ENGINEERING TRIPOS PART IB

Tuesday 1 June 1999 2 to 4

Paper 4

FLUID MECHANICS AND HEAT TRANSFER

Answer not more than **four** *questions.*
Answer at least **one** *question from each section.*
All questions carry the same number of marks.

Answers to questions in each section should be tied together and handed in separately.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

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SECTION A

1 A centrifugal atomiser is shown in Fig. 1. A highly viscous fluid is supplied from above the centre of a spinning disk and is centrifuged towards the edge of the disk in the form of a thin film. The film is sufficiently thin for the pressure in the fluid to be taken as uniform and equal to atmospheric pressure, while the fluid is sufficiently viscous to ensure that the flow is laminar and that the radial component of velocity, v_r, is much less than the local disk speed, Ωr. (Here Ω is the disk angular velocity and r is the radial coordinate.) In addition, it may be assumed that the circumferential velocity of the fluid, v_θ, is everywhere equal to the local disk speed, and that gravitational forces are negligible.

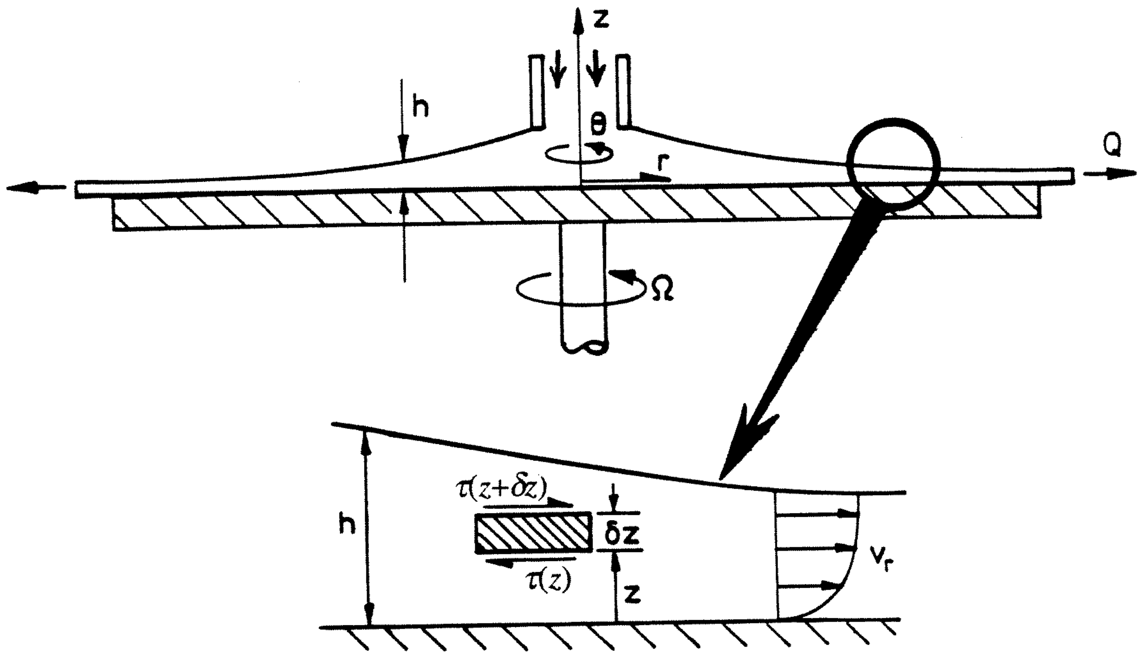

Fig. 1 A centrifugal disk atomiser

(a) Apply Newton's second law of motion to the small element of fluid shown in Fig. 1, and show that,

$$\tau(z+\delta z)-\tau(z)=-(\rho\delta z)(\Omega^2 r)$$

where τ is the radial shear stress, z is the distance measured from the surface of the disk and δz is the thickness of the element of fluid.

(Cont.

(Each fluid particle follows a near-circular path when viewed from above, and so you may assume that the radial acceleration of a typical fluid particle is simply its centripetal acceleration.) [6]

(b) The fluid is Newtonian and so τ is related to v_r in the usual way. Show that the radial velocity distribution in the fluid is given by

$$v_r = \frac{\rho \Omega^2 r}{\mu}\left(hz - \frac{z^2}{2}\right)$$

where μ is the viscosity of the fluid and $h(r)$ is the thickness of the film at radius r. (Hint; the surface boundary condition at $z = h$ is $\tau = 0$.) [10]

(c) Derive an expression for the volumetric flow rate, Q, and show that h varies as $r^{-2/3}$. [4]

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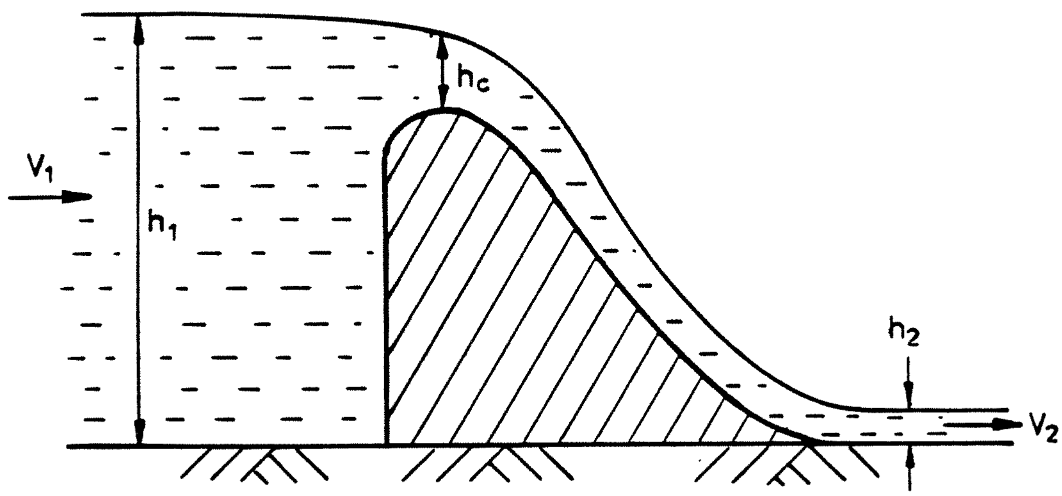
2 (a) The water flow over the spillway shown in Fig. 2 has uniform velocity at sections 1 and 2, and the fluid depths at these points are $h_1 = 5$m and $h_2 = 0.7$m. Neglecting losses, compute the velocities, V_1 and V_2, at sections 1 and 2. [8]

(b) Explain why the pressure distribution in the fluid at 1 and 2 may be treated as hydrostatic, and derive an expression for the net horizontal force (per unit width), F, exerted by the water on the dam in terms of h_1, h_2, V_1, V_2, ρ and g.

Indicate clearly the control volume used in your derivation and state any assumptions you make. Compute the force per unit width exerted on the dam using the results from (a). [9]

(c) In practice, friction is important to the extent that it causes a significant loss of mechanical energy between 1 and 2, although the frictional forces are negligible compared with the hydrostatic forces. For given values of h_1 and h_2, will your calculation in (b) overestimate or underestimate F? Explain your answer. [3]



Section 1 Section 2

Fig. 2 Flow over a dam spillway

3 (a) Explain how Bernoulli's equation is conventionally extended to allow for mechanical energy losses. Write down the extended Bernoulli equation, defining all terms which appear in the equation. [2]

(b) Water flows through an abrupt expansion in a pipe. The Reynolds number is high and the flow may be taken to be uniform upstream and downstream of the expansion (sections 1 and 2 in Fig. 3 below). The flow is steady on average and it is observed that the pressure on the back-face of the expansion is uniform and equal to the upstream pressure p_1. Use the force-momentum equation to derive an expression for $p_1 - p_2$ in terms of V_1 and V_2, where V is the velocity of the fluid at any one section. State any assumptions you make. [Hint; use the control volume shown below.] [6]

(c) Now use the extended Bernoulli equation, in conjunction with the results of (b), to show that the mechanical energy loss per unit volume, ΔC, is given by

$$\Delta C = C_1 - C_2 = \tfrac{1}{2}\rho(V_1 - V_2)^2$$

(Here C is the total mechanical energy per unit volume, i.e. Bernoulli's constant.) [6]

(d) The flow rate in the pipe is 10 kg/s, the area ratio, A_2/A_1, is equal to 2 and V_1 is 10 m/s. Calculate the rate of dissipation of mechanical energy (in watts) caused by the pipe expansion. What happens to this 'lost' mechanical energy? [4]

(e) Note that the fluid viscosity does not appear in any of the above expressions. The implication is that we can make the viscosity as small as we wish, yet still obtain a finite rate of dissipation of energy. Why is this, in fact, quite reasonable? [2]

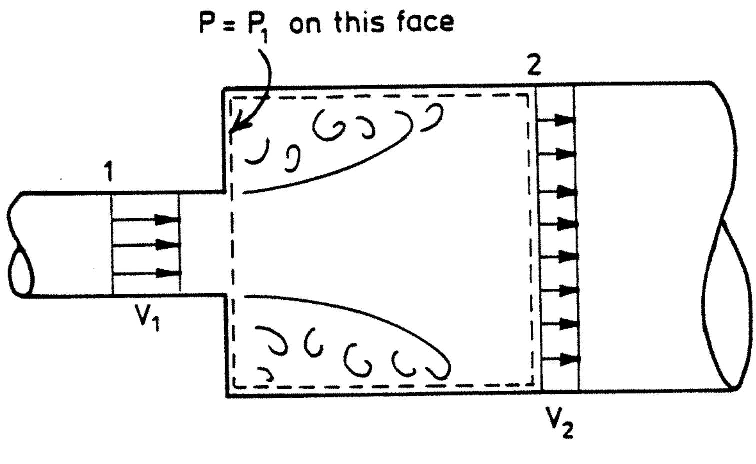


Fig. 3 Sudden expansion in a pipe

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4 (a) A fluid with density ρ , viscosity μ and mean velocity V passes through a long, smooth pipe of length L and diameter d. Using only dimensional and geometric arguments, show that the pressure drop, Δp, must be of the form

$$\Delta p = \left(f \frac{L}{d} \right) \left(\frac{1}{2} \rho V^2 \right)$$

where f is an arbitrary function of Reynolds number, $f = f(\rho V d/\mu)$. [You may ignore entrance effects.]

When the flow is laminar the inertia and hence density of the fluid is unimportant. Show that, when density is irrelevant, f must be of the form $f = k(\rho V d / \mu)^{-1}$, where k is a constant. Hence show that, for laminar flow,

$$\Delta p = \frac{k}{2} \frac{\mu V L}{d^2}$$ [14]

(b) When the pipe wall is rough, with a typical roughness height of ε , show that f is now a function of both Reynolds number and relative roughness, $\frac{\varepsilon}{d}$.

Sketch the variation of f with Reynolds number and relative roughness. [6]

SECTION B

5 A satellite in the shadow of the moon loses energy by radiation. Its shape is a cylinder of radius 1 m and length 10 m. The heat lost at the ends of the satellite may be neglected. The metallic surface has a low emissivity $\varepsilon = 0.1$ and is at temperature 300 K. You may take the temperature of space as 2.4 K.

(a) Calculate the rate of heat loss from the satellite to space. [4]

(b) N equally spaced, concentric metal cylinders are fixed around the satellite. Their thickness is negligible and emissivity is 0.1 on both sides. Express the equivalent thermal resistance to radiation heat transfer as a function of N. (The gap between the cylinders is narrow, so you may assume that the effective radius of all of the cylinders is 1 m.) [10]

(c) What should N be if the heat loss calculated in (a) is to be reduced by a factor of at least 20? [6]

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6 A copper sheet is to be used as a cooling fin. A temperature difference is maintained along the length, L, of the copper sheet, T_1 at one end and room temperature, T_0, at the other end. The width of the sheet is considered to be infinite and its thickness is 1 mm. Both faces of the sheet are characterised by a convection heat transfer coefficient $h = 10\ \mathrm{Wm^{-2}K^{-1}}$, and the length of the fin is sufficiently long that the temperature of the fin reaches room temperature well before the end of the fin.

(a) Determine the temperature distribution along the length of the sheet, assuming that it is uniform across the thickness. Sketch this distribution. [The thermal conductivity of copper, λ_c, is $400\ \mathrm{Wm^{-1}K^{-1}}$.] [6]

(b) In order to increase the thermal resistance to lateral heat transfer, the sheet is now coated on both faces with an insulating material of thermal conductivity $\lambda_i = 10^{-3}\mathrm{Wm^{-1}K^{-1}}$ and of thickness 1 mm as shown in Fig. 4. Sketch the transverse temperature distribution across the thickness of the insulated fin and express the local rate of heat loss in terms of the local copper temperature and T_0. (You may neglect axial conduction in the insulation.) [8]

(c) Determine the new temperature distribution along the length of the sheet. [6]

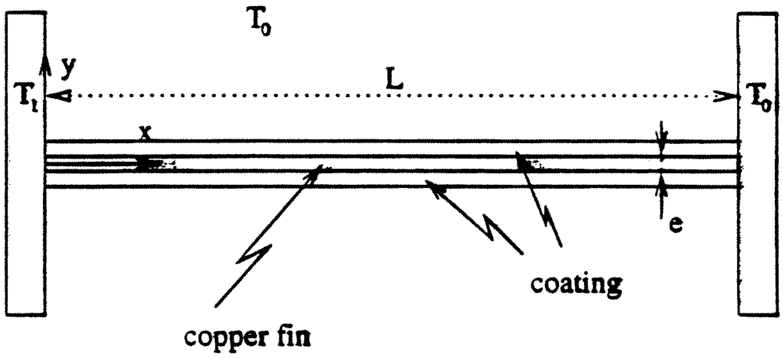

Fig. 4 Cooling fin

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