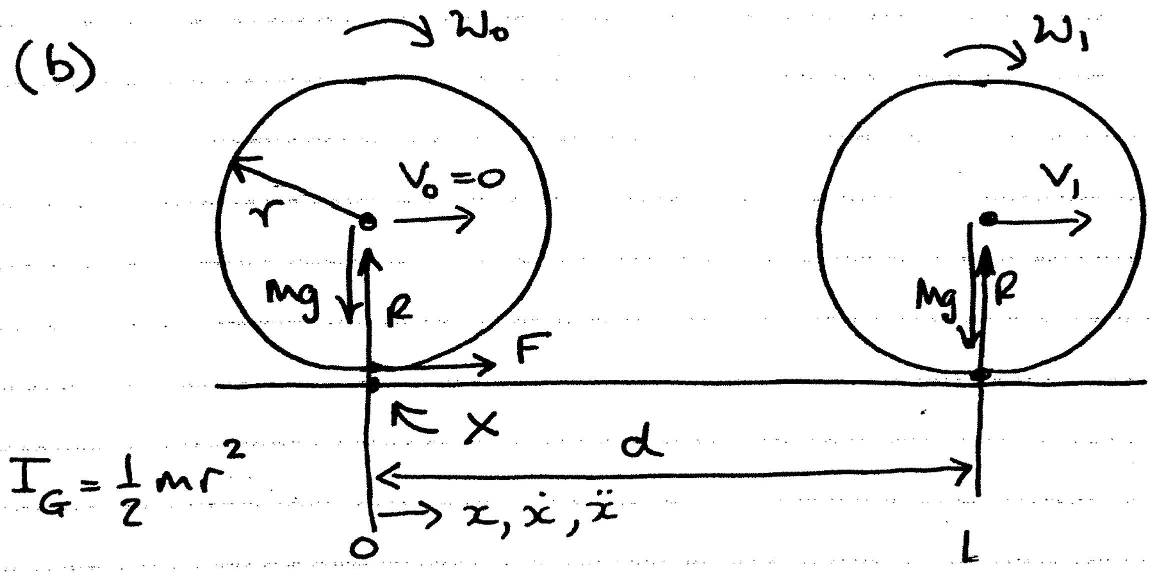
PART IB MECHANICS VERSION 2 Q1/1

1 (b)



Moment of momentum about X :

$$\left(\tfrac{1}{2} mr^2\right)\omega_0 = \left(\tfrac{1}{2} mr^2\right)\omega_1 + (mv_1)\, r$$

When slipping stops, $v_1 = r\omega_1$

so $\frac{1}{2} r^2 \omega_0 = \frac{1}{2} r^2 \omega_1 + r^2\omega_1 = \frac{3}{2} r^2 \omega_1$

$$\therefore \omega_1 = \frac{\omega_0}{3} \quad \text{and} \quad \underline{\underline{v_1 = \frac{r\omega_0}{3}}}$$

During slip, friction force const $F = \mu mg$

$\mu mg = m\ddot{x}$ $\quad \therefore$ const $\ddot{x} = \mu g$

$v^2 = u^2 + 2as$ so $V_1^2 = \underbrace{\cancel{V_0^2}}_{=0} + 2\mu g d$

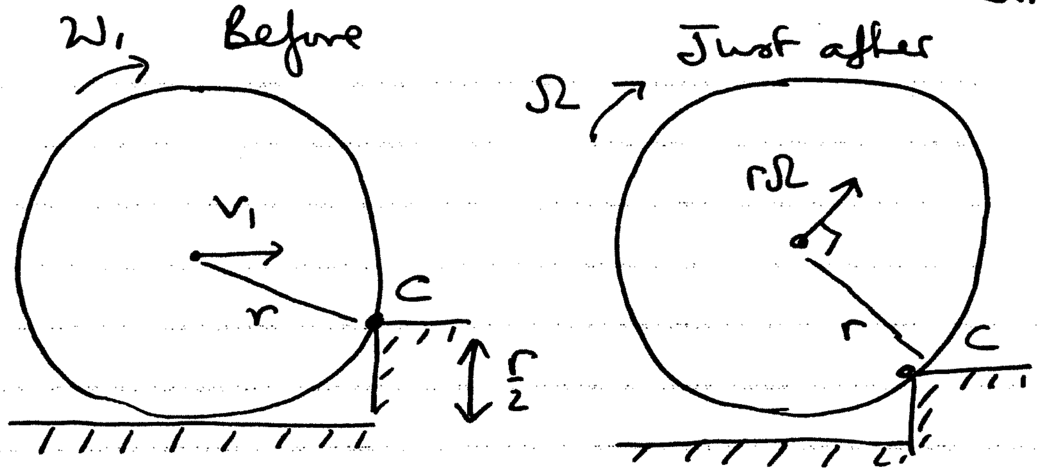
$$\frac{r^2\omega_0^2}{9} = 2\mu g d$$

$$\therefore d = \underline{\underline{\frac{1}{18} \frac{r^2\omega_0^2}{\mu g}}}$$

(c)

$I_G = \frac{1}{2} m r^2$

$V_1 = r\omega_1$



Moment of momentum conserved about C

$$(m \underset{r\omega_1}{v_1})\frac{r}{2} + \left(\frac{1}{2} m r^2\right)\omega_1 = (m r \Omega) r + \left(\frac{1}{2} m r^2\right)\Omega$$

$$\frac{\omega_1}{2} + \frac{\omega_1}{2} = \Omega + \frac{\Omega}{2}$$

$$\therefore \underline{\Omega = \frac{2}{3}\omega_1 = \frac{2}{9}\omega_0}$$

Energy conserved as disc rotates about C

$$\frac{1}{2}\left(\frac{1}{2} m r^2\right)\Omega^2 + \frac{1}{2} m (r\Omega)^2 = \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega_2^2 + \frac{1}{2} m (r\omega_2)^2 + m g \frac{r}{2}$$

$$\frac{\Omega^2}{\cancel{4}} + \frac{{}^{2}\Omega^2}{\cancel{4}} = \frac{\omega_2^2}{\cancel{4}} + \frac{{}^{2}\omega_2^2}{\cancel{4}} + \frac{2g}{\cancel{2}r}$$

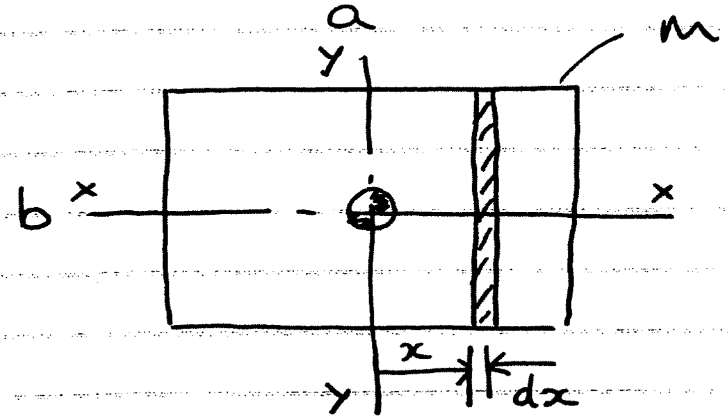
$$\cancel{3}\Omega^2 = \cancel{3}\omega_2^2 + \frac{2}{3}\frac{g}{r}$$

$$\therefore \omega_2^2 = \Omega^2 - \frac{2}{3}\frac{g}{r} = \frac{4}{81}\omega_0^2 - \frac{2}{3}\frac{g}{r}$$

$$\underline{\underline{V_2 = r\sqrt{\frac{4}{81}\omega_0^2 - \frac{2}{3}\frac{g}{r}}}}$$

For $\omega_2 = 0$, $\underline{\underline{\omega_0 = \sqrt{\frac{27}{2}\frac{g}{r}}}}$

2 (a)



Mass/unit area $= \dfrac{M}{ab}$

$I_{yy} = \sum x^2 dm$; $I_{xx} = \sum y^2 dm$

and $I_{zz} = I_{xx} + I_{yy}$. (Perp. axis)

$$I_{yy} = \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dm = \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 \left(\frac{M}{ab}\right) b\, dx$$

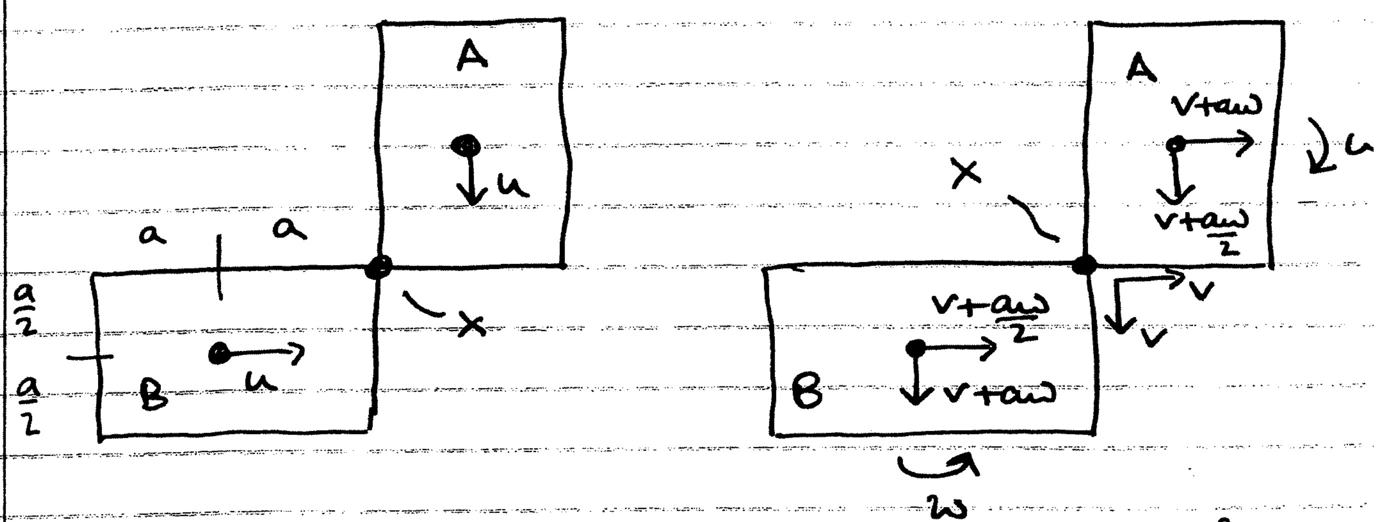
$$= \frac{M}{a}\left[\frac{x^3}{3}\right]_{-\frac{a}{2}}^{a/2} = \frac{M}{3a}\left[\frac{a^3}{8} + \frac{a^3}{8}\right] = \frac{Ma^2}{12}$$

Similarly I_{xx}

$$\therefore I_{zz} = \frac{M}{12}(a^2 + b^2)$$

(b) (i) Equal in magnitude; opposite in direction – from symmetry

(ii) Before After



$$I_G = \frac{M}{12}(4a^2 + a^2) = \frac{5}{12} Ma^2$$

Linear momentum $\xrightarrow{+}$

$$\cancel{m}u = \cancel{m}\left(v + \frac{a\omega}{2}\right) + \cancel{m}(v + a\omega)$$

$$\underline{u = 2v + \frac{3}{2}a\omega} \quad \text{——} \ (1)$$

Moment of ~~momentum~~ for B about X $\circlearrowright +$

$$(\cancel{m}u)\frac{\cancel{a}}{\cancel{2}} = \cancel{m}\left(v + \frac{a\omega}{2}\right)\frac{\cancel{a}}{\cancel{2}} + 2\cancel{m}(v + a\omega)\cancel{a} + \frac{5}{\cancel{12}\,6}\cancel{m}a^{\cancel{2}}\omega$$

$$u = v + \frac{a\omega}{2} + 2v + 2a\omega + \frac{5}{6}a\omega$$

$$\underline{u = 3v + \frac{10}{3}a\omega} \quad \text{——} \ (2)$$

From (1) & (2) : $\quad \underline{v = a\omega\left(\frac{3}{2} - \frac{10}{3}\right) = -\frac{11}{6}a\omega}$ —(3

From (1) $\quad u = -\frac{11}{3}a\omega + \frac{3}{2}a\omega = -\frac{13}{6}a\omega$

$$\therefore \quad \underline{\omega = -\frac{6}{13}\frac{u}{a}} \ ; \quad \underline{v = \frac{11}{13}u}$$

So the velocity of C of G of B :

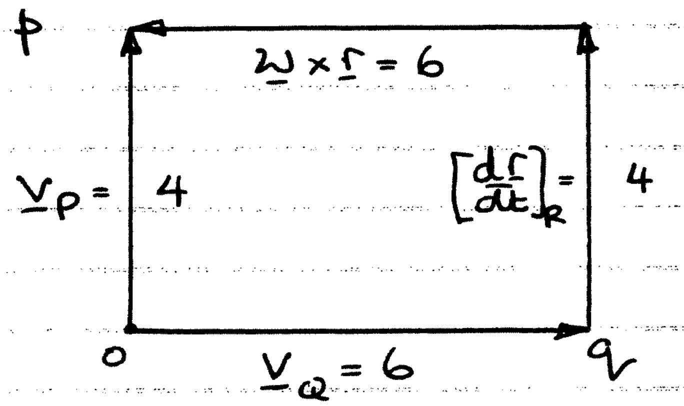
$$\underline{V}_{GB} = \left[\frac{11}{13}u + \frac{a}{2}\left(-\frac{6}{13}\frac{u}{a}\right)\right]\underline{i} \left[= \frac{8}{13}u\underline{i}\right]$$

$$- \left[\frac{11}{13}u + a\left(-\frac{6}{13}\frac{u}{a}\right)\right]\underline{j} \left[= -\frac{5}{13}u\underline{j}\right]$$

$$\underline{\underline{\underline{V}_{GB} = \frac{8}{13}u\underline{i} - \frac{5}{13}u\underline{j}}}$$

3 (a)(i) $\underline{V}_P = \underline{V}_Q + \left[\frac{d\underline{r}}{dt}\right]_R + \underline{\omega} \times \underline{r}$ (Data Book)

Vel diag

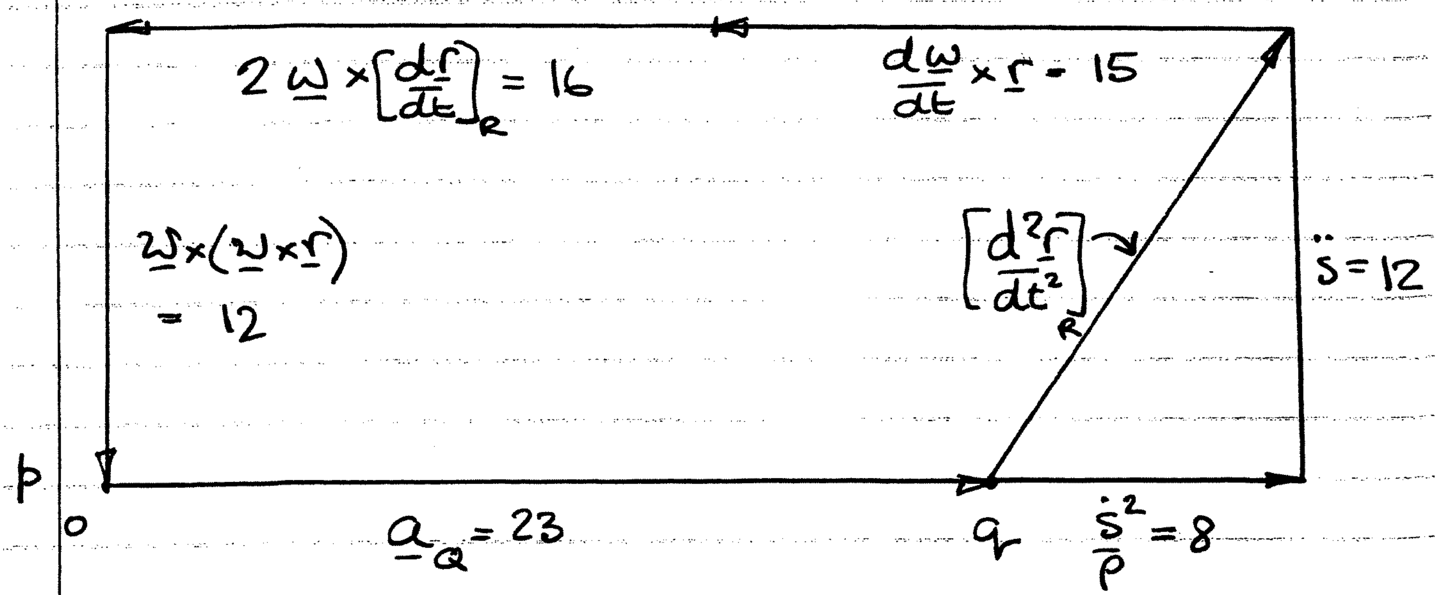


$$\underline{V}_P = 4\underline{j}$$

(ii) $\underline{a}_P = \underline{a}_Q + \left[\frac{d^2\underline{r}}{dt^2}\right]_R + \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \left[\frac{d\underline{r}}{dt}\right]_R + \underline{\omega} \times (\underline{\omega} \times \underline{r})$

(Data Book)

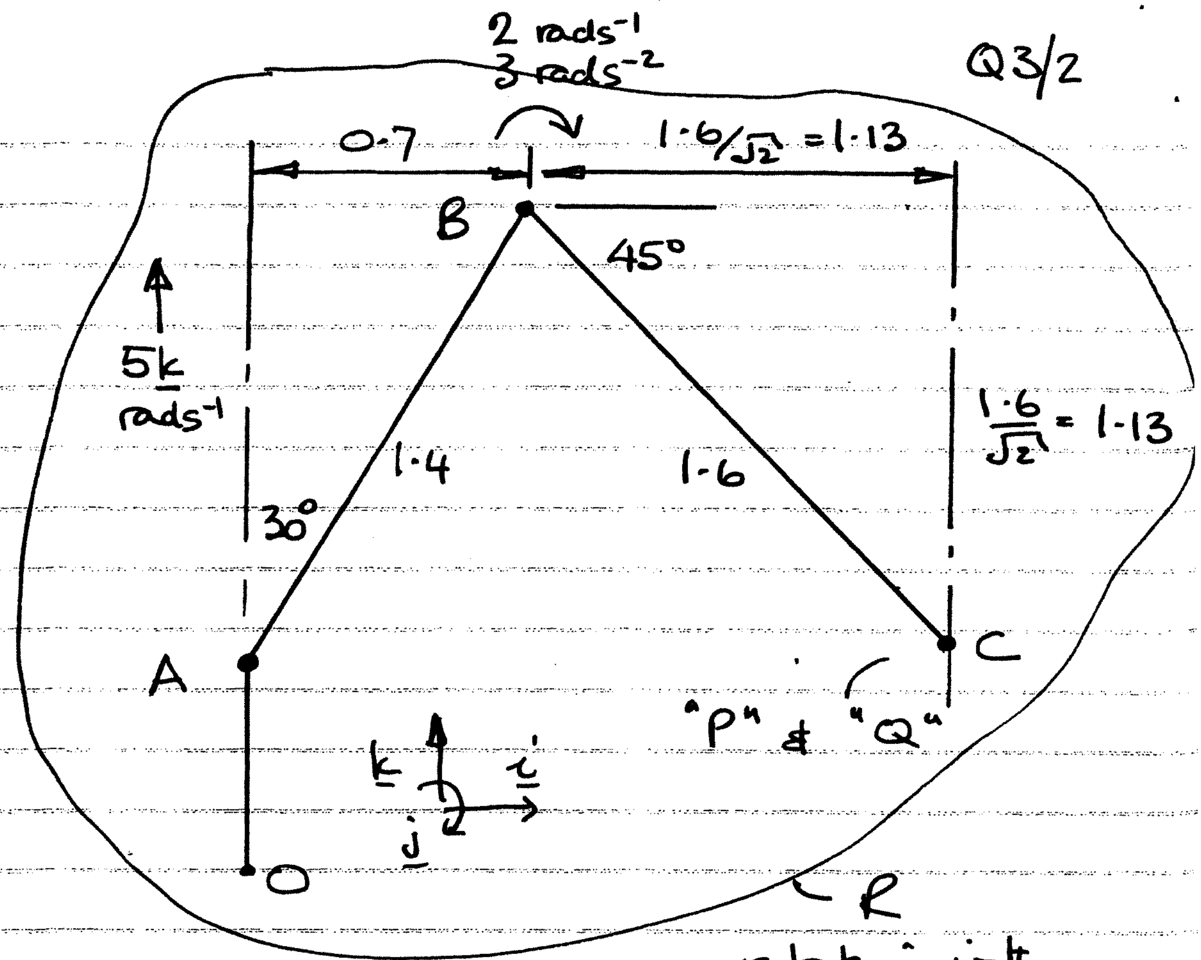
Accn diag



$$\underline{a}_P = 0$$

Q3/2

(b)



rotating with $\underline{\omega} = 5\underline{k}$

$\frac{d\omega}{dt} = 0$

In data book terms, place both P & Q at C, hence $\underline{r} = 0$

A, B, C lie in body rotating with $\underline{\omega}$

$$\underline{V}_P = \underline{V}_Q + \left[\frac{d\underline{r}}{dt}\right]_R + \underbrace{\cancel{\underline{\omega} \times \underline{r}}}_{=0\text{, since } \underline{r}=0}$$

$$\underline{V}_Q = (0.7 + 1.13)\,5\,\underline{j} = 9.15\underline{j}$$

$$\left[\frac{d\underline{r}}{dt}\right]_R = -1.13 \times 2\,(\underline{i} + \underline{k}) = -2.26\underline{i} - 2.26$$

$$\therefore \underline{V}_P = -2.26\underline{i} + 9.15\underline{j} - 2.26\underline{k} \text{ ms}^{-1}$$

$$\underline{a}_P = \underline{a}_Q + \left[\frac{d^2\underline{r}}{dt^2}\right] + \underbrace{\cancel{\frac{d\underline{\omega}}{dt}}}_{=0} \times \underline{r} + 2\underline{\omega} \times \left[\frac{d\underline{r}}{dt}\right]_R + \underbrace{\cancel{\underline{\omega} \times (\underline{\omega} \times \underline{r})}}_{=0}$$

$$\underline{a}_Q = 1.83 \times 5^2 (-\underline{i}) = \underline{-45.75\,\underline{i}}$$

$$\left[\frac{d^2\underline{r}}{dt^2}\right] = 3 \times 1.13(-\underline{i} - \underline{k}) \qquad [\ddot{s}]$$

$$+ 1.13 \times 2^2 (-\underline{i} + \underline{k}) \qquad \left[\frac{\dot{s}^2}{\rho}\right]$$

$$= \underline{-7.91\,\underline{i} + 1.13\,\underline{k}}$$

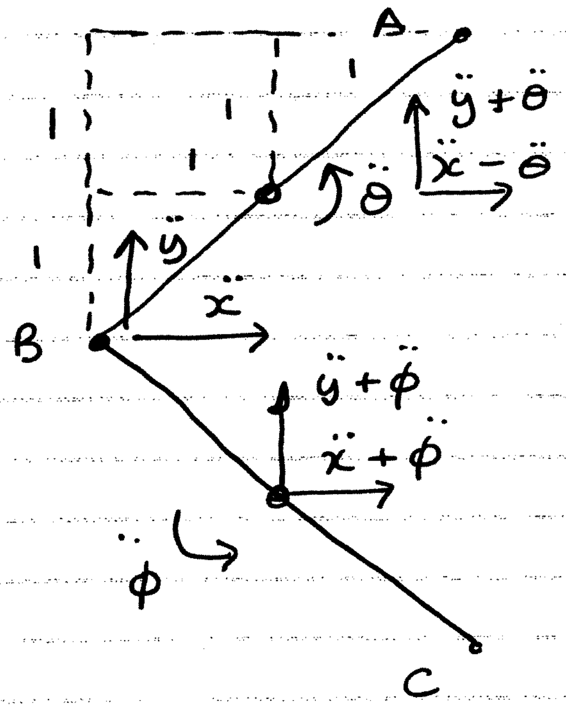
$$2\underline{\omega} \times \left[\frac{d\underline{r}}{dt}\right]_R = 2\left\{5\underline{k} \times (-2.26\,\underline{i} - 2.26\,\underline{k})\right\}$$

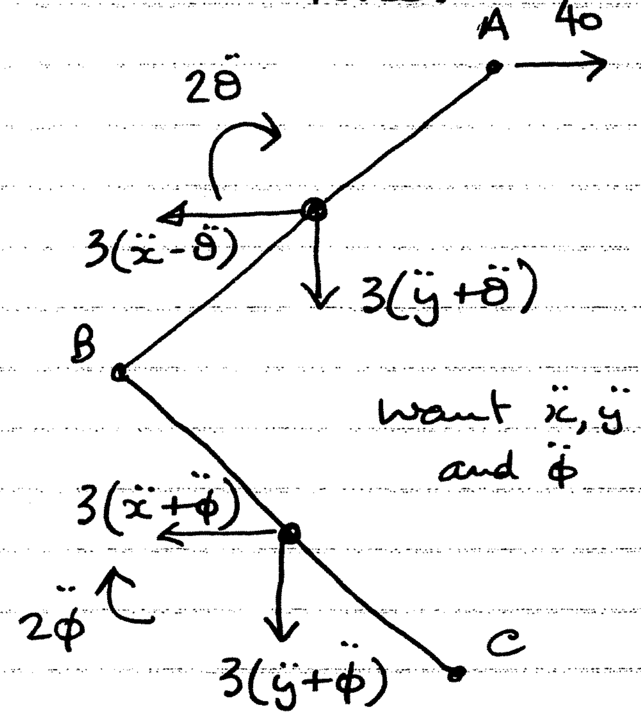
$$= \underline{-22.6\,\underline{j}}$$

$$\underline{\underline{\underline{a}_P = (-53.7\,\underline{i} - 22.6\,\underline{j} + 1.1\,\underline{k})\ \mathrm{ms}^{-2}}}}$$

See attached for parts (a) & (b) Q4/1

4 (c) Accelerations Forces





want $\ddot{x}, \ddot{y}$ and $\ddot{\phi}$

$$I_G = \frac{1}{12} m L^2 = \frac{1}{12} 3 (2\sqrt{2})^2 = \frac{24}{12} = \underline{2 \text{ kgm}^2}$$

For the two rods:

$$\rightarrow \quad 40 = 3\ddot{x} - 3\ddot{\theta} + 3\ddot{x} + 3\ddot{\phi} = 6\ddot{x} - 3\ddot{\theta} + 3\ddot{\phi} \quad (1)$$

$$\downarrow \quad 0 = 6\ddot{y} + 3\ddot{\theta} + 3\ddot{\phi} \quad (2)$$

For AB about B $\curvearrowright$

$$80 = 3(\ddot{x}-\ddot{\theta}) - 2\ddot{\theta} - 3(\ddot{y}+\ddot{\theta})$$

$$80 = 3\ddot{x} - 3\ddot{y} - 8\ddot{\theta} \quad (3)$$

For BC about B $\curvearrowright$

$$0 = 3(\ddot{x}+\ddot{\phi}) + 3(\ddot{y}+\ddot{\phi}) + 2\ddot{\phi}$$

$$0 = 3\ddot{x} + 3\ddot{y} + 8\ddot{\phi} \quad (4)$$

$$
\begin{array}{lrrrrl}
(1) & 6\ddot{x} & & -3\ddot{\theta} & +3\ddot{\phi} & = 40 \\
(2) & & 6\ddot{y} & +3\ddot{\theta} & +3\ddot{\phi} & = 0 \\
(3) & 3\ddot{x} & -3\ddot{y} & -8\ddot{\theta} & & = 80 \\
(4) & 3\ddot{x} & +3\ddot{y} & & +8\ddot{\phi} & = 0
\end{array}
$$

(1) + (2) gives:

$$6\ddot{x} + 6\ddot{y} + 6\ddot{\phi} = 40 \quad \text{— (5)}$$

$2 \times (4)$ gives

$$6\ddot{x} + 6\ddot{y} + 16\ddot{\phi} = 0 \quad \text{— (6)}$$

(6) − (5) gives $10\ddot{\phi} = -40$, $\underline{\ddot{\phi} = -4 \text{ rad s}^{-2}}$

(1) − (2) gives: $\longrightarrow$ $\underline{\underline{\ddot{\underline{\phi}} = -4\underline{k} \text{ rad s}^{-2}}}$

$$6\ddot{x} - 6\ddot{y} - 6\ddot{\theta} = 40 \quad \text{— (7)}$$

$2 \times (3)$ gives:

$$6\ddot{x} - 6\ddot{y} - 16\ddot{\theta} = 160 \quad \text{— (8)}$$

(7) − (8) gives: $10\ddot{\theta} = -120$ $\quad \underline{\ddot{\theta} = -12 \text{ rad s}^{-2}}$

$\ddot{\underline{\theta}} = -12\underline{k} \text{ rad s}^{-2}$

From (1): $6\ddot{x} = 40 - 3\times 12 + 3 \times 4$

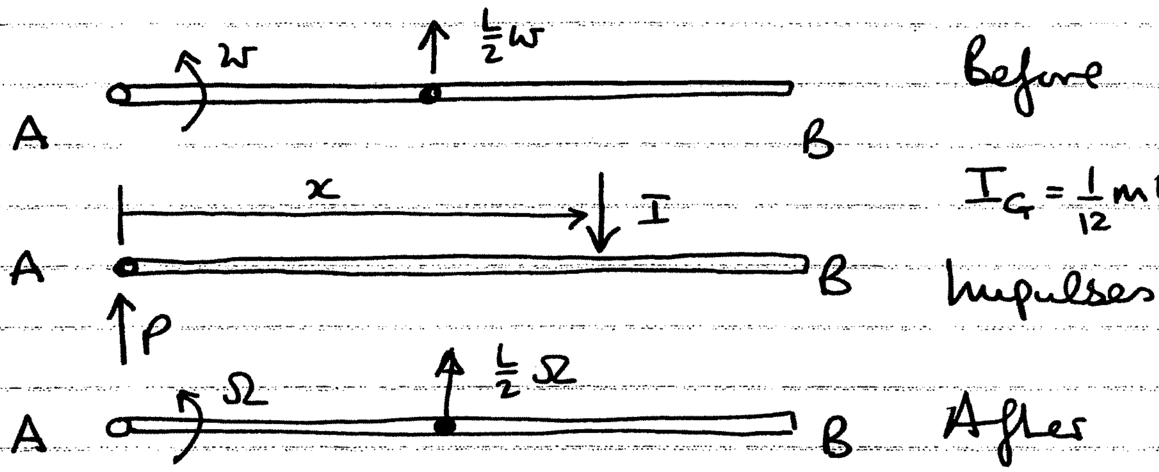
$$\therefore \ddot{x} = \frac{16}{6} = \underline{\frac{8}{3} \text{ ms}^{-2}}$$

From (2): $6\ddot{y} = 3\times 12 + 3\times 4 = 48$

$$\therefore \ddot{y} = 8 \text{ ms}^{-2}$$

$$\underline{\underline{\underline{a}_B = \left(\frac{8}{3}\underline{i} + 8\underline{j}\right) \text{ms}^{-2}}}$$

5 (a)



$I_G = \frac{1}{12} mL^2$

Linear momentum ↑ +ve

$$P - I = M\left(\frac{L}{2}\Omega\right) - M\left(\frac{L}{2}\omega\right)$$

For no P, $\underline{I = \frac{ML}{2}(\omega - \Omega)}$ —— (1)

Moment of Momentum about A ↺ +ve

$$-Ix = \left[M\frac{L\Omega}{2}\frac{L}{2} + I_G\Omega\right] - \left[M\frac{L\omega}{2}\frac{L}{2} + I_G\omega\right]$$

$$-Ix = \frac{ML^2}{4}\left[\Omega + \frac{\Omega}{3} - \omega - \frac{\omega}{3}\right]$$

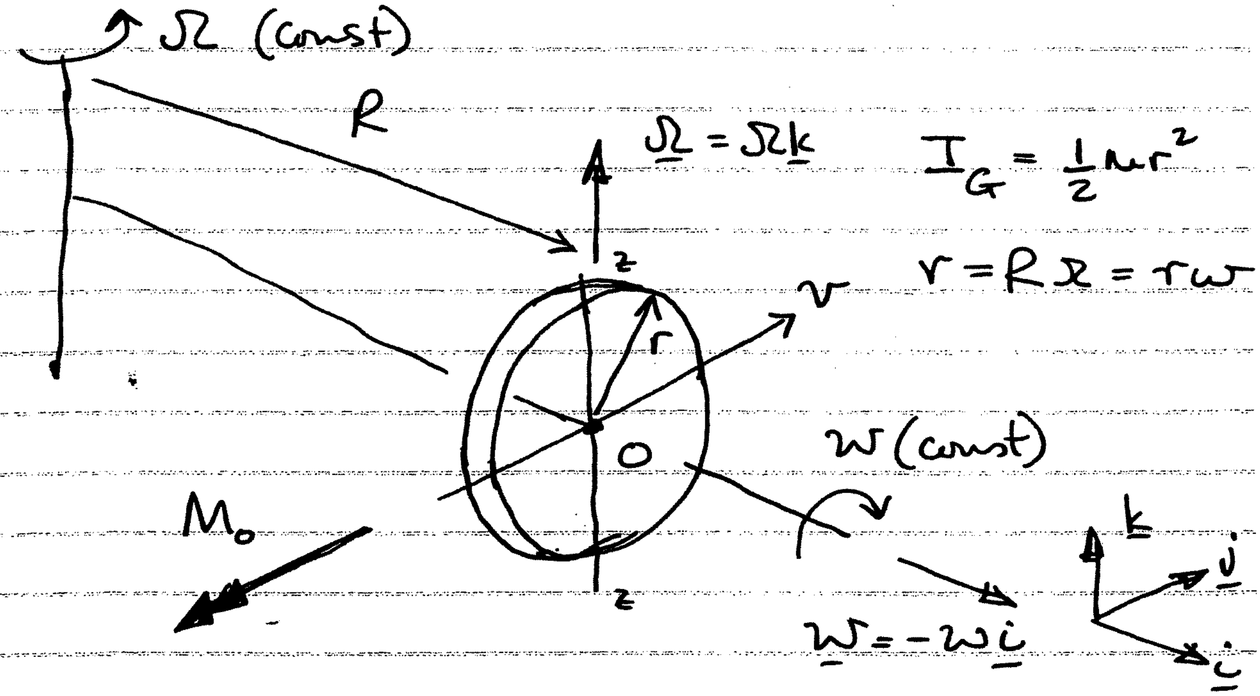
$$\therefore \underline{Ix = \frac{ML^2}{3}(\omega - \Omega)} \quad \text{—— (2)}$$

From (1) and (2)

$$x\,\frac{\cancel{M}L}{2}\cancel{(\omega-\Omega)} = \frac{\cancel{M}L^2}{3}\cancel{(\omega-\Omega)}$$

$$\underline{\underline{x = \frac{2}{3}L}}$$

(b)



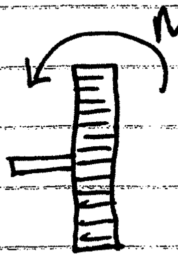
$I_G = \frac{1}{2} m r^2$

$v = R\Omega = r w$

$$\underline{H}_o = I_G \underline{w} + I_{zz} \underline{\Omega} = -I_G w \underline{i} + I_{zz} \Omega \underline{k}$$

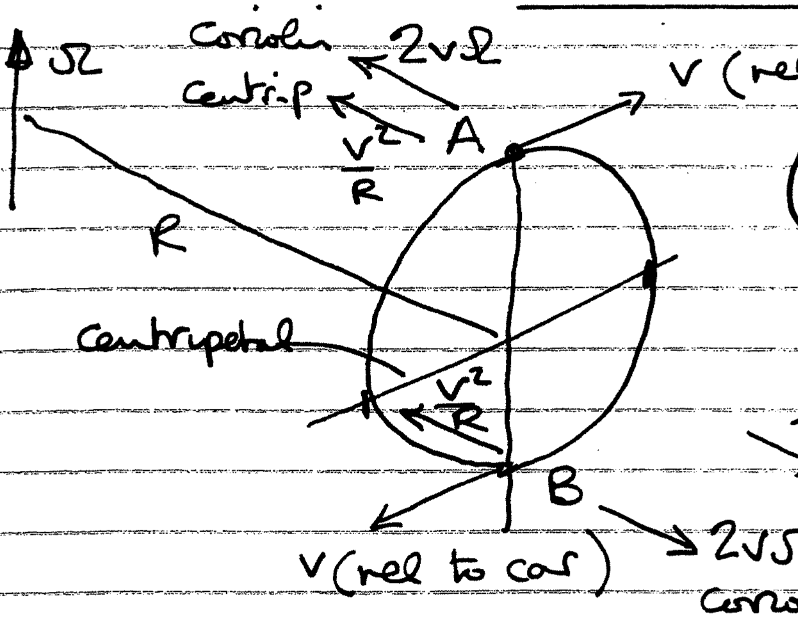
$$\dot{\underline{H}}_o = \underline{M}_o = -I_G(\underbrace{\cancel{\dot{w}}}_{=0}\underline{i} + w\dot{\underline{i}}) + I_{zz}(\underbrace{\cancel{\dot{\Omega}}}_{=0}\underline{k} + \Omega \underbrace{\cancel{\dot{\underline{k}}}}_{=0})$$

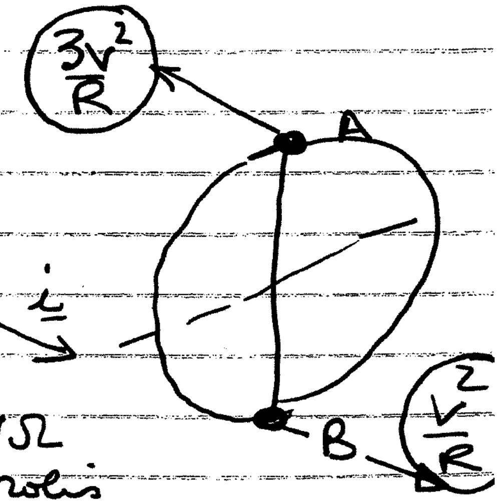
$$\underline{M}_o = -I_G w(\Omega \underline{j}) = -\frac{1}{2} m r^2 \cdot \frac{v}{r}\frac{v}{R}\underline{j} = \underline{\underline{-\frac{m r v^2}{2R}\underline{j}}}$$



Looking from the rear

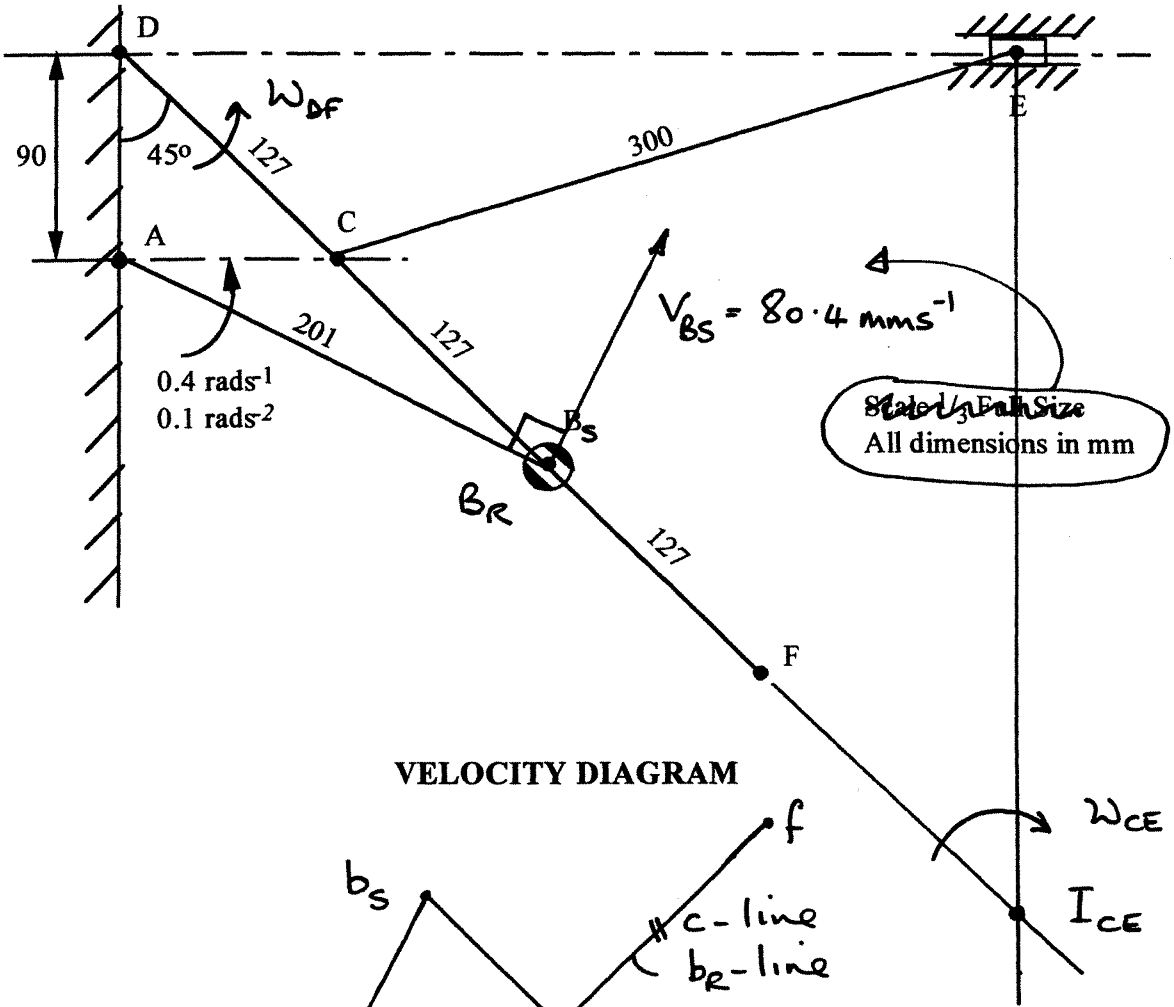
<u>Accelerations of A&B in $\underline{i}$-di</u>



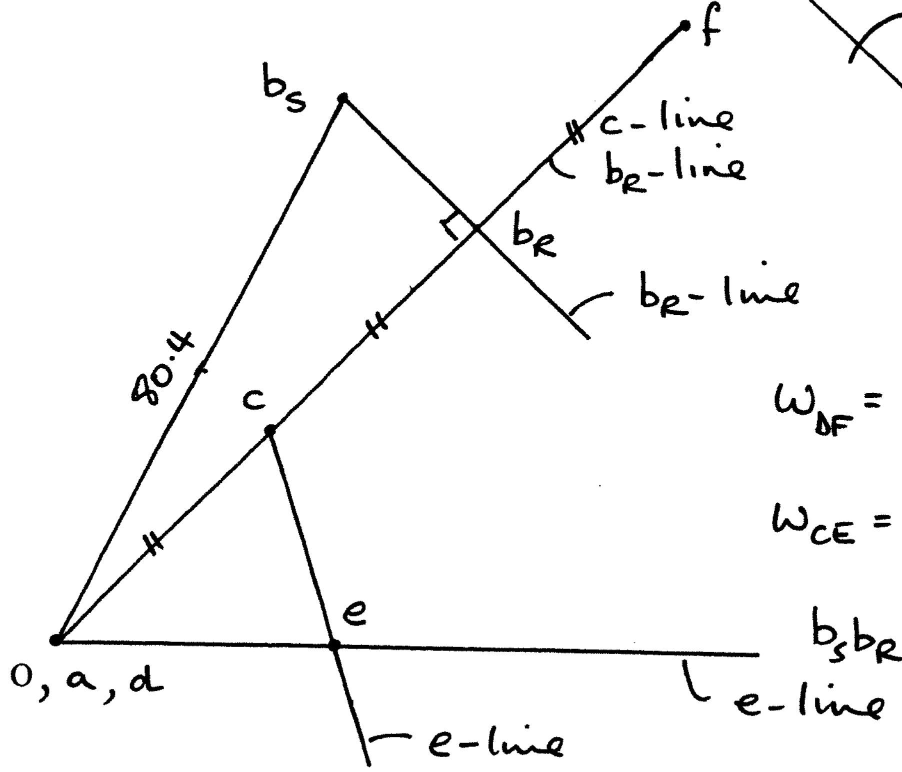


ENGINEERING TRIPOS PART IB Paper 1 MECHANICS Question 6

This sheet is to be attached to your script



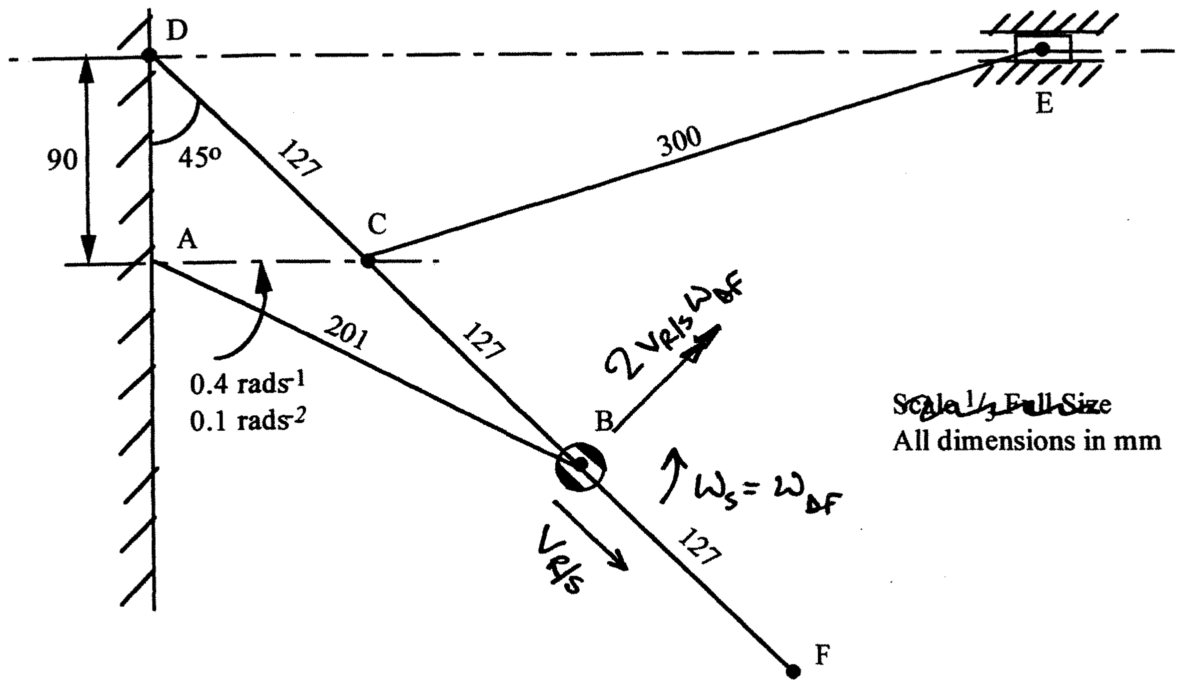
VELOCITY DIAGRAM



$$\omega_{DF} = \frac{df}{DF} = \frac{114}{381} = \underline{\underline{0.3 \text{ rads}}}$$

$$\omega_{CE} = \frac{ce}{CE} = \frac{30}{300} = \underline{\underline{0.1 \text{ rads}}}$$

$$b_s b_R = \underline{\underline{24 \text{ mms}^{-1}}}$$

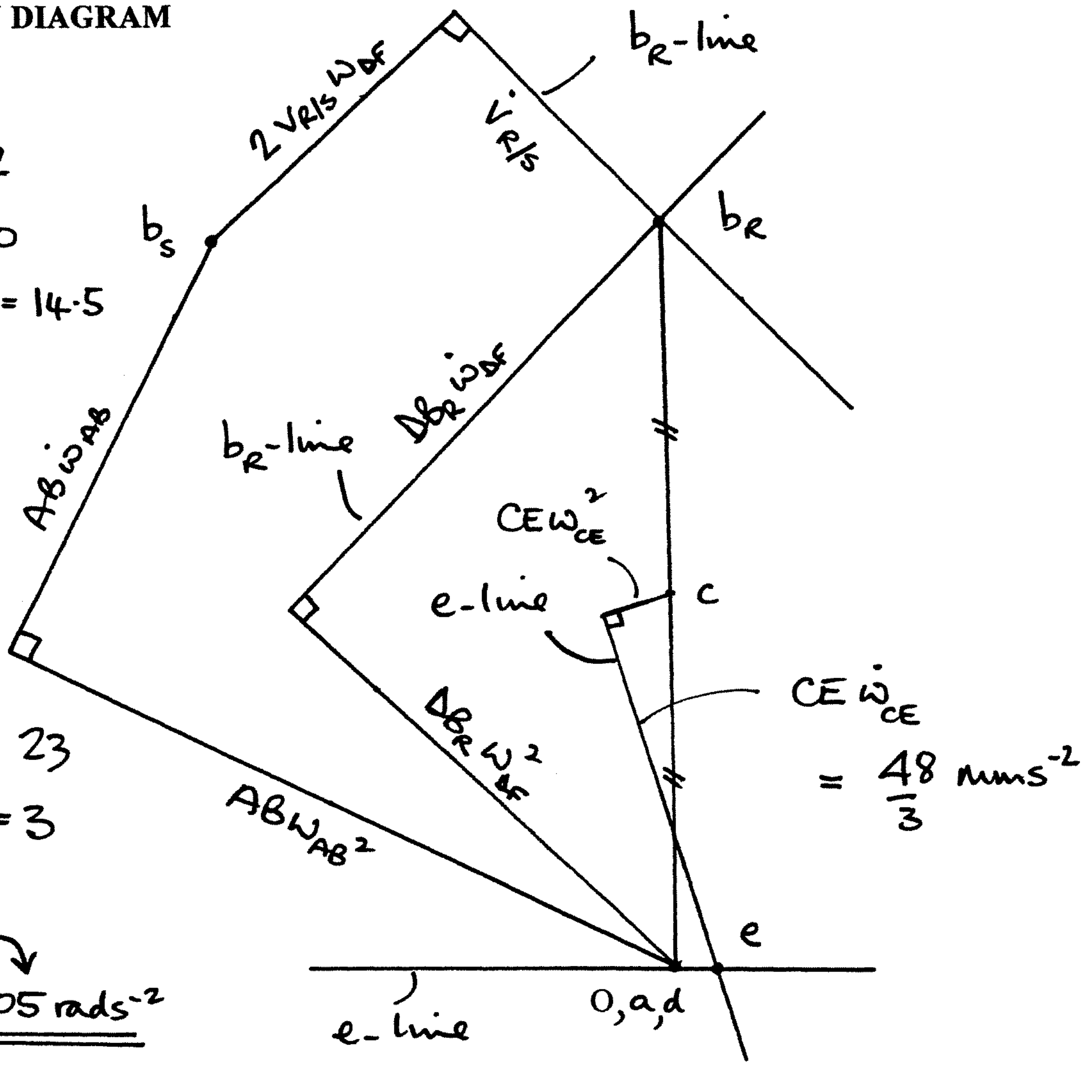


ACCELERATION DIAGRAM

$AB\omega_{AB}^2 = 201 \times 0.4^2 = 32$

$AB\dot{\omega}_{AB} = 201 \times 0.1 = 20$

$2V_{R/S}\,\omega_{DF} = 2 \times 24 \times 0.3 = 14.5$



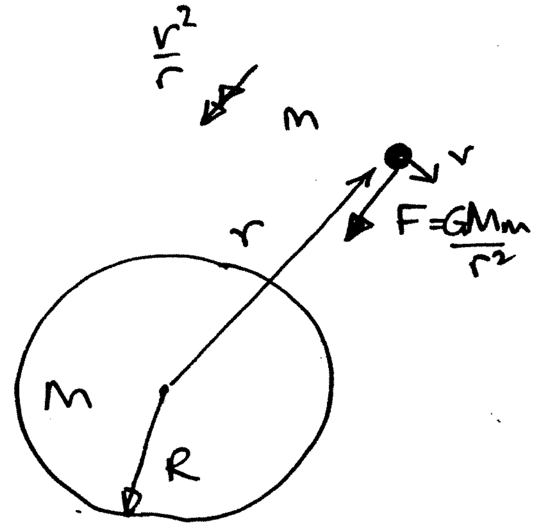
$DB_R\omega_{DF}^2 = 254 \times 0.3^2 = 23$

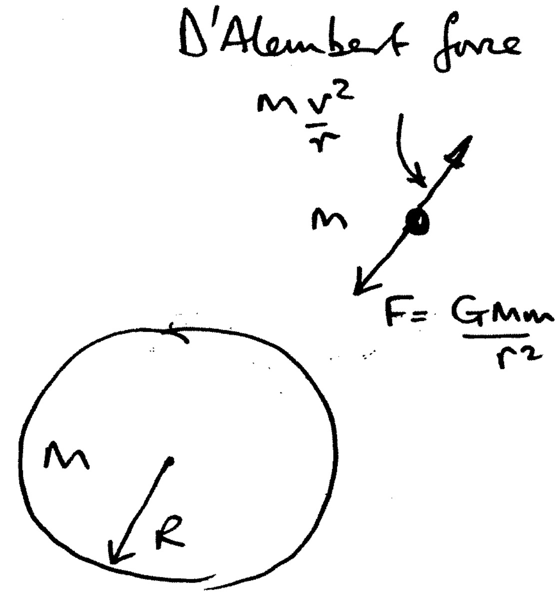
$CE\,\omega_{CE}^2 = 300 \times 0.1^2 = 3$

$\dot{\omega}_{CE} = \dfrac{16}{300} = \underline{\underline{0.05 \text{ rads}^{-2}}}$

$oe = \dfrac{5}{3} = \underline{\underline{1.7 \text{ mms}^{-2}}} \rightarrow$

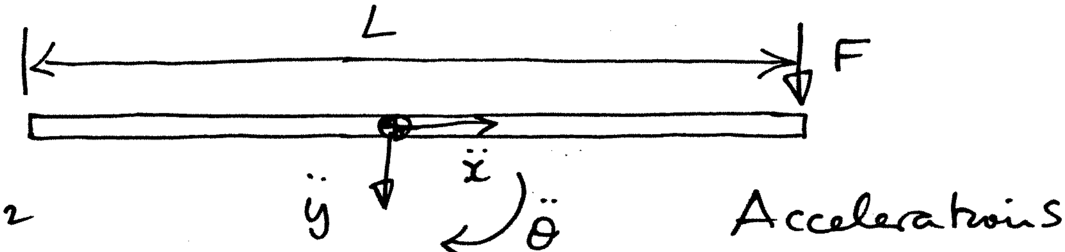
4 (a)



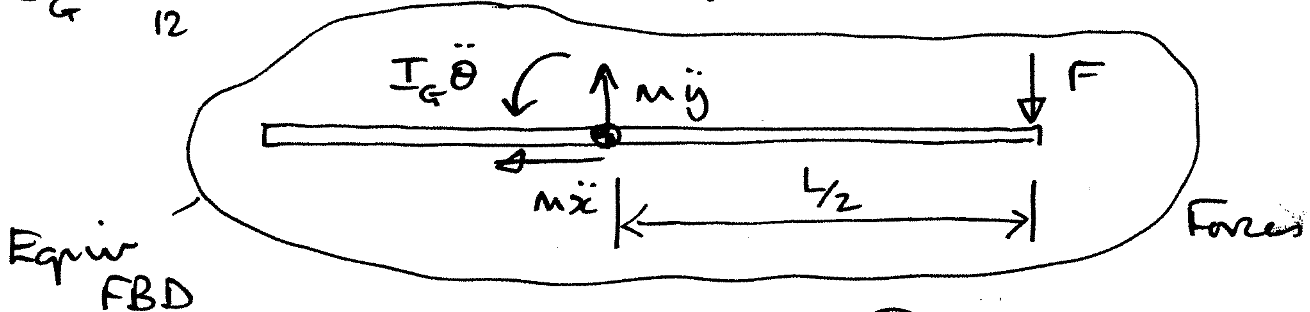


Dynamics	Statics equiv
$\underline{F} = m\underline{a}$	$\sum \underline{F} = 0$
+ve $\quad \frac{GMm}{r^2} = m\left(\frac{v^2}{r}\right)$	$\frac{GMm}{r^2} - m\frac{v^2}{r} = 0$

(b)



$I_G = \frac{1}{12} mL^2$



Moments about C of G : (+)

$F\frac{L}{2} - \frac{1}{12} mL^2 \ddot{\theta} = 0 \; ; \quad F = \frac{1}{6} mL\ddot{\theta} \quad \therefore \; \underline{\underline{\ddot{\theta} = \frac{6F}{mL}}}$

$\overset{+}{\leftarrow} \quad m\ddot{x} = 0 \quad \therefore \; \underline{\underline{\ddot{x} = 0}}$

$\uparrow + \quad m\ddot{y} - F = 0 \quad \therefore \; \underline{\ddot{y} = \frac{F}{m}}$