ENGINEERING TRIPOS PART IB

Monday 31 May 1999 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

(TURN OVER

1 Fig. 1(a) shows a cantilever AB constructed using a hollow steel box-beam of uniform cross-section as shown in Fig. 1(b). The cantilever is rigidly built-in to the wall at A. A small rigid arm BC extends out horizontally 1 m from the vertical centreline of the box-beam and supports a single vertical point load P of 100 kN at C.

(a) Calculate the bending moment, shear force and torque at A. Assume the self-weight of the box-beam is 2 kN/m and the self-weight of the end arm BC is negligible. [3]

(b) Determine the angle through which the end section at B rotates about the longitudinal axis of the box-beam due to this loading. [3]

(c) Calculate the maximum longitudinal bending stress immediately adjacent to the support at A. [4]

(d) At this same cross-section adjacent to the support at A, determine the maximum shear stress in the weld at S located at the top of the web of the box-beam (Fig. 1(b)), immediately below the top flange due to:

(i) the shear force alone; [5]

(ii) the applied torque alone. [5]

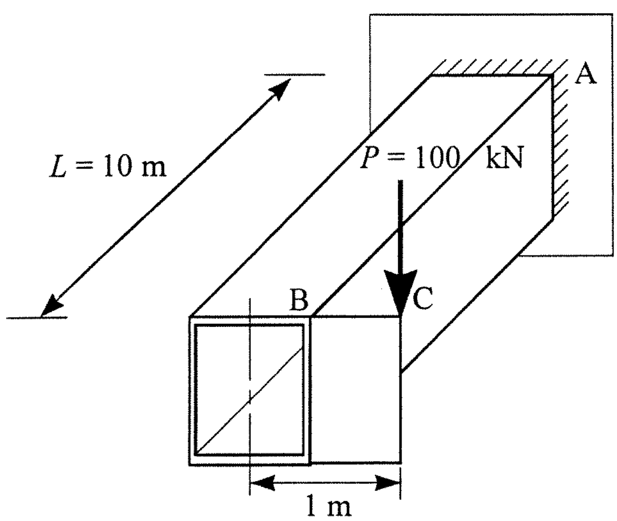

Fig. 1(a)

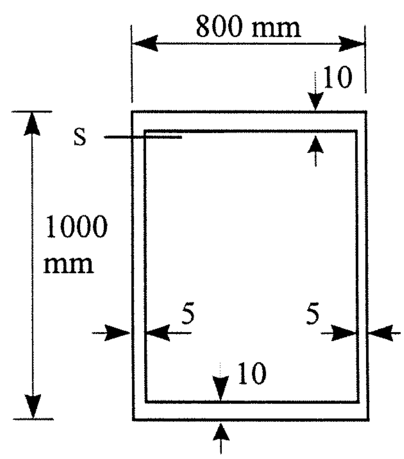

Fig. 1(b)

2 Fig. 2 shows the cross-section of a submarine's long, hollow, thin-walled steel propeller shaft with radius 250 mm and wall thickness 10 mm. When the submarine is travelling at a normal cruising speed of 10 knots the shaft is subjected to a torque of 250 kNm and axial compression of 1500 kN.

(a) Assuming the submarine is on the surface and hence the effect of the water pressure surrounding the shaft is negligible:

(i) calculate the shear stress and longitudinal stress acting at a typical location on the shaft; [4]

(ii) draw Mohr's circle of stress at this point; [5]

(iii) determine the principal stresses at this point and their orientation to the longitudinal axis of the shaft. [5]

(b) The submarine continues to travel at 10 knots, having submerged to a depth of 300 m. Calculate the maximum shear stress in the shaft. (Assume the cylindrical propeller shaft has closed ends and its interior remains unpressurised). [6]

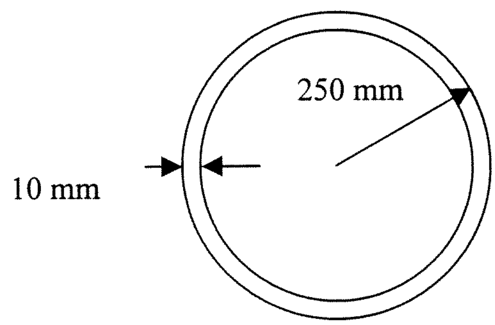

Fig. 2

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3 (a) Which, if any, of the four yield-line patterns shown in Fig. 3(a) are *not* geometrically compatible failure mechanisms? For any invalid mechanism sketch a compatible failure mechanism which closely resembles that shown in the diagram. [4]

(b) A rectangular reinforced concrete bridge slab ABCD shown in Fig. 3(b) is simply supported along abutments BC and AD. The bridge is required to carry a single lorry axle, represented by two point loads each of magnitude P, at mid-span in the position shown. The slab is reinforced so that its moment capacity in sagging and hogging anywhere in the slab is m per unit width. The self-weight of the slab may be ignored.

Calculate the collapse load, P of the bridge in terms of the moment capacity m by considering a failure mechanism comprising a single transverse yield-line HJ across the full-width of the bridge at mid-span, as shown in Fig. 3(b). [3]

(c) The alternative failure mechanism shown in Fig. 3(c) is now considered. Point G is fixed whereas the position of the yield-line EF is defined by the parameter α.

(i) For this pattern show that the collapse load is given by: [4]

$$P = m(1 + \alpha + \frac{2}{\alpha})$$

(ii) Find the least upper bound value for P in terms of the moment capacity m for this mechanism. [5]

(iii) What is your best estimate of the collapse load, P of this bridge? [1]

(d) If the reinforced concrete slab was replaced by a solid granite stone slab with identical dimensions but with moment capacity everywhere equal to $m/2$, would you still be able to apply yield-line theory to determine the collapse load? Briefly explain your answer. [3]

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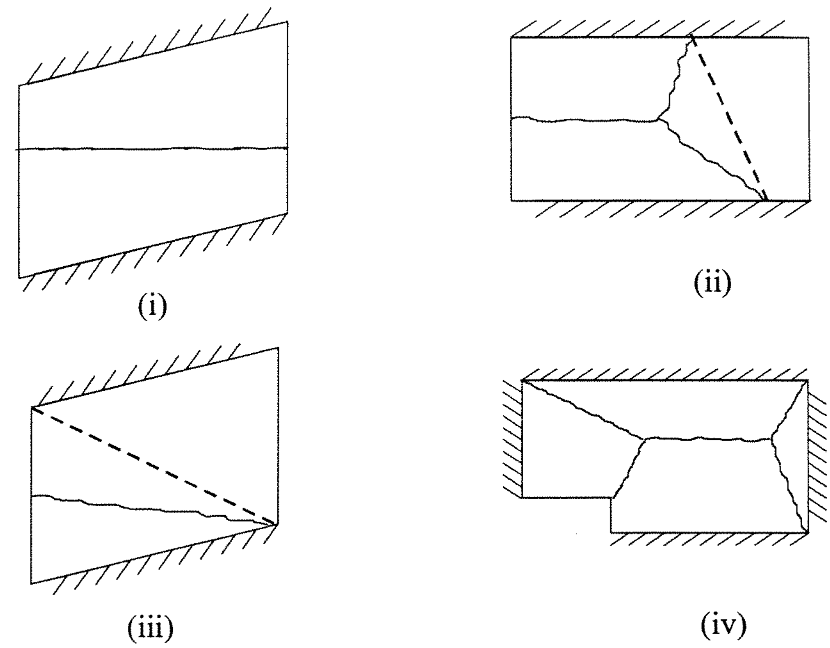

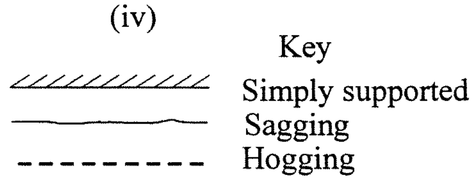

Fig. 3(a)

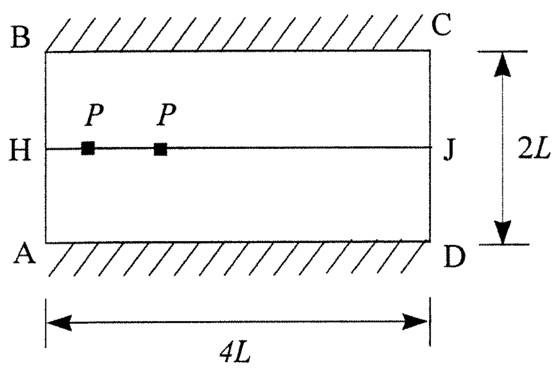

Fig. 3(b)

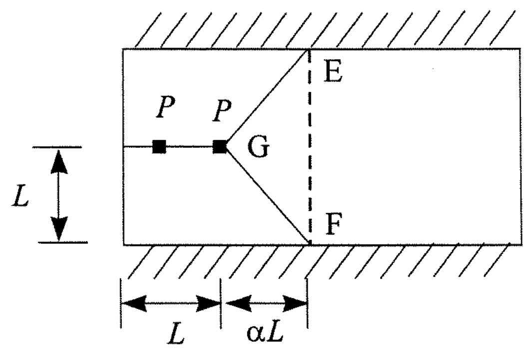

Fig. 3(c)

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4 The pin-ended steel strut shown in Fig. 4(a) is required to carry an axial load P of 1500 kN without failing due to either yielding anywhere in the cross-section or buckling in any lateral direction. For this strut the yield stress, σ_y is 350 N/mm^2, Young's Modulus, E is 210 GPa and the length, L is 6 m.

(a) Find the smallest universal column section from the tables in the Data Book that will satisfy this requirement, assuming a perfectly straight strut with no initial lateral imperfections. [5]

(b) The required minimum failure load is increased to 3750 kN. The following two strengthening options have been suggested:

Option 1: Both ends of the column are fully fixed against rotation in any direction as shown in Fig. 4(b).

Option 2: The bottom support is fully fixed against rotation in any direction as in Fig. 4(c). (For this option assume the effective length of the strut is $0.7L$).

For each of these two options:

(i) Sketch the likely buckling mode and show the effective length of the strut. [2]

(ii) What would be the buckling load of the strut, assuming the same column section as determined in part (a) above? [2]

(iii) Would failure be governed by buckling or yielding? [2]

(iv) Would the strengthening option be adequate for the increased load? [2]

(c) The original pin-ended strut in Fig. 4(a) was found to have an initial lateral imperfection at mid-height of δ_0 = 10 mm as shown in Fig. 4(d). The initial lateral imperfection is assumed to be of the form $v = \delta_0 \sin(\pi z/L)$. Assume the same column section as in parts (a) and (b) above.

(i) At what axial load would the lateral deflection at mid-height reach 20 mm? [4]

(ii) Determine the maximum stress in the column at this load. [3]

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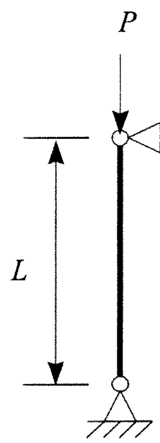

Fig. 4(a)

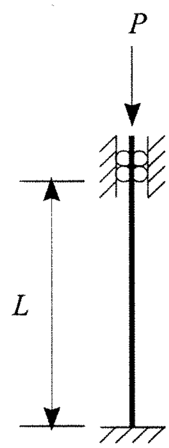

Fig. 4(b)

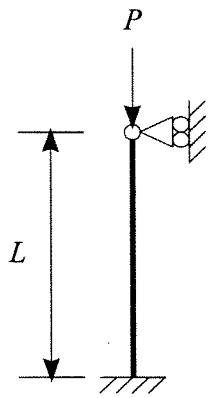

Fig. 4(c)

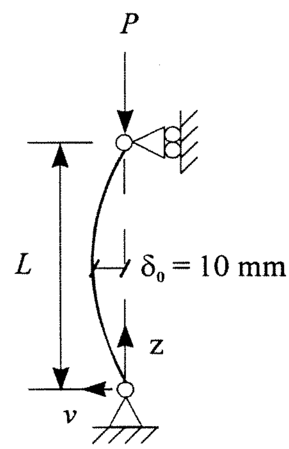

Fig. 4(d)

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5 (a) Determine the number of force redundancies in each of the two-dimensional structures shown in Fig. 5(a). [3]

(b) The beam shown in Fig. 5(b) has flexural rigidity *EI*. Calculate the vertical reactions at the supports and draw the shear force diagram, marking in salient values on the plot. Assume the beam is initially stress free. [8]

(c) Due to consolidation of the supporting ground the support at B settles downwards by a distance δ where

$$\delta = \frac{17WL^3}{12EI}$$

By how much will the reaction at B be reduced? [5]

(d) In addition to the settlement of support B by δ, the ground movement results in a rotation about A of the wall supporting the beam. What rotation of the support at A will result in the reaction at B being equal to zero? [4]

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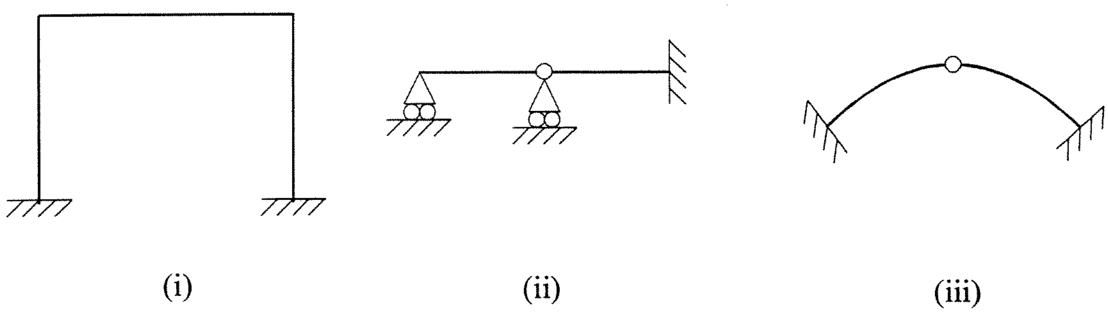

Fig. 5(a)

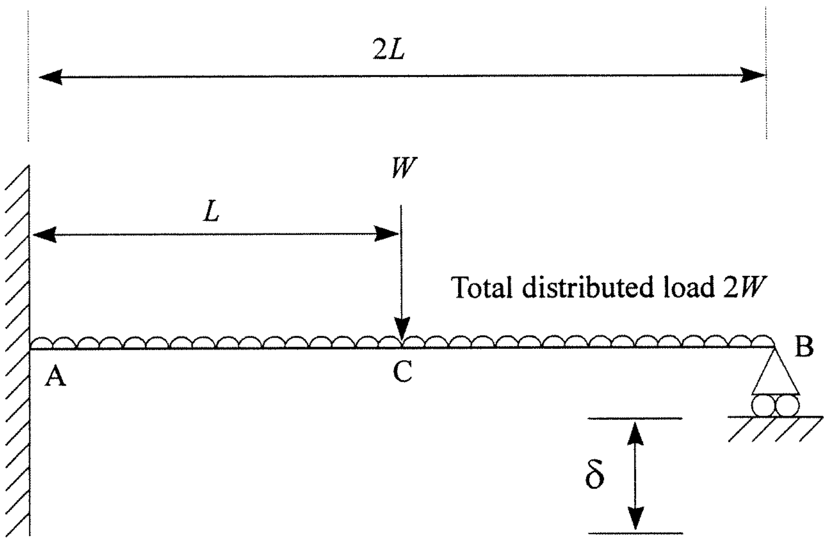

Fig. 5(b)

(TURN OVER

6 The structure shown in Fig. 6(a) is constructed using a single steel section which runs horizontally from A to B and then vertically from B to C. The plastic moment capacity of the section is assumed to be constant and equal to M_p along its entire length from A to C, including the joint at B. Point loads of magnitude W are applied vertically at D and horizontally at C. A total load of $2W$ is uniformly distributed along the beam from A to B.

(a) Derive expressions for the collapse load, W for the following two postulated mechanisms:

(i) A single plastic hinge forms at B. [2]

(ii) A single plastic hinge forms at D under the point load. For this second mechanism find the critical position x of the point load W along the beam AB for the lowest estimate of collapse load. [9]

(b) Draw the resulting bending moment diagram at collapse for the critical failure mechanism. [9]

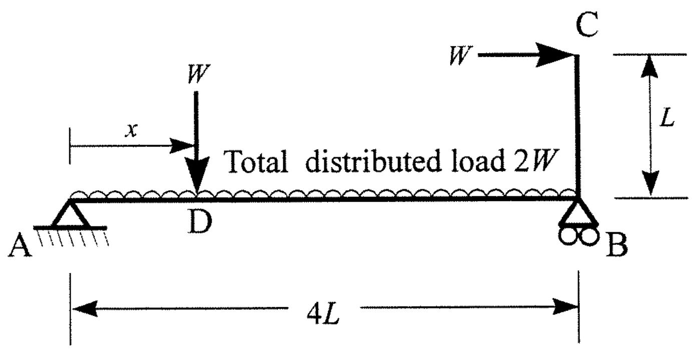

Fig. 6(a)

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