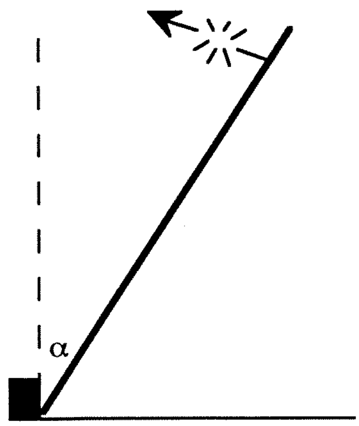

Fig. 2a

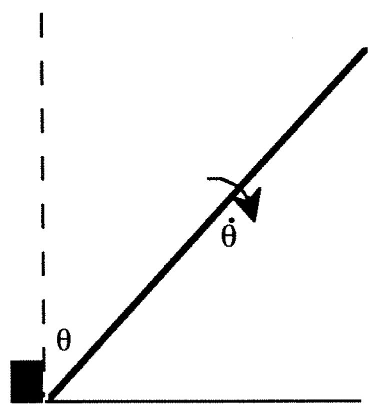

Fig. 2b

3 A bead of mass m slides on a frictionless ring of radius R : while the ring is rotated about a *vertical* axis through a diameter with a constant angular velocity Ω.

By using the rotating unit vectors $\mathbf{e}, \mathbf{e}^*$ as shown in Fig. 3, where $\mathbf{e}$ is in the plane of the ring and $\mathbf{e}^*$ normal to it, or otherwise, obtain the kinematic equations for the acceleration of the particle when it is at a position θ measured from the downward vertical. Find the force exerted by the bead on the hoop in the $\mathbf{e}^*$ direction, and show that the angular acceleration satisfies [5]

$$\ddot{\theta} = \Omega^2 \sin\theta \cos\theta - \frac{g}{R}\sin\theta$$ [5]

Hence obtain an equation for the angular velocity $\dot{\theta}$, given that the particle is initially at rest at the point $\theta = \alpha$. [3]

Verify that the equation for $\dot{\theta}^2$ contains a factor $(\cos\alpha - \cos\theta)$, and obtain an expression for any other positions at which the particle could be at rest. Discuss your answer for the cases of very small and very large values of the ring angular velocity Ω. [3] [4]

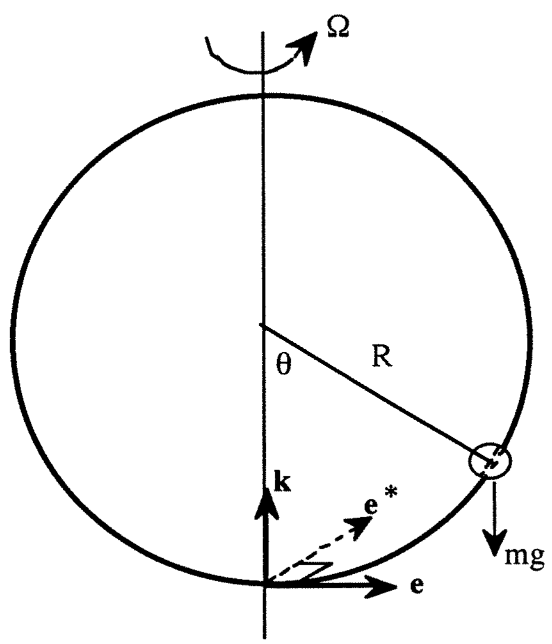

Fig. 3

4 During the erection of a box girder bridge, a horizontal bridge deck AB of length 2ℓ , which may be treated as a uniform bar, is standing on top of two vertical piers. Because of a construction error, end A slips off its pier, and the deck then rotates about the top B of the other pier until A hits the ground, at which point its angular velocity is ω_1.

After the impact, end A remains in contact with the ground, and the deck now rotates about A. Show that if the only impulsive reaction were at A, then the rotation of the deck would be *in the opposite sense* to that before impact. What does this result imply for the supporting tower? [9]

In fact the failure resulted from a buckle occurring at mid-span. Assume now that during the fall there is enough residual strength at the buckle for the deck to behave as a rigid body (Fig. 4a); but that on impact the buckle C becomes effectively a frictionless pivot (Fig. 4b).

Analyse the motion immediately after impact, again assuming no impulse at B. [11]
Assume all angles are small in your analyses.

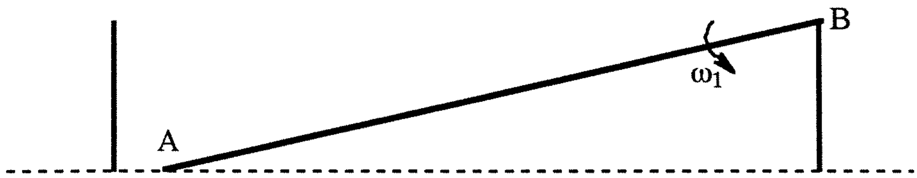

Fig. 4a

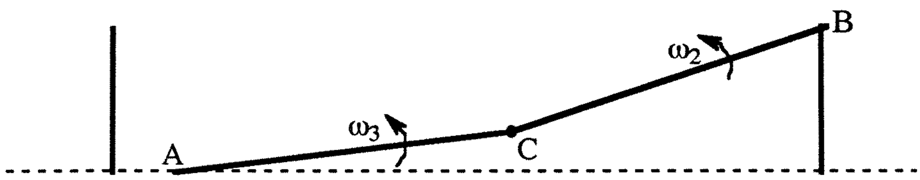

Fig. 4b

(TURN OVER

5 Show from first principles that the moment of inertia about a diameter of a thin-walled hollow sphere of mass m' and radius r is equal to $\frac{2}{3}m'r^2$. Deduce the moment of inertia of a solid sphere of mass m and radius R. [5]

A billiard ball of mass m and radius a receives an impulse P at the end of a horizontal diameter but inclined at an angle θ to the horizontal as shown in Fig. 5. If the coefficient of friction between the ball and the table is μ , show that the condition that the ball *initially* rolls is $\mu \geq \frac{5}{7} + \frac{2}{7}\cot\theta$. [5]

If the angle at which the impulse is applied is less than this value, find the total amount of energy lost by the ball before it begins to roll. [10]

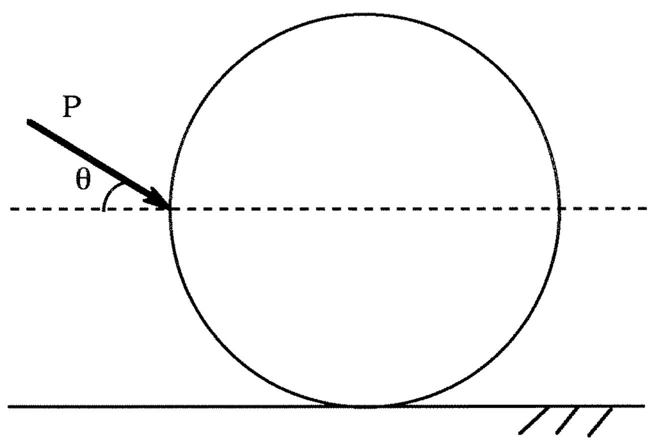

Fig. 5

6 (a) Three rotors A, B and C are fixed to a shaft, each separated by a distance of 100 mm, as shown in Fig. 6. The "out-of-balance" of rotors A, B and C are respectively 0.008 kg m, 0.015 kg m and 0.017 kg m. Determine the relative angular position of the rotors on the shaft required in order to achieve static balance. In this statically-balanced configuration, what is the magnitude of the dynamic out-of-balance couple when the shaft is spinning at 400 rad/s? How could this be reduced, and to what value? [6] [4]

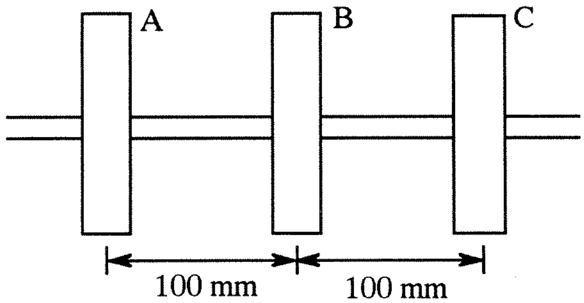

Fig. 6

(b) Explain the existence of a *gyroscopic moment,* and derive its value. Indicate clearly on a sketch the direction in which it acts. [5]

The shaft of the thrust bearing shown in Fig. 7 rotates with angular velocity Ω. The balls, of radius r, roll without slip round a circle of radius R. Find an expression for the gyroscopic moment on a ball. [5]

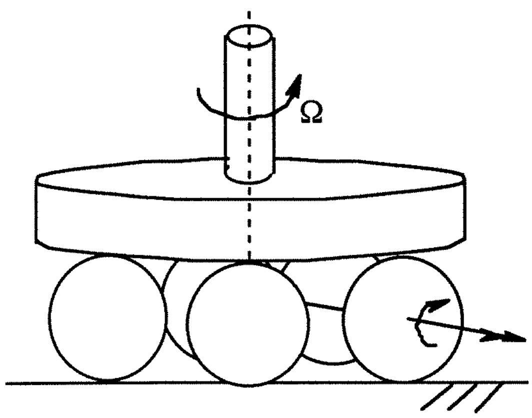

Fig. 7a

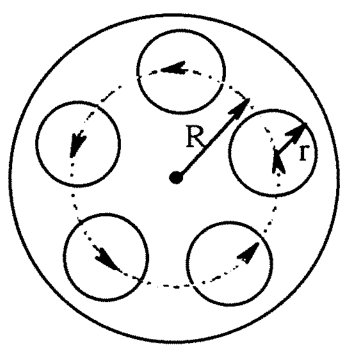

Fig. 7b

END OF PAPER