ENGINEERING TRIPOS   PART IB

Tuesday 1 June 1999   9 to 11

Paper 3

MATERIALS

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

(TURN OVER

1 (a) Explain briefly the difference between *heterogeneous* and *homogeneous* nucleation in liquids. Discuss the factors which determine the rate at which a liquid transforms *homogeneously* to a solid, using graphs of the key variables to illustrate your answer. [8]

(b) Show that the critical radius r^* for homogeneous growth of a spherical nucleus of a solid phase in a liquid is given by

$$r^* = \frac{-2\gamma T_E}{\Delta H} \frac{1}{(T_E - T)}$$

where γ is the surface energy per unit area of the solid phase, ΔH is the change in enthalpy on solidification per unit volume of solid, T_E is the equilibrium temperature and T is the temperature. [4]

(c) The critical number of molecules required to achieve *homogeneous* nucleation of ice in clean water contained in a beaker is 175. Given that the surface energy between ice and water is 0.025 Jm^{-2}, the enthalpy change *per unit mass* of water on freezing is –335 $kJkg^{-1}$ and the density of ice is 0.92 Mgm^{-3}, determine the undercooling at which *heterogeneous* nucleation of ice takes place on the beaker walls if the angle of contact between the ice and the beaker is 10°, as shown in Fig. 1. You may assume that the volume of a spherical cap, V, is given by

$$V = \frac{2\pi r^3}{3}\left(1 - \frac{3}{2}\cos\theta + \frac{1}{2}\cos^3\theta\right)$$

where r is the radius of the sphere and θ is the angle between the base of the cap and the arc. The relative atomic weights of hydrogen and oxygen are 1.01 and 16.00, respectively. Avagodro's Number is 6.02×10^{23} mol^{-1}.

Comment on the difference between the degree of undercooling required for heterogeneous and homogeneous nucleation of ice in water. [6]

(d) Describe briefly one example of how controlled nucleation is used to determine material properties in a practical fabrication process. [2]

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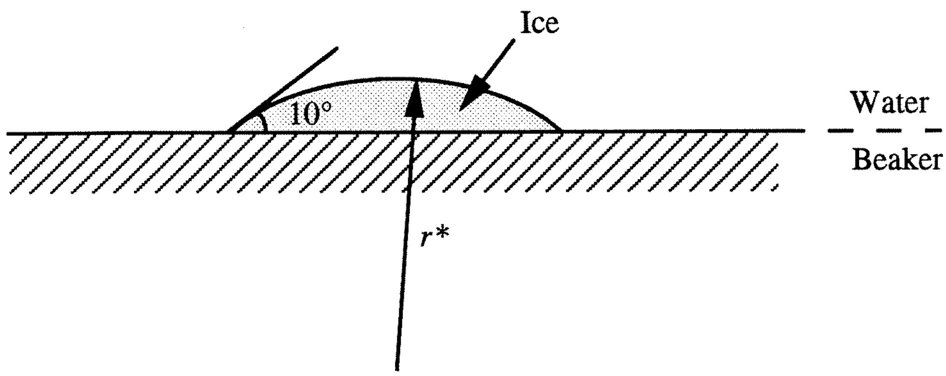

Fig. 1

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2 (a) Describe in detail the factors which account for the differences in hardness and ductility between metals and ceramics. Why are statistics required to characterise failure in ceramics? [6]

(b) Figure 2 shows a Weibull plot of survival probability $P_s(V)$ against fracture stress for SiC samples of volume 50 mm^3 and identical geometry under tensile stress. Describe carefully an experiment to generate this data and explain why the axes of the plot are constructed in this way. Use Fig. 2 to estimate the Weibull constants m and σ_0 for SiC. Determine the mean strength of SiC samples which have a volume 5 times greater than those used in the test. [6]

(c) Determine the survival probability for a batch of SiC samples of uniform cross-section 5 mm × 5 mm and length 10 mm when subject to a tensile load of 4 kN, as shown in Fig. 3(a). [4]

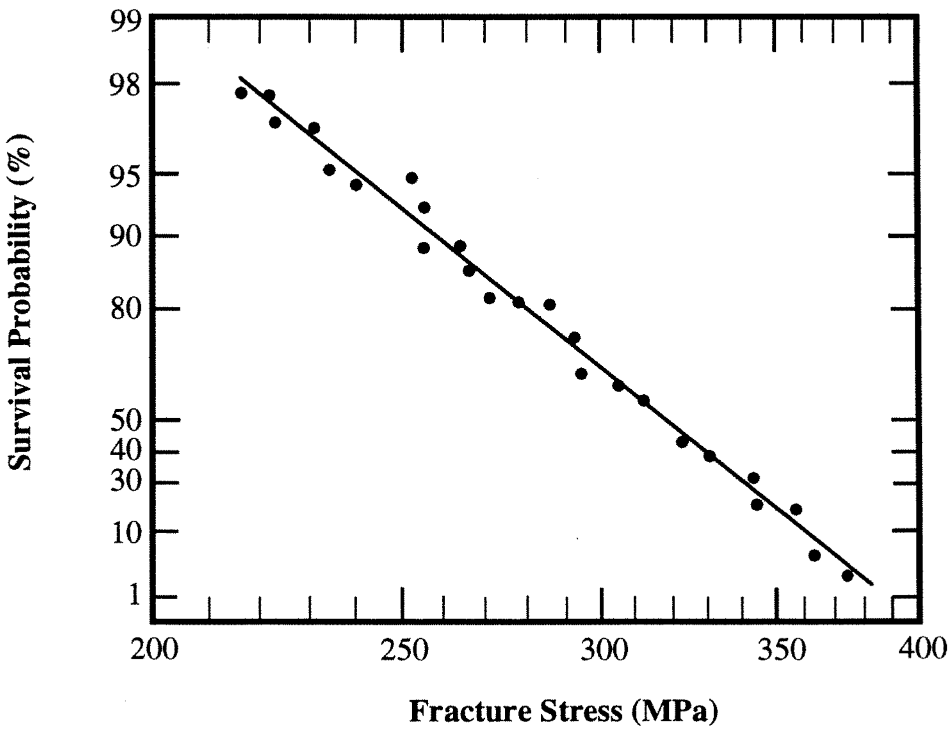

Fig. 2

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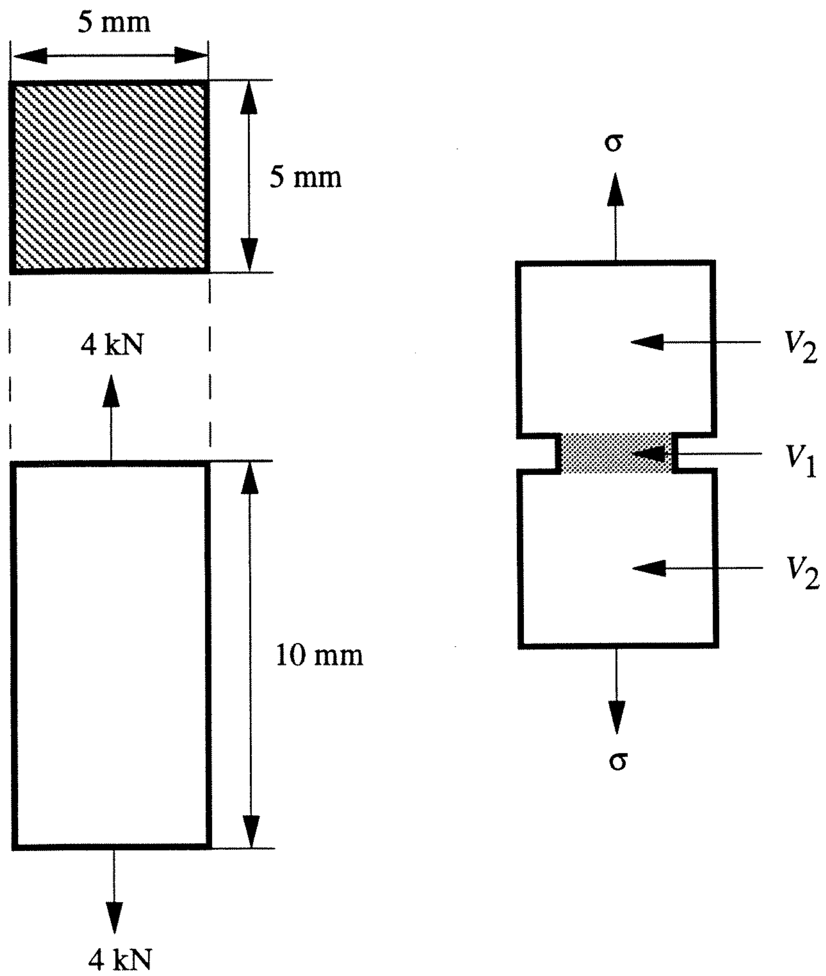
(d) A second batch of SiC samples are machined with rectangular uniform notches on two opposite sides, as shown in Fig. 3(b). The ligament between the notches has a cross-sectional area which is half that of the unnotched part of the specimen. The volume of the ligament is V_1 and that of the specimen ends is V_2. The specimen is loaded in tension with a remote stress σ.

Assuming that the stress in the ligament and at the ends of the specimen is uniform, write down an expression for the probability of survival of the specimen. Explain carefully why in practice this calculation is invalid and the survival probability is actually much lower for a given applied stress. [4]



(a) (b)

Fig. 3

3 (a) Describe the typical microstructures and properties of low, medium and high plain carbon steels. In each case indicate the percentage carbon content of the steel and give two examples of its application. Use the iron-carbon phase diagram in the Data Book to estimate the proportion of Fe_3C in 1.5 wt. % carbon steel at room temperature. [8]

(b) Figure 4 shows the isothermal transformation diagram for a 0.45 wt. % C alloy steel. Indicate the position of the carbide line on a sketch of this figure and identify the phases and microstructures labelled A to E. Hence determine the final microstructural composition of a small specimen of this steel which has been subject to the following time-temperature heat treatments before being quenched to room temperature;

(i) Cool rapidly to 250 °C, hold for 1000 s;
(ii) Cool rapidly to 400 °C, hold for 500 s;
(iii) Cool rapidly to 430 °C, hold for 10 s;
(iv) Cool rapidly to 625 °C, hold for 10 s.

Assume that the specimen is heated initially to 845 °C and held at this temperature for 1 hour prior to each thermal cycle.

What is the significance of the critical cooling rate and why can it not be measured accurately from a TTT diagram? [8]

(c) Explain the significance of the carbide line in determining the microstructure of hypo-eutectoid steel heat treated towards the knee of the TTT diagram. [2]

(d) By reference to the TTT diagram, what are the implications for the weldability of 0.45 wt. % C steel compared with that of low carbon steel? [2]

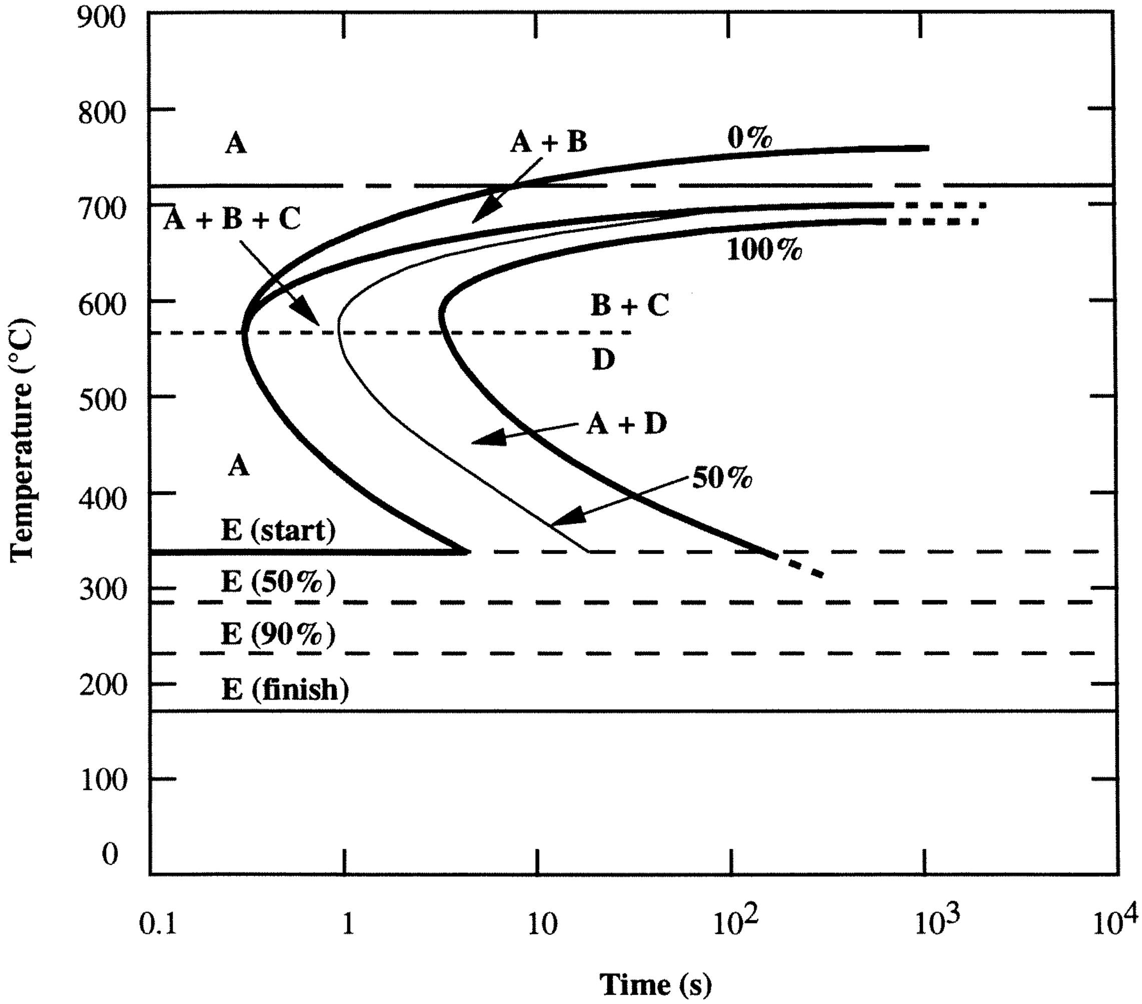


Fig. 4

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4 (a) Define the term *process model.* Identify the advantages of process modelling and explain carefully how these can aid the manufacturing of components and devices. [6]

(b) A long rectangular metal bar of width $2w$ and height $2h$ is forged between two open parallel dies as shown in Fig. 5. For equilibrium, the incremental change in transverse compressive stress σ_x across an elemental strip of material of width δx is balanced exactly by the friction stress $\tau(x)$ at the surface of the dies. By considering equilibrium conditions and positive values of x only, derive an equation relating the pressure gradient $\dfrac{d\,p(x)}{d\,x}$ across each die to $\tau(x)$ and h. Hence derive the following expression for $p(x)$

$$p(x) = \sigma_Y \left[1 + \frac{m}{2}\left(\frac{w - x}{h}\right)\right] \quad \text{for } x > 0$$

Assume that the friction stress is given by $\tau = \dfrac{m\,\sigma_Y}{2}$, where m is a constant of the forging process ($0 < m < 1$) and σ_Y is the yield stress of the bar material.

State clearly any further assumptions you make in your derivation.

Sketch the variation of pressure with distance across the entire width of the bar. [8]

(c) Calculate the average die pressure and the force required to forge a bar of aluminium alloy of yield stress 40 MPa, width 3 cm, height 1 cm and length 20 cm if the pressure varies only along the width of the bar and $m = 0.8$ for the forging arrangement. [4]

(d) Describe the conditions under which the above analysis is most relevant to the forging of aluminium. [2]

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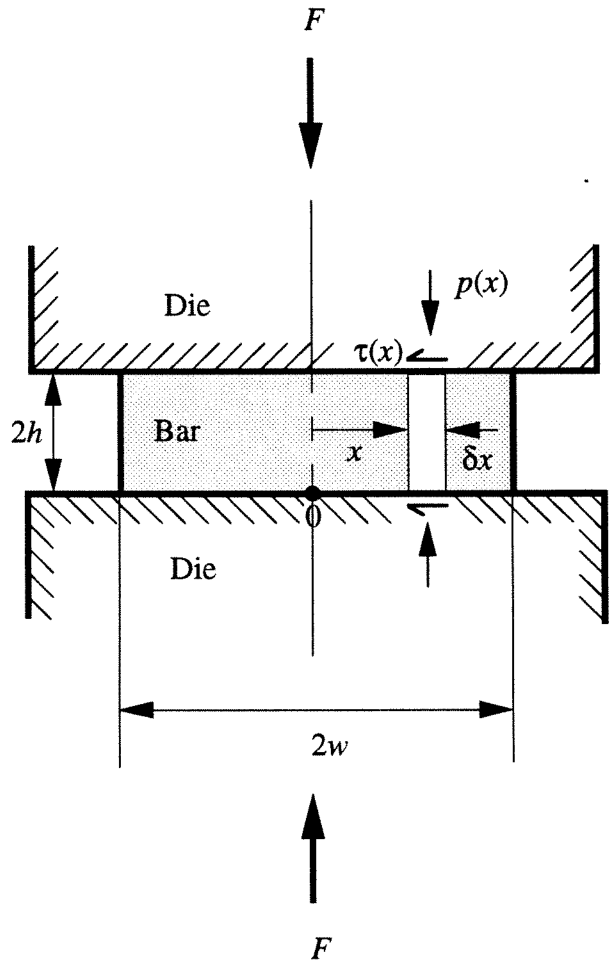

Fig. 5

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5 (a) The drive shaft of a lorry consists of a solid cylinder of fixed length L which, under operation, is required to withstand a specified torque T without yielding. The maximum shear stress τ_{max} generated by the torque is given by

$$\tau_{max} = \frac{2T}{\pi R^3}$$

where R is the radius of the cylinder, which may be varied. Derive a performance index M1 for the selection of a light and strong drive shaft material. [6]

(b) Quenched and tempered low alloy steel with a yield strength of 1560 MPa is to be considered as a candidate material for manufacture of the drive shaft. On the basis of M1, identify the region on the selection chart in Fig. 6 corresponding to material performance at least as good as low alloy steel for this application, given that a further constraint on the drive shaft is that it must be able to support a tensile load of at least 300 MPa. **A separate copy of Fig. 6 is provided which should be annotated and handed in with your answer.** [4]

(c) Using the Structures Data Book where appropriate, derive a second performance index M2 in terms of the shear modulus G of the drive shaft material if it is also required to provide a specified torsional stiffness T/ϕ, where ϕ is the twist per unit length. Hence rank the candidate materials given in Table 1 in order of performance for both M1 and M2. What additional information is required in order to be able to select the lightest material? [6]

(d) Comment on the factors which limit the potential of each material listed in Table 1 as a replacement for alloy steel for drive shaft applications. [4]

Material	**Density, ρ (kg m^{-3})**	**σ_y (MPa)**	**E (GPa)**
Titanium Alloy	4500	960	115
GFRP	1800	200	30
Aluminium Alloy	2800	600	70

Table 1

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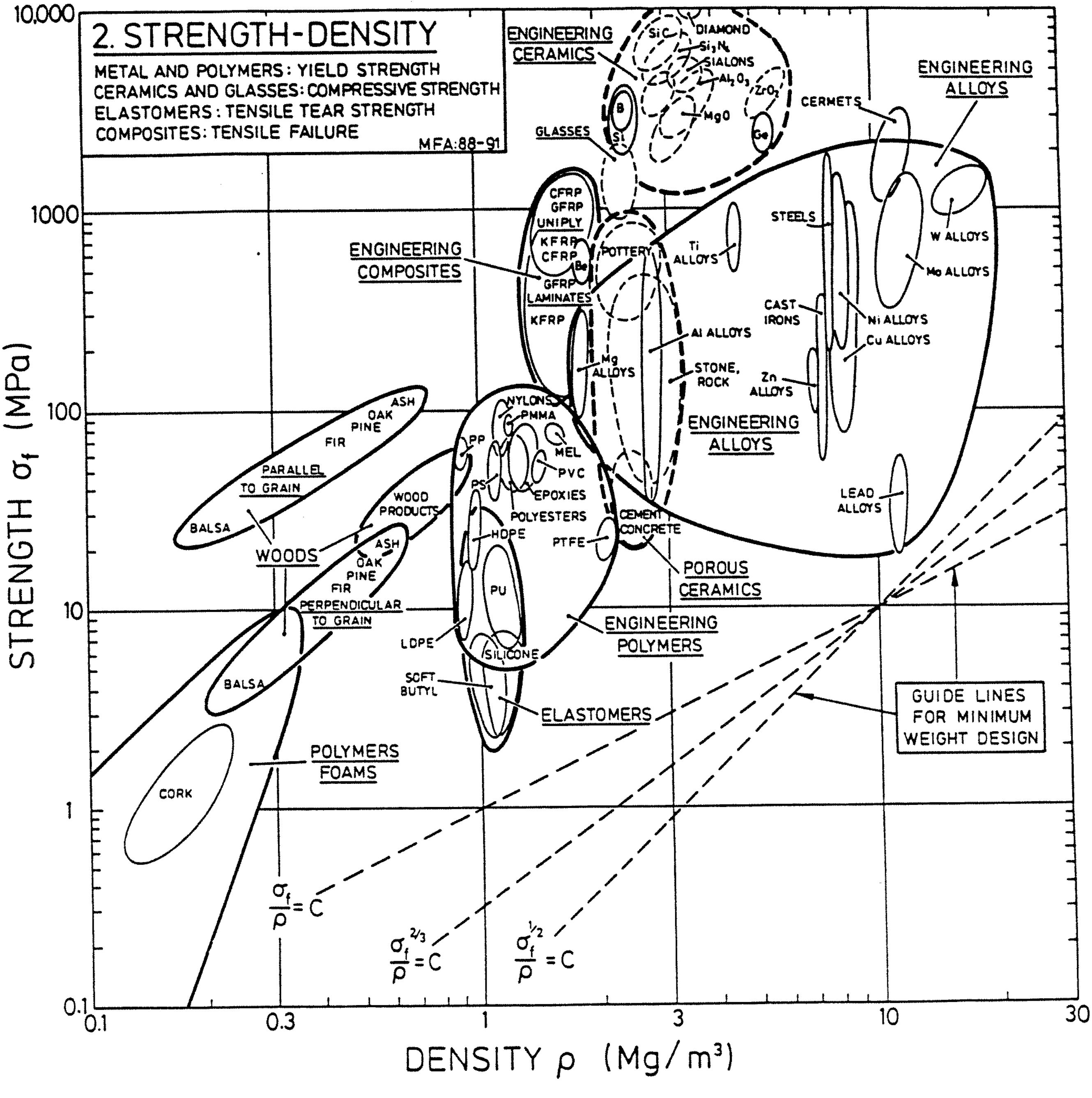

2. STRENGTH-DENSITY
METAL AND POLYMERS: YIELD STRENGTH
CERAMICS AND GLASSES: COMPRESSIVE STRENGTH
ELASTOMERS: TENSILE TEAR STRENGTH
COMPOSITES: TENSILE FAILURE
MFA:88-91
ENGINEERING CERAMICS
SiC
DIAMOND
Si3N4
SIALONS
Al2O3
ZrO2
MgO
B
Si
Ge
GLASSES
ENGINEERING ALLOYS
CERMETS
STEELS
W ALLOYS
Mo ALLOYS
Ti ALLOYS
CFRP
GFRP
UNIPLY
KFRP
CFRP
Be
GFRP
LAMINATES
KFRP
ENGINEERING COMPOSITES
POTTERY
CAST IRONS
Ni ALLOYS
Cu ALLOYS
Al ALLOYS
Mg ALLOYS
STONE, ROCK
Zn ALLOYS
ENGINEERING ALLOYS
NYLONS
PMMA
MEL
PVC
EPOXIES
POLYESTERS
PP
PS
HDPE
PU
LDPE
SILICONE
PTFE
CEMENT
CONCRETE
POROUS CERAMICS
LEAD ALLOYS
ASH
OAK
PINE
FIR
PARALLEL TO GRAIN
BALSA
WOODS
WOOD PRODUCTS
ASH
OAK
PINE
FIR
PERPENDICULAR TO GRAIN
BALSA
ENGINEERING POLYMERS
SOFT BUTYL
ELASTOMERS
CORK
POLYMERS FOAMS
GUIDE LINES FOR MINIMUM WEIGHT DESIGN
$\frac{\sigma_f}{\rho} = C$
$\frac{\sigma_f^{2/3}}{\rho} = C$
$\frac{\sigma_f^{1/2}}{\rho} = C$
STRENGTH σ_f (MPa)
10,000
1000
100
10
1
0.1
DENSITY ρ (Mg/m^3)
0.1
0.3
1
3
10
30

Fig. 6

6 (a) Outline the strength-limiting mechanisms by which thermoplastics commonly fail at different temperatures. Identify the temperature range relative to the glass transition temperature for each process and give examples of materials to support your answer. [6]

(b) The mechanical properties of a thermoplastic can be represented by a series combination of a spring of modulus E and a linear dash-pot of damping constant η, as shown in Fig. 7. Write down an equation for the strain rate in the thermoplastic when subject to an instantaneous stress. Show that the stress in the material relaxes with time t according to the following equation if the strain is subsequently held constant;

$$\sigma(t) = \sigma(0)\exp\left[-\frac{t}{t_0}\right]$$

where $\sigma(t)$ and $\sigma(0)$ represent the time-dependent and initial stresses, and t_0 is a time-independent constant of the material. Explain the significance of this equation and identify the conditions under which it is most applicable. [6]

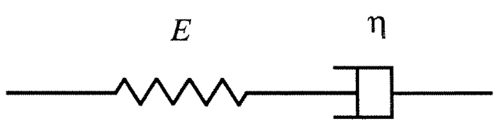

Fig. 7

(c) In a stress relaxation test, a cylindrical rod of a thermoplastic of radius 3 mm and length 10 cm elongates by 2.6 mm when subjected to an instantaneous tensile force of 150 N. A force of 28 N is required to maintain this strain after 10 s. Given that η = 1.2 GPas for the thermoplastic, determine $\sigma(0)$, t_0 and E. Hence identify the subject of the test. [6]

(d) Comment briefly on the usefulness of the spring dash-pot arrangement for modelling the microscopic properties of thermoplastics. [2]

END OF PAPER

ENGINEERING TRIPOS PART IB PAPER 3 MATERIALS

Materials Selection Chart for Question 5

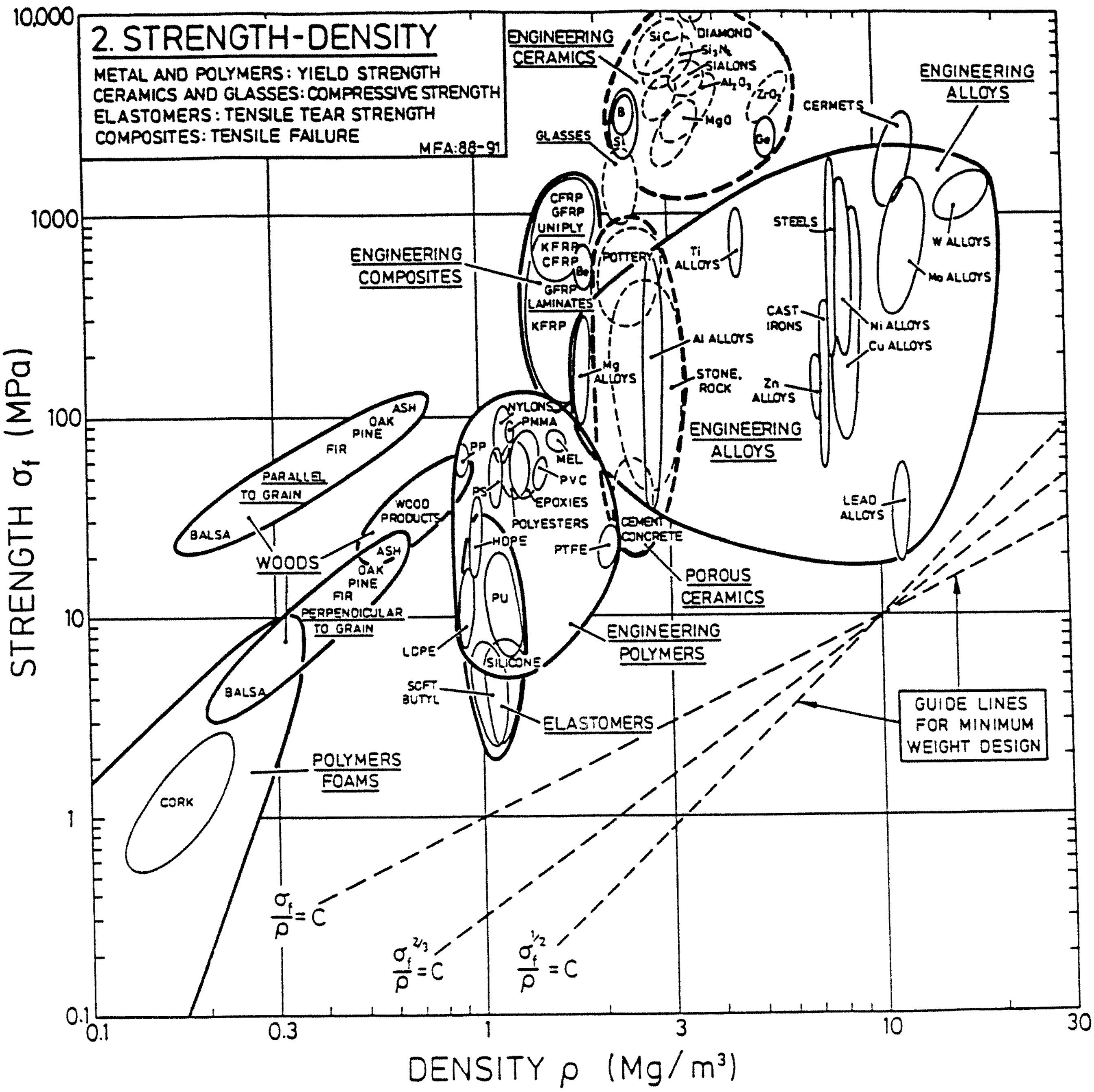

Fig. 6. This figure should be annotated and handed in as part of your solution to question 5.