IB 2000 Answers

ENGINEERING TRIPOS   PART IB   2000

ANSWERS FOR PAPER 1 - MECHANICS

Q1 (c) $v_2 = r\sqrt{\left(\frac{4}{81}\omega_0^2 - \frac{2}{3}\frac{g}{r}\right)}$; $\omega_0 = \sqrt{\left(\frac{27}{2}\frac{g}{r}\right)}$

Q2 (b) (i) Since the situation is symmetrical, the angular velocities after the collision are equal in magnitude and opposite in direction.

(ii) $\underline{v}_{GB} = \frac{u}{13}\left(8\underline{i} - 5\underline{j}\right)$

Q3 (a) (i) $\underline{v}_P = 4\underline{j}$ ms^{-1}

(ii) $\underline{a}_P = 0$

(b) $v_C = -2.3\underline{i} + 9.2\underline{j} - 2.3\underline{k}$ ms^{-1} ; $a_C = -53.7\underline{i} - 22.6\underline{j} + 1.1\underline{k}$ ms^{-2}

Q4 (b) $\ddot{x} = 0$ perpendicular to F; $\ddot{y} = \frac{F}{m}$ parallel to F; $\ddot{\theta} = \frac{6F}{mL}$

(c) $\underline{\ddot{\phi}} = -4\underline{k}$ rads^{-2} ; $\underline{a}_B = 8\left(\frac{1}{3}\underline{i} + \underline{j}\right)$ ms^{-2}

Q5 (a) $x = \frac{2}{3}L$

For an ideal impulse, i.e. an infinite force for infinitesimal time, the analysis would not change if the plane containing the bat and ball was not horizontal.

(b) The magnitude of the couple is $M_G = \frac{1}{2}\frac{mrv^2}{R}$

The direction of the couple on the rear wheel is anticlockwise about the longitudinal axis of the car, looking at the wheel from the rear of the car.

The couple can be explained in terms of providing the coriolis accelerations to the particles around the perimeter of the wheel.

6 (b) $\omega_{DF} \approx 0.3$ rads^{-1} anticlockwise; $\omega_{CE} \approx 0.1$ rads^{-1} clockwise;

$v_{B(sliding)} \approx 24$ mms^{-1}

(c) $\dot{\omega}_{CE} \approx 0.05$ rads^{-2} clockwise; $a_E \approx 2$ mms^{-2} to the right

Part IB 2000, Paper 2, Structures, Answers

1. (a) (i) $\pm\frac{L^2\rho g}{8t}$ at the centre, top or bottom of the tank.

(ii) Longitudinal $\frac{pD}{4t}$; hoop $\frac{pD}{2t}$ everywhere.

(b) 9.0 mm

(c) Because the bending and pressure stresses are in the same sense

2. (a) 85.3 mm (b) 29430 Nmm, 98.1 N, 29430 Nmm; 116 horizontal, 0 vertical, 167 shear (all N/mm^2) (c) +235 N/mm^2 inclined at 35.4 degree below the horizontal axis , -119 N/mm^2

3. (c) $M_p/3$ (d) $0.3166M_p$

4. (b) $12m/L^2$ (c) $14.25\ m/L^2$

5. (a) .1065qL (c) $0.00291qL^4/EI$

6. (b) 1.13 m (c) L/500

Engineering Tripos Part IB 2000

Paper 3, Materials

Answers

1(c) Cooling rate = 17.4 Ks^{-1}. Choose steel D.

2(c) Injection moulding: £5.42 for 1000, £0.52 for 50000
Rotational moulding: £1.70 for 1000, £0.72 for 50000
Choose RM for 1000, IM for 50000.

3(b) Carbon steel, Al alloys

5(c) 2041 MPa

6(b) 190^{o}C: 76% (Pb), 24% L
175^{o}C: 90% (Pb), 10% (Sn)
In eutectic region: 60% (Sn), 40% (Pb)

2000 Part IB Paper 4

Answers

Fluid Mechanics and Heat Transfer

1.b	(i) $49.94\ kNm^{-2}$ above atmospheric
	(ii) $119.7\ kN$
	(iii) $64.85\ kN$
2.a	$V_1 = 8\,ms^{-1}, V_2 = 10\,ms^{-1}, V_3 = 17.8\,ms^{-1}$
c	$-53\ kN$ (to the left)
3.a	(i) $U_0 = -\frac{dP}{dx}R^2/(4\mu)$, with $\frac{dP}{dx}$ pressure gradient and μ dynamic viscosity
	(ii) $\overline{u} = U_0/2$
b	(i) $Re = \rho\overline{u}D/\mu = 1270$ is below 2000, so that the laminar results in **a** apply
	(ii) $U_0 = 0.254\ ms^{-1}$
4.a	ρ density and σ surface tension (force per unit length)
b	$v \sim \sqrt{g\lambda}$
c	$v \sim \sqrt{\sigma/(\rho\lambda)}$
d	$\lambda_{min} \sim \sqrt{\sigma/(\rho g)}$ and $v_{min} \sim [\sigma g/\rho]^{1/4}$
5.b	Using the chain rule $\frac{dF}{dr} = \frac{dF}{dT}\frac{dT}{dr}$
c	$F = K\ln r + L$, where K and L are constants
d	$3.995\ kW$
6.a	$P_s = 1.01325\ bar$ and $P_{air} = 0.9867\ bar$
b	0.3187 (kg of steam per kg of mixture)
c	$4.639\ kJ\,K^{-1}kg^{-1}$
d	$T = 80.9\ ^oC$ and steam mass fraction: 0.3006 (still saturated)

2000 Part 1B - PAPER 5 - ANSWERS

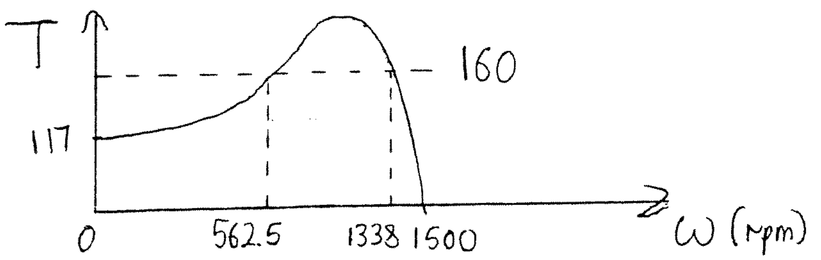
1. a) CMRR=100 dB b) R_C = 500 Ω; R_T = 2.49 kΩ c) V_{CC} = 19.3 V, V_{EE} = 100.3 V
d) Use a 40 mA constant current source in place of R_T.

2. a) R_B = 96.5 kΩ; R_L = 500 Ω b) Gain = -10; R_{in} = 5 kΩ; R_{out} = 500 Ω
c) C_1 = 1.59 μF d) f_{3dB} = 390 kHz

3. b) I_{line} = 306.2 A; P_{lines} = 56.2 kW; V_S = 3.72 kV c) C = 109 μF; I_{line} = 272.2 A
d) cosφ = 0.999 leading; I_{line} = 245 A. Even better power factor, but capacitors may not be adequately rated.

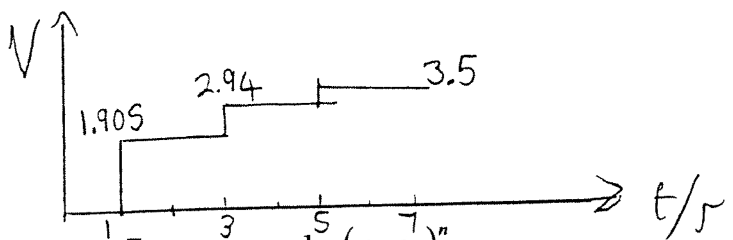
4. a) E = 14.87 kV; I = 3.94 kA b) For 11 kV generator, E = 16.39 kV ∠20.5°;
For 33 kV generator, E = 49.17 kV ∠20.5° c) Prime-mover input power controls generator output real power, rotor field current controls generator output reactive power.

5. b) i) Torque = 117 Nm ii) Speed = 562.5 rpm or 1338 rpm



c) η = 77.5 % at 1338 rpm, 20 % at 562.5 rpm. Therefore, operating at 1338 rpm is preferable. d) Connect additional rotor resistance via slip-rings.

6. a) Z_0 = 50 Ω; τ = 25 ns b) ρ_S = 0.6; ρ_L = 0.905
c)



d) $V = V_{dc}\dfrac{Z_0}{Z_0+R_0}(1+\rho_L)\dfrac{1-(\rho_L\rho_S)^n}{1-\rho_L\rho_S}$; No. of reflections = 6; Total delay = 325 ns;
V_∞ = 4.17 V

7. b) ***E***, ***H*** and Poynting vector ($\boldsymbol{E}\times\boldsymbol{H}^*/2$) are mutually orthogonal, $\eta_0 = \dfrac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$;

$\boldsymbol{E} = \boldsymbol{e}_\theta \dfrac{\hat{I}l\beta\eta_0}{4\pi r}\sin\theta\exp(\omega t-\beta r)$ d) $P = \dfrac{\beta^2\hat{I}^2l^2\eta_0}{12\pi}$ e) Need large R_a for efficient radiation otherwise current is large for given radiated power ⇒ large ω for small l.

ENGINEERING TRIPOS PART IB Thursday 8 June 2000

Paper 6 INFORMATION ENGINEERING

Brief answers:

1 (a) $\bar{p}(s) = \frac{1}{s}(1 - e^{-sT})$

(c) $c(t) = 0.5(1 - e^{-0.01t}) - H(t - 10) \cdot 0.5(1 - e^{-0.01(t-10)})$
where $H(t)$ is the Heavyside (unit) step function.

(d) $c(5) = 0.02439$ kg m^{-3}; $c(100) = 0.01935$ kg m^{-3}.

2 (a) Outer loop = position feedback = accurate control;
inner loop = velocity feedback = better damping.

(b) $c(s) = \dfrac{K_c G_1 G_2}{1 + K_v G_1 + K_c G_1 G_2} r(s) + \dfrac{-K_v G_1 G_2}{1 + K_v G_1 + K_c G_1 G_2} d(s)$

(c) (i) 0; (ii) 0.11; (iii) -0.001 .

(d) Add integral action to K_c .

3 (b) Gain margin = 9.6 dB; phase margin = 33 deg.

(c) Gain margin = 12.6 dB; phase margin = 54 deg.

4 (c) $K = 4.243$; peak CLFR ≈ 1.38 at $\omega \approx 1.2$ rad s^{-1}.

5 (a) $T_1 = 0.53$ ms; $T_2 = 0.047$ ms; shunt inductance of transformers, and shunt capacitance and series resistance of the line.

(b) $h(t) = A(e^{-t/T_1} - e^{-t/T_2})$

(d) $h(t)$ decays too rapidly. Overcome by modulation onto a mid-band carrier.

6 (b) r.m.s. noise $= \dfrac{\delta V}{\sqrt{12}}$

(c) SNR = 90.3 dB.

Part IB 2000

Paper 7: Mathematical Methods

Answers

1. (a) $\dfrac{\partial}{\partial t}\iiint\limits_V (\rho c T)\,\mathrm{dV} = -\iiint\limits_V \rho c\nabla.(T\mathbf{u})\mathrm{dV} - \iiint\limits_V -\lambda\nabla^2 T\mathrm{dV}.$

2. (a) $4\pi a^2$.
   (b) $4\pi a^2$, 0, $4\pi a^2$.
   (c) 0.

3. (a) $F = -(\nu/\omega)^2 F^{(4)}$, $G = -(\nu/\omega)^2 G^{(4)}$.
   (b) $u = V\exp(-y/\delta)\,[\cos(y/\delta)\cos\omega t + \sin(y/\delta)\sin\omega t]$.

4. (a) (ii) $x_0 > 2$.
   (b) $x_1 = 2$, $x_2 = 3$, $x_3 = 4$.

5. (a) (ii) When the points are collinear or in some other degenerate configuration.

6. (c) (i) $\frac{T}{2}\left[A\,\mathrm{sinc}\left(\frac{(\omega-\omega_1)T}{2}\right) + B\,\mathrm{sinc}\left(\frac{(\omega-\omega_2)T}{2}\right) + A\,\mathrm{sinc}\left(\frac{(\omega+\omega_1)T}{2}\right) + B\,\mathrm{sinc}\left(\frac{(\omega+\omega_2)T}{2}\right)\right]$.

7. (a) (ii) $\begin{cases} \dfrac{e^{i\omega T/2}}{T\mathrm{sinc}(\omega T/2)} & \text{for } |\omega| < \omega_m, \\ 0 & \text{otherwise.} \end{cases}$
   (b) (i) $\mathrm{E}[X] = \displaystyle\int_{-\infty}^{\infty} x f(x)\,dx.$

8. (b) 1022.26 g.
   (c) (iii) 0.107, (iv) £54.72.

Paper 8

Q1	(a)	4.93	
	(b)	2.97	
	(c)	1.12	
Q3	(b)	750 mm	8 bars of 40 mm dia. spaced 125 mm apart
Q5	(a)	$l_{eff} = 0.785l$	
		Forebody: 2153 N/m, 8.6 Hz	
		Afterbody: 7268 N/m, 19.3 Hz	
	(b)	A = new forebody	
		B = wood forebody	
		C = new afterbody	
		D = wood afterbody	
Q6	(a)	±5.2 mm	26.65 mm at 12 Hz
	(b)	±1.73 mm	
Q7	(a)	242.5 K	187 m/s
	(b)	324 m/s	375 m/s
	(c)	30.4 deg.	
	(d)	28210 J/kg	27.9 K
	(e)	1.38	
Q8	(b)	1600 kPa	773 K
	(c)	1016 K	338 kPa
	(d)	9.2	
	(e)	362 m/s	
Q10	(b)	$V = -\int_0^h \frac{Nex}{\varepsilon} dx = \frac{Neh^2}{2\varepsilon}$	
	(c)	$5\text{x}10^{22}$ m^{-3}	
	(d)	$5\text{x}10^{-7}$ m	
	(e)	-2.5 V	
	(f)	$2\text{x}10^{7}$ V/m	
Q11	(a)	$2\text{x}10^{29}$ m^{-3}	
	(b)	$3.5\text{x}10^{-10}$	
	(c)	379 K	
	(e)	$2\text{x}10^{-6}$ m	$2\text{x}10^{-11}$ s
Q12	(c)	474.982 μm	475.022 μm