ENGINEERING TRIPOS PART IB

Thursday 3 June 1999 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer at least one question from this section

1 (a) If a linear system has an impulse response $g(t)$, state the condition under which the system is *asymptotically stable.*

Determine whether or not the systems with the following impulse responses are asymptotically stable.

(i) $g(t) = \delta(t-1)$;

(ii) $g(t) = \begin{cases} \dfrac{1}{1+t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$. [8]

(b) Consider a linear system with impulse response

$$g(t) = \begin{cases} 0, & t \leq 0 \\ 1, & 0 < t \leq 1 \\ 0, & t > 1 \end{cases} .$$

(i) Show that the system is asymptotically stable and find directly, using convolution, an expression for its response to the input $u(t) = \cos(\omega t)$ that is valid for $t > 1$.

(ii) Determine the transfer function of the system, and find from this its steady-state response to the same input, in the form $A\cos(\omega t + \phi)$. Verify that your answer agrees with that determined in part (b)(i).

[12]

2 Figure 1 shows an electric motor driving a flywheel of inertia J. The motor has negligible inertia. A disturbance torque T_d acts on the flywheel. A simple feedback loop is being used in an attempt to maintain a constant angular velocity. A sensor attached to the motor produces a voltage of $k_t\omega$, where $\omega = \dot{\theta}$ and θ is the angular position of the shaft. The voltage V_r is a reference input, applied by an operator. The amplifier converts voltage to current according to the differential equation

$$\tau \dot{I} + I = k_a V_i$$

where V_i is its input voltage, τ and k_a are constants and I is the current at the amplifier output. The motor exerts a torque T equal to $k_m I$.

(a) Draw a block diagram of the feedback system, and find its open-loop transfer function. Show that the closed-loop transfer functions from $\bar{T}_d(s)$ to $\bar{\omega}(s)$ and from $\bar{V}_r(s)$ to $\bar{\omega}(s)$ are given respectively by:

$$H_1(s) = \frac{-(s\tau + 1)}{Js(s\tau + 1) + k_a k_m k_t} \quad \text{and} \quad H_2(s) = \frac{k_m k_a}{Js(s\tau + 1) + k_a k_m k_t}$$ [8]

(b) Taking $\tau = 0.2$, choose k_a (in terms of the other parameters) to be as large as possible, subject to the constraint that the damping ratio of the closed-loop system be no less than 0.5. [6]

(c) Assuming that $\lim_{t\to\infty} V_r = \alpha$, determine the eventual angular velocity of the flywheel if $T_d = 0$. How does your answer change if a unit step disturbance torque $T_d = H(t)$ is applied? [6]

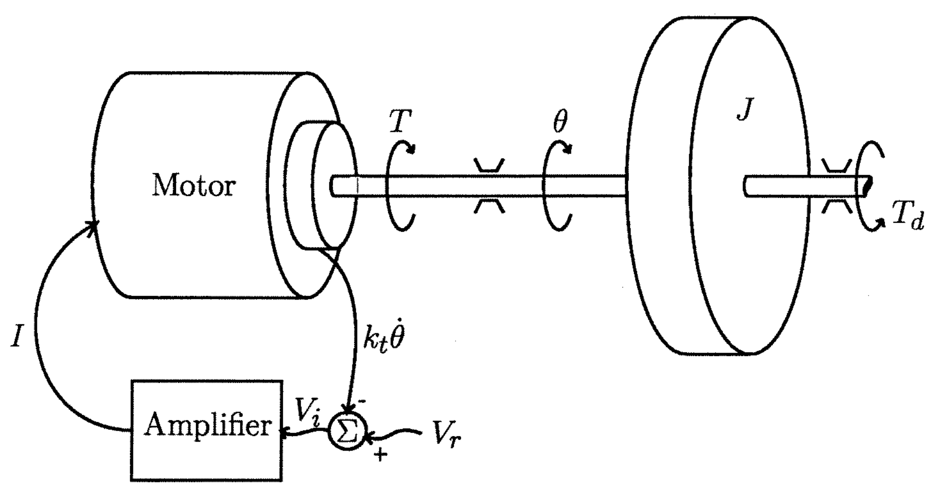

Fig. 1

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3 (a) Define the terms *closed-loop characteristic equation* and *closed-loop poles* for the feedback system of Fig. 2. [5]

(b) If

$$G(s) = \frac{1}{s^2+1} \qquad \text{and} \qquad K(s) = k\,\frac{s+1}{s+a}$$

find the values of k and a for which two of the closed-loop poles are located at

$$s = -1 \pm 2\mathrm{j}$$ [7]

(c) Using the new values $k = 13$ and $a = 7$, sketch the Bode diagram of $K(s)G(s)$, and estimate the phase margin of the feedback system. [8]

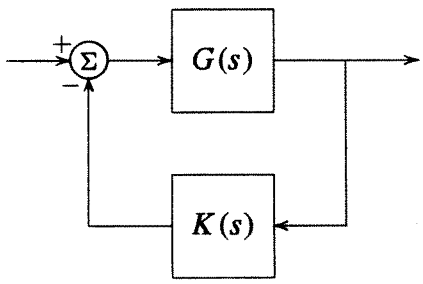

Fig. 2

Note: the Bode Diagram of $G(s)$ alone is given as Fig. 3. An extra copy is provided on a separate sheet, and should be handed in with your answer if constructions are made on it.

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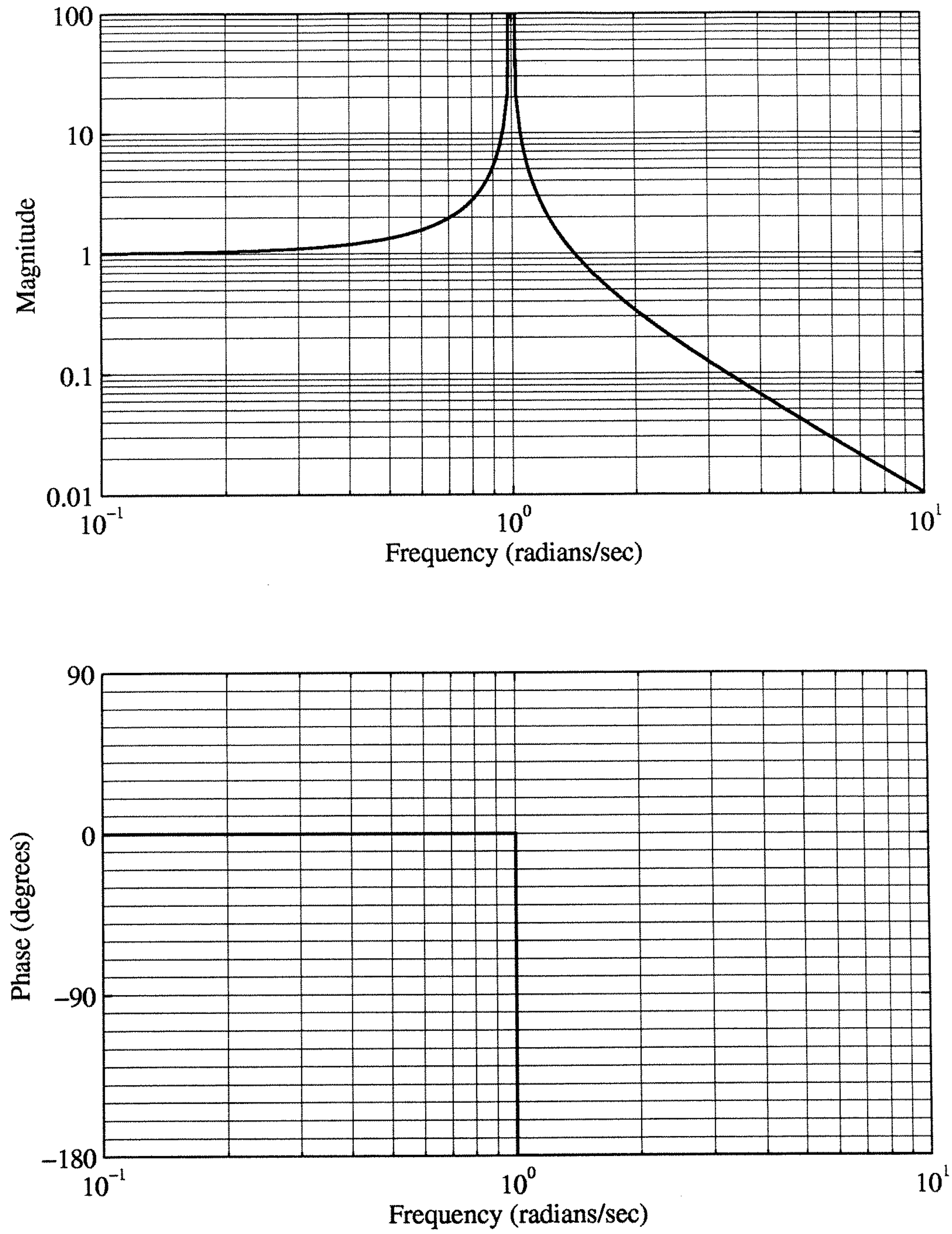
100
10
1
0.1
0.01
Magnitude
10^{-1}
10^{0}
10^{1}
Frequency (radians/sec)
90
0
−90
−180
Phase (degrees)
10^{-1}
10^{0}
10^{1}
Frequency (radians/sec)

Fig. 3

4 (a) State the Nyquist stability criterion as it applies to the feedback system of Fig. 4, where $K(s)$ and $G(s)$ are asymptotically stable linear systems. [4]

(b) Consider the feedback system of Fig. 4, with

$$G(s) = \frac{1}{s+1}, \quad K(s) = 2e^{-s\tau}$$

(i) Sketch the Nyquist diagrams of the feedback system for $\tau = 0$ and for $\tau = 0.5$. [5]

(ii) Determine the largest value of τ for which the closed-loop system remains stable. [6]

(iii) For $\tau = 0.5$, estimate the range of frequencies for which the magnitude of the steady-state gain from r to y is greater than unity. [5]

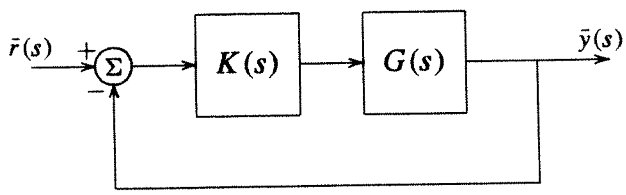

Fig. 4

SECTION B

Answer at least one question from this section

5 The UK standard for analogue TV signals is 625 lines/frame, 25 frames/s, and 2:1 interlacing, with an aspect ratio of 4:3.

(a) Describe how the image information is carried in the TV signal. Your answer should include the scanning process, the meaning of the terms lines/frame, frames/s, interlacing and aspect ratio, and explain why the quoted numerical values of these terms were chosen. What is the purpose of *sync pulses* and *blanking periods*? [5]

(b) Assuming that equal horizontal and vertical resolution are required, calculate the pixel transmission rate for the above TV standard. Ignore sync pulses and blanking periods. [3]

(c) A UK standard TV image consists of a single vertical white line, one pixel wide, on a black background. Find a Fourier series representation for the resulting TV signal, assuming that white is transmitted as 1 V and black as 0 V. Ignore sync pulses and blanking periods, and ignore colour information, which is transmitted independently. [6]

(d) Estimate the bandwidth required for satisfactory transmission of this TV signal, justifying your estimate by reference to its Fourier series representation. [3]

(e) A UK standard TV signal is transmitted through a channel with an upper frequency cutoff of 3 MHz. What effect will this channel have on (i) horizontal edges and (ii) vertical edges in the picture? [3]

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6 (a) Telephone channels have a passband which extends from 300 Hz to 3400 Hz. What effect does this have on the transmission of (i) speech and (ii) music over telephone channels? [5]

(b) Amplitude modulation (AM) is to be used for modulation of a telephone signal $x(t)$ on to a carrier signal with frequency 100 kHz. The resulting AM signal may be written as

$$s(t) = [a_0 + x(t)] \cos(\omega_c t)$$

State the value of ω_c, and explain what restrictions there are on the value of a_0. [3]

(c) Assuming that $x(t)$ is a unit amplitude cosine wave, $x(t) = \cos(\omega_M t)$, derive an expression for the AM signal which shows the presence of a carrier frequency component and sidebands. If $\omega_M = 2\pi \times 10^3$ rad/s, what are the sideband frequencies? [5]

(d) For the general telephone signal discussed in parts (a) and (b), what passband does the AM signal occupy, and what would be a suitable spacing of carrier frequencies in a practical Frequency Division Multiplexing (FDM) system? [3]

(e) Describe what operations would be required to extract and demodulate the AM signal in the receiver in order to produce the desired telephone signal $x(t)$. [4]

END OF PAPER

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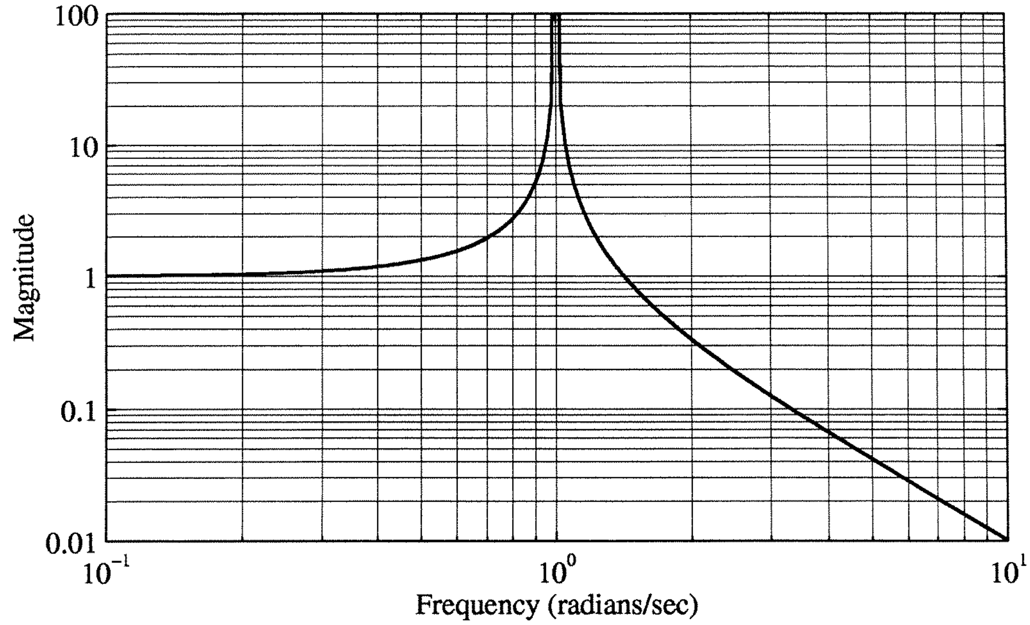

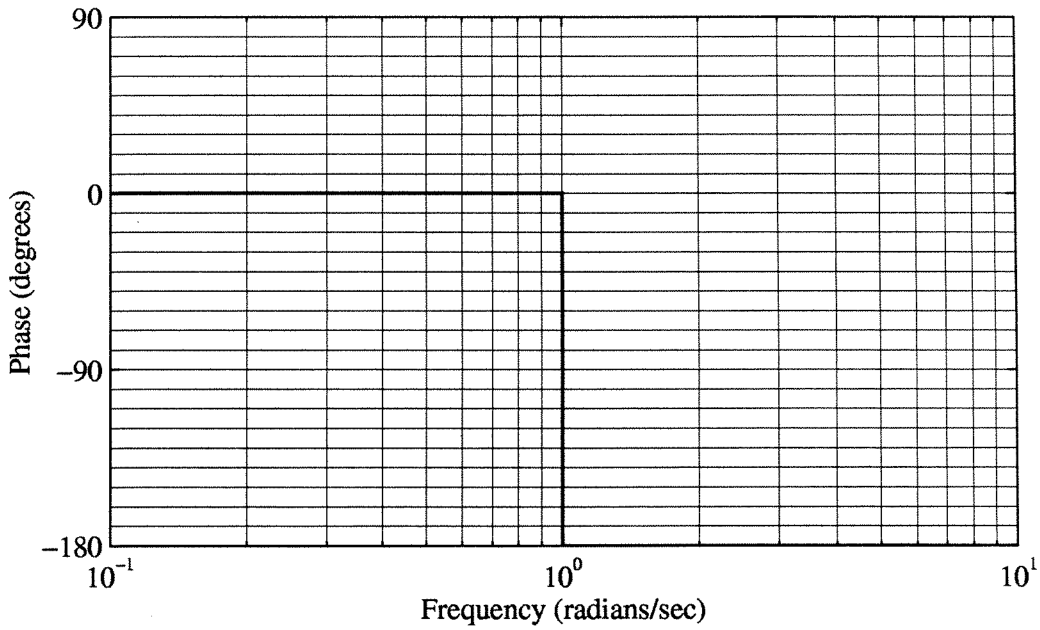

Note: this extra copy of Fig. 3 should be handed in with your answer to Question 3 if constructions are made on it.