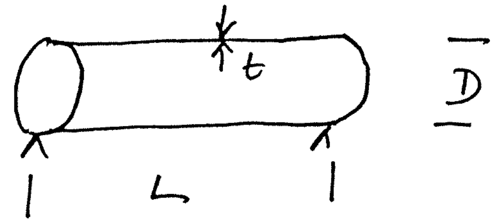
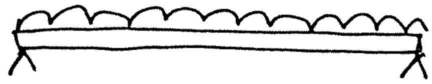
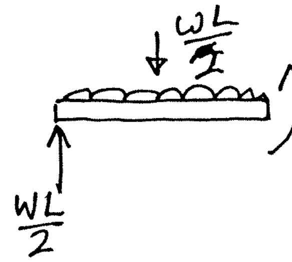
1B 2000 P2/1/1

STRUCTURES PAPER — SOLUTIONS + EXAMINERS COMMENTS.

1. (a) (i)







$M = \frac{wL^2}{2} - \frac{wL}{4}\cdot\frac{L}{2}$

Volume $= \frac{\pi D^2 L}{4}$

Weight $= \frac{\pi D^2 L}{4} \rho g$

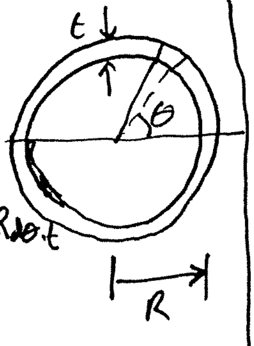
$\therefore$ Bending moment at centre (sagging) $= \frac{wL}{8} = \frac{\pi D^2 L^2}{32} \rho g$

Simple bending theory $\frac{\sigma}{y} = \frac{M}{I}$ (worst at top or bottom)

$I = \int y^2 dA$

$= \int_0^{2\pi} R^2 \sin^2\theta . R d\theta . t$

$= \pi R^3 t$



$\therefore \sigma = \frac{My}{I} = \frac{\cancel{\pi} D^2 L^2 \rho g}{\cancel{32}_{4}\ \cancel{\pi} D^3 t} \cdot \frac{D}{2} \cdot 8$

$= \frac{L^2 \rho g}{8t}$

(4 marks)

Alternative calculation of I. I for circular rod $= \frac{\pi R^4}{4}$ (data book)

$\therefore$ For ring of thickness t

$I = \frac{\pi R^4}{4} - \pi\frac{(R-t)^4}{4} = \cancel{\frac{\pi R^4}{4}} - \left[\cancel{\frac{\pi R^4}{4}} - \frac{\cancel{4}\pi R^3 t}{\cancel{4}} + O.t^2\right]$

$= \pi R^3 t$

(ii) Longitudinal stress due to pressure $p = \frac{pD}{4t}$

Hoop stress due to pressure $p = \frac{pD}{2t}$

(4 marks)

(b) At the underside of the tank in the centre the shear stress is zero.

$\therefore$ The axial stress is given by (i)

$$\sigma = \frac{L^2 \rho g}{8t} = \frac{(20.\cancel{10^3})^2 . (\cancel{10^3}.\cancel{10^{-9}}).9.81}{8t} \quad (N/mm^2)$$

(t in mm)

$$= \underline{\underline{490/t}}$$

Shear stress is zero.

Axial stress due to radial pressure

$$= \frac{2.2000}{4t} = 1000/t \quad (N/mm^2)$$

Hoop stress due to radial pressure $= 2000/t \quad (N/mm^2)$

Von Mises Yield criterion

$\sigma_1 = 1490/t$

$\sigma_2 = 2000/t$

$\sigma_3 = (-2)$ — could assume this was zero.

$$\left(\frac{1490}{t} - \frac{2000}{t}\right)^2 + \left(\frac{2000}{t} + \cancel{2}\right)^2 + \left(\frac{1490}{t} + \cancel{2}\right)^2 = 2.400^2$$

≈ 0 ≈ 0

$$\Rightarrow \left(\frac{510}{t}\right)^2 + \left(\frac{2000}{t}\right)^2 + \left(\frac{1490}{t}\right)^2 = 2 \cdot 400^2$$

$$\Rightarrow t^2 = 20 \cdot 25 \text{ mm}^2 \quad \Rightarrow \underline{\underline{t = 4 \cdot 5 \text{ mm}}}$$

(9 marks)

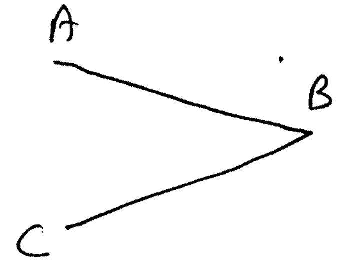
To apply a factor of safety of 2, $\underline{\underline{t = 9 \cdot 0 \text{ mm}}}$

Alternatively, use an allowable stress of $\frac{400}{2} = 200 \text{ N/mm}^2$ in the von Mises expression. Leads to the same answer

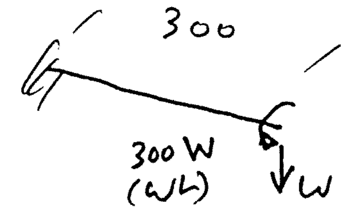
(c) Because the bending stresses add to the axial tension caused by the internal pressure on the base. At the top they are in the opposite sense (3 marks)

A very popular question. They could determine the stresses due to the internal pressure, and could combine the stresses using the von Mises yield condition well, but they could not determine the bending stresses. Very few drew a proper free-body diagram and very few got the caculation of I correct. Many said that $I = \int y^2 dA$ but then wrote down something spurious; no one tried to do the integration directly and many did not apply the fact that t<<D to simplify the calculation. Many left g out of their calculation of self-weight, and M meaning bending moment was confused with m meaning mass by several. Virtually none knew the distinction between *principle* and *principal*. When asked to apply a safety factor of 2 only 44% correctly halved the allowable stress or doubled the resulting thickness. Many *doubled* the yield stress – others said that the 2 in the von Mises expression for $2\sigma_y^2$ *was* the safety factor, or added another factor of 2. Two candidates went so far as to work out the thickness with and without a safety factor but made no comment when they found that the structure was supposedly thinner when a safety factor was used.

(a)



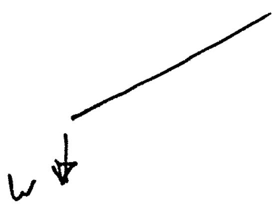
Consider AB first



End deflection at B due to $W = \dfrac{WL^3}{3EI}$

End twist due to $WL = \dfrac{WL}{GJ}.L = \dfrac{WL^2}{GJ}$

for BC



End deflection due to bending of $BC = \dfrac{WL^3}{3EI}$

$\therefore$ Total end deflection $= \dfrac{2WL^3}{3EI} + L.\dfrac{WL^2}{GJ}$

What is I? $= \dfrac{10.20^3}{12} - \dfrac{9.18^3}{12} = \underline{\quad 2292 \text{ mm}^4 \quad}$

$$J = \frac{4\ Ae^2}{\oint \frac{ds}{t}} = \frac{4.(19 . 9.5)^2}{\underbrace{\left(\frac{2.19}{0.5} + \frac{2.9.5}{1}\right)}_{95}} = \underline{1372 \text{ mm}^4}$$

For aluminium $E = 70$ KN/mm^2

$G = 26$ KN/mm^2

$\therefore$ Tip deflection $= WL^3 \left(\dfrac{2}{3EI} + \dfrac{1}{GJ} \right)$

$= 10 \cdot 9.81 \cdot (300)^3 \left(\dfrac{2}{3 \cdot 70 \cdot 10^3 \cdot 2292} + \dfrac{1}{26 \cdot 10^3 \cdot 1372} \right)$

$= \underline{\underline{85.3 \text{ mm}}}$ (6 marks)

(b) Torque $= 29430$ Nmm $\quad (= 10 \cdot 9.81 \cdot 300)$

Bending moment $= 29430$ Nmm

Shear force $= 98.1$ N

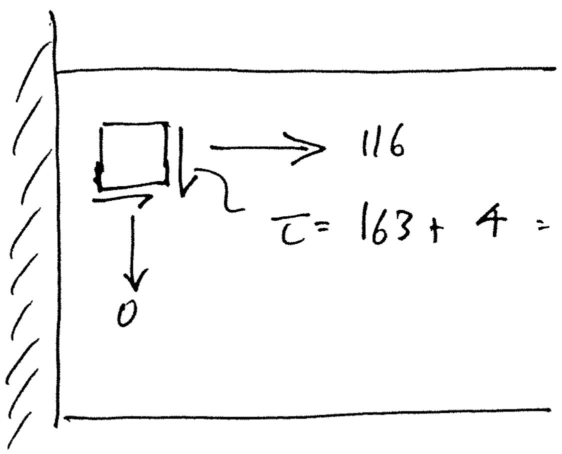
Direct stress $= \dfrac{My}{I} = \dfrac{29430 \cdot 9}{2292} = 116 \text{ N/mm}^2$

Shear stress due to shear force $= \dfrac{FA\bar{y}}{I \cdot 2t}$

$= \dfrac{98.1 \cdot 10 \cdot 1 \cdot 9.5}{2292 \cdot 2 \cdot 0.5} = 4.07 \text{ N/mm}^2$

Shear stress due to torque $= \dfrac{T}{2A_e t} = \dfrac{29430}{2 \cdot 19 \cdot 9.5 \cdot 0.5}$

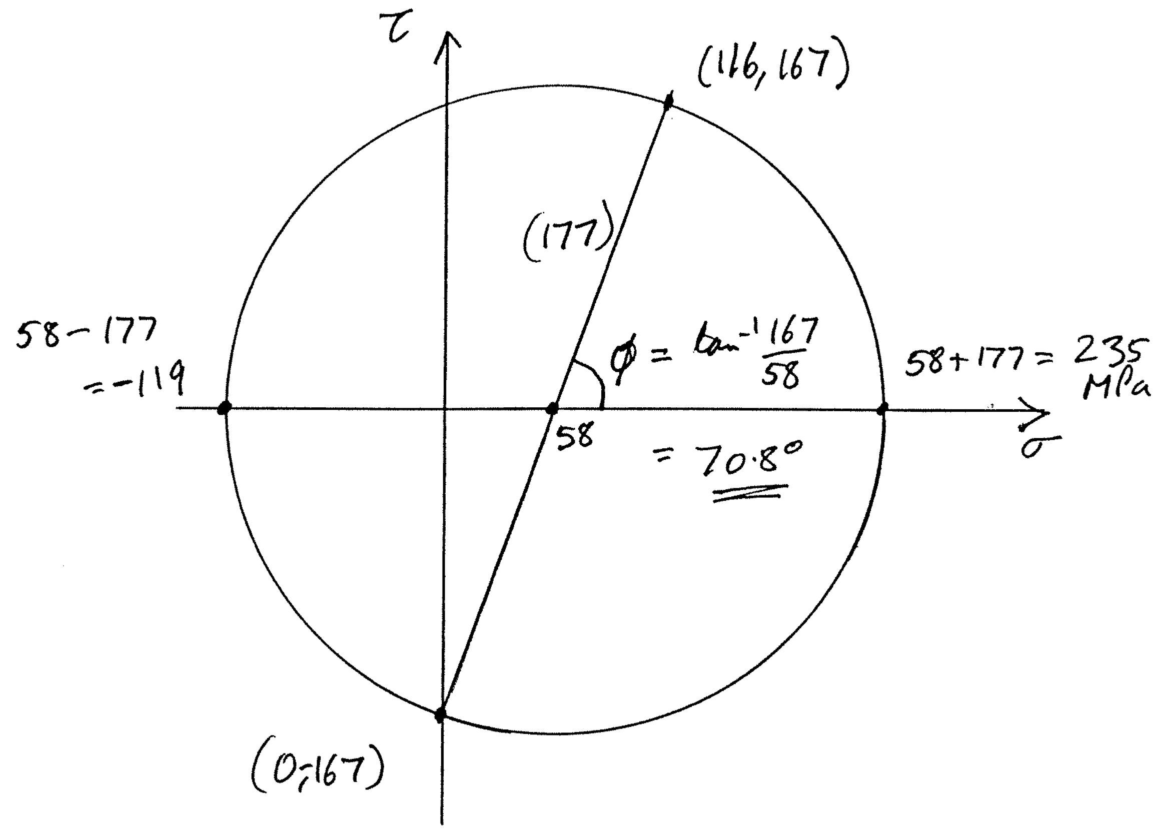
$= 163 \text{ N/mm}^2$

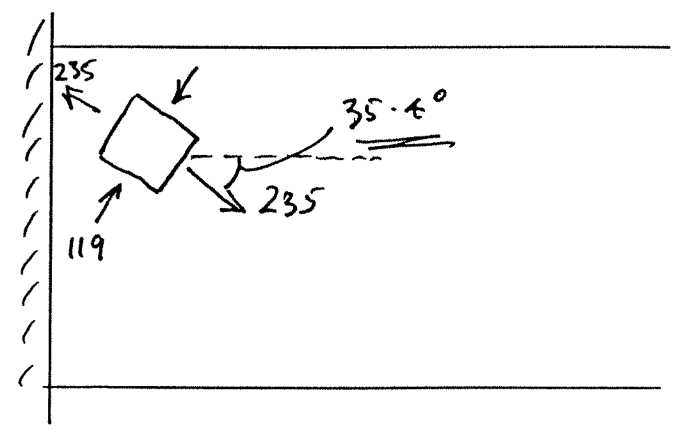


$\tau = 163 + 4 = 167 \text{ N/mm}^2$ (important that they add!)

(8 marks)

(c)





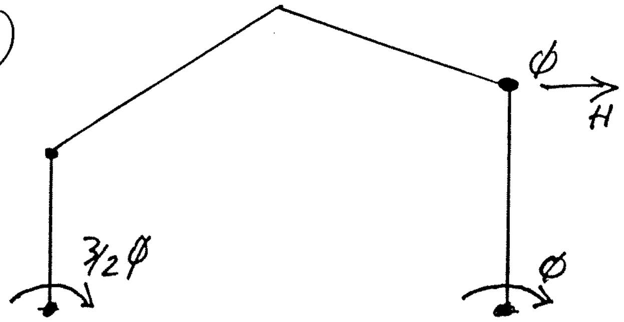
(6 marks)

Very popular. No real pattern to the errors. Most of them realised the deflections could be found by combining three separate components but several left one out or assumed that the outer portion applied bending moment to the inner portion, rather than torque. The stress calculations were mainly done correctly, but they had trouble making sense of the shear stresses and very few of them bothered to think about the *direction* of the shear stress – most showed that the vertical shear stresses applied by the beam to the clamp were upwards, with the consequence that their Mohr's circles were upside down. A significant number assumed that the bending stresses were vertically upwards. The most common error of all was the inconsistent use of units, with little or no check on N or kN, m or mm, and a significant number used suffixes m, k, μ, M and G, which their calculators display, in place of a proper set of units, and lost significant marks because they made consequential errors.

3 (a), + (b)

↑ I at ∞

(I)

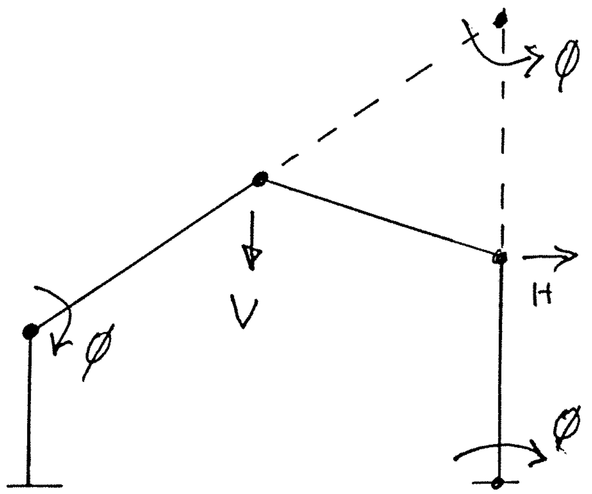


$$15 \cdot H \cdot \phi = 5 M_p \cdot \phi$$

$$\therefore H = \frac{M_p}{3}$$

(2 marks)

(II)

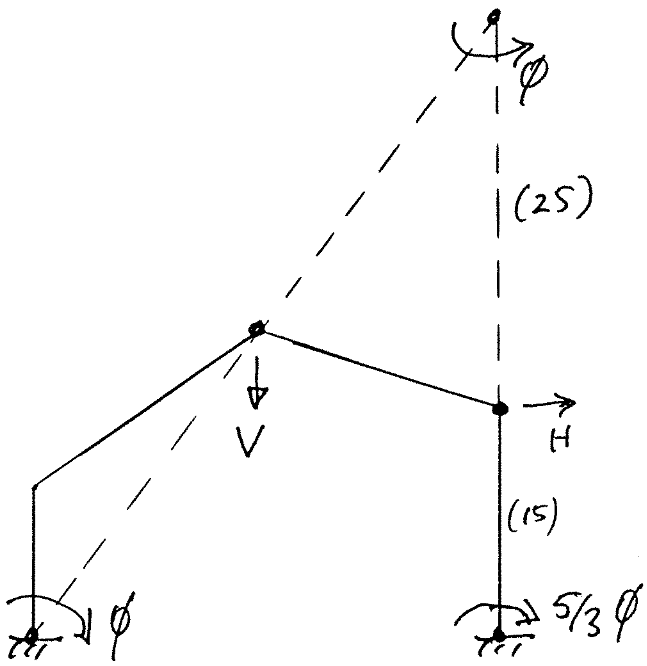


$$15 \cdot V \cdot \cancel{\phi} + 15 H \cancel{\phi} = 6 M_p \cancel{\phi}$$

$$\therefore V + H = 0.4 M_p$$

(2 marks)

(III)



$$15 V \cancel{\phi} + \frac{5}{3} \cdot 15 \cdot H \cdot \cancel{\phi} = \frac{22}{3} \cancel{\phi} M_p$$

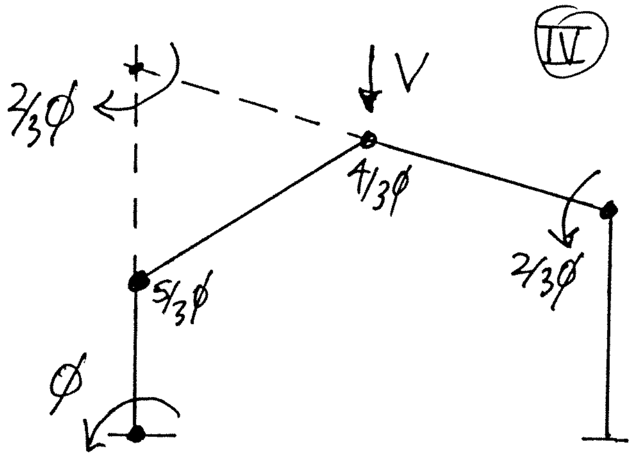
$$\therefore V + \frac{5}{3} H = \frac{22}{45} M_p$$

(2 marks)

3 marks for sketched mechanisms

10 marks for calculations plus diagram overleaf.

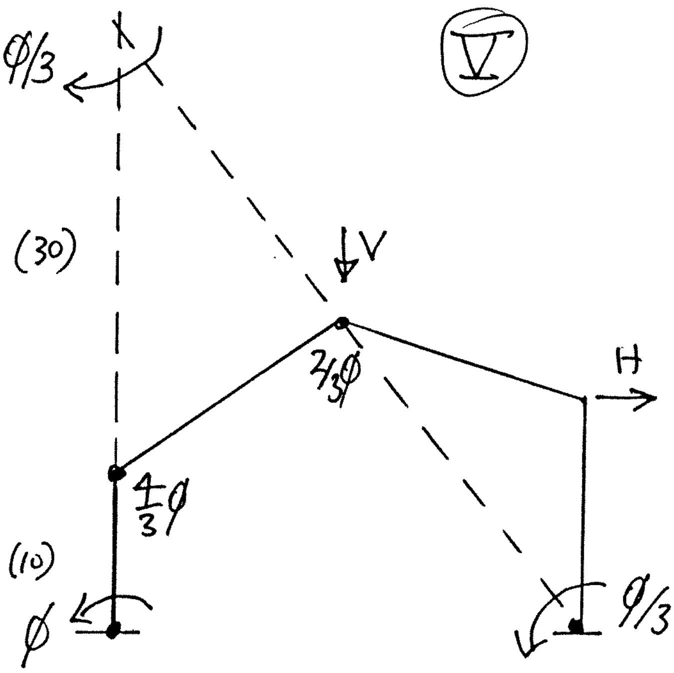
Two other mechanisms are possible and due credit was given if these were used



H does no work

$$\underbrace{\left(1+\frac{5}{3}+\frac{4}{3}+\frac{2}{3}\right)}_{\frac{14}{3}} M_p.\phi = 15V.\frac{2}{3}\phi$$

$$\Rightarrow \underline{\underline{V = \frac{7}{15} M_p}}$$



H does negative work

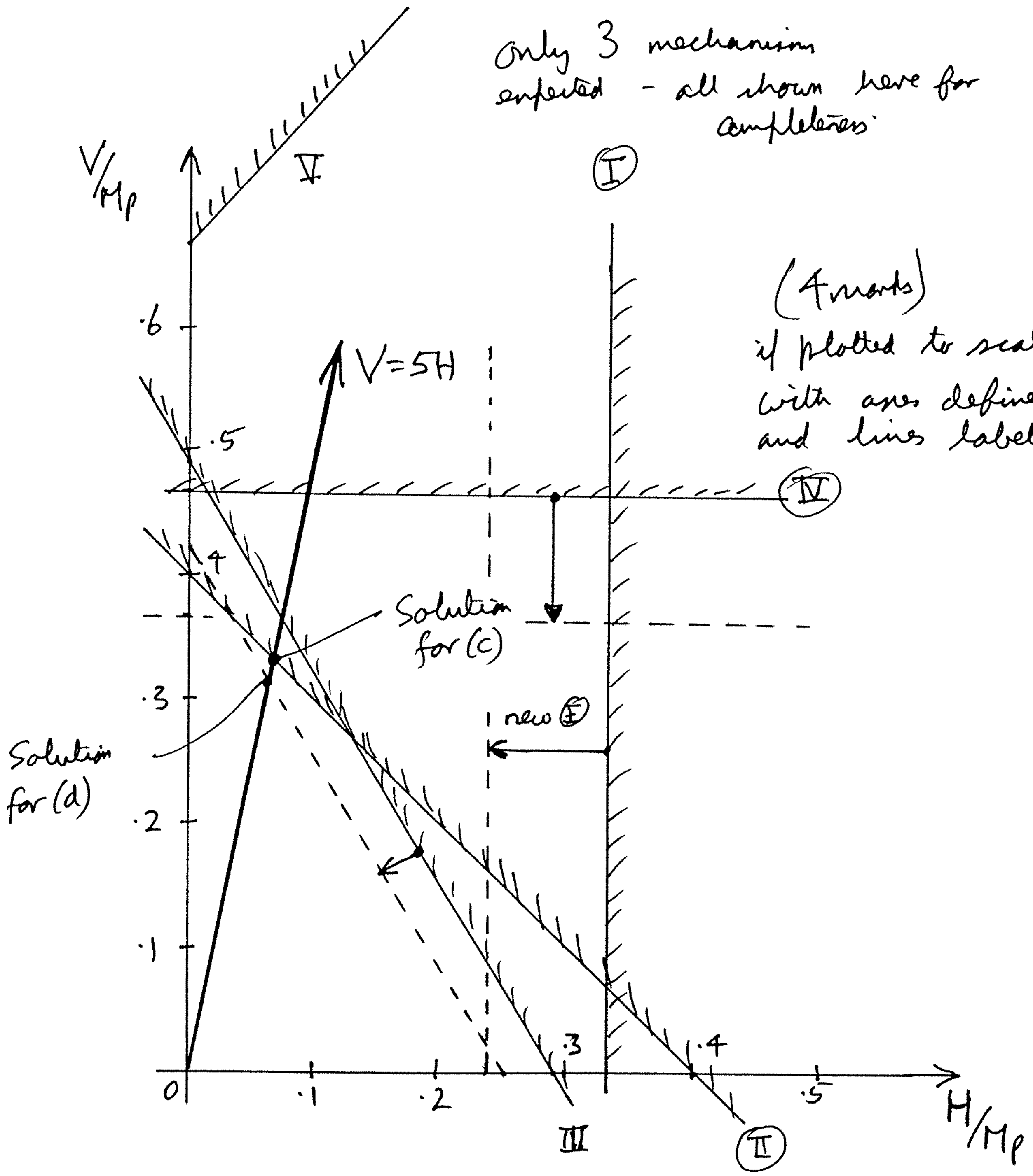
$$\left(1+\frac{4}{3}+\frac{2}{3}+\frac{1}{3}\right)M_p\phi$$

$$= 15V.\frac{\phi}{3} - 15H.\frac{\phi}{3}$$

$$\frac{10}{\cancel{3}} M_p \cancel{\phi} = \frac{15}{\cancel{3}}(V-H)\cancel{\phi}$$

$$\Rightarrow \underline{\underline{V - H = \frac{2}{3} M_p}}$$

Only 3 mechanisms expected - all shown here for completeness.



(4 marks)
if plotted to scale, with axes defined and lines labelled.

(c)

From figure, for $V = 5H$, mode Ⅱ governs

$$V + H = 6H = 0\cdot4\,M_p \qquad H = M_p/15\ ;\ V = M_p/3$$

(3 marks)

(d) If pin at A, no work done there

I becomes $15H = \frac{7}{2} M_p \quad \Rightarrow H = 0{\cdot}233 M_p$

II cannot occur since it now has 5 hinges which is one more than needed.

III becomes $15V + \frac{5}{3}.15H = \frac{19}{3} M_p$

IV becomes $V = \frac{11}{30}.M_p$

V becomes $V - H = \frac{7}{15} M_p$

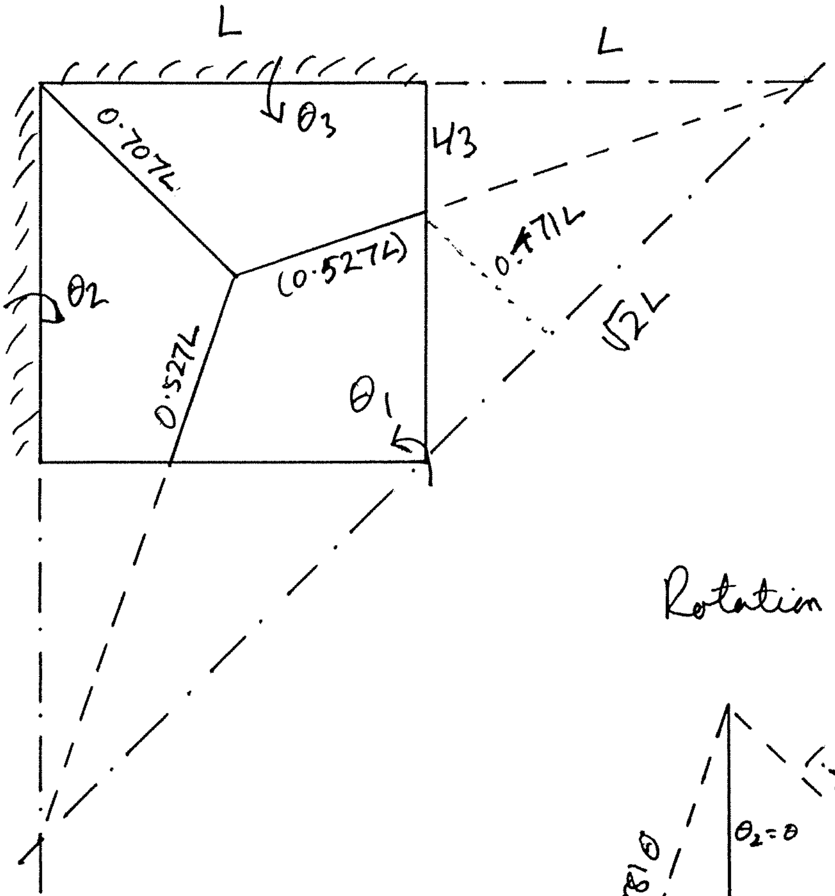
Replot these lines as dashed lines on plot.

From revised plot, solution is now mode III
$H = {\cdot}0633 M_p$; $\underline{\underline{V = 0{\cdot}3166 M_p}}$.

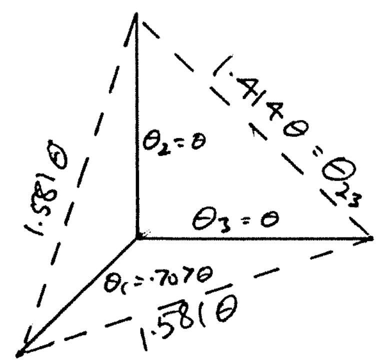
Those who drew a reasonably accurate sketch of the collapse mechanism, showing the instantaneous centres and the dimensions, got it right. Those who didn't got their angles and lengths wrong, and more importantly led themselves astray in what was going on.

"Sketch the collapse mechanism" does not mean draw a few wavy lines – it means draw a diagram from which I can determine where you are assuming the hinges to be. Similarly "plot the interaction diagram" means plot to scale, preferably on graph paper, using scales with values marked. It does not mean draw something the size of a postage stamp, in the bottom left hand corner of a sheet of paper, with no indication of what the various lines mean.

3 marks for basic mechanism



Rotation hodograph



(4 marks)

Compatability requires

$\theta_2 = \theta_3 = \theta$

and $\theta_1 .707 = \theta . 0.5$

$\Rightarrow \theta_1 = 0.707\,\theta$

$\therefore$ Work done in hinges

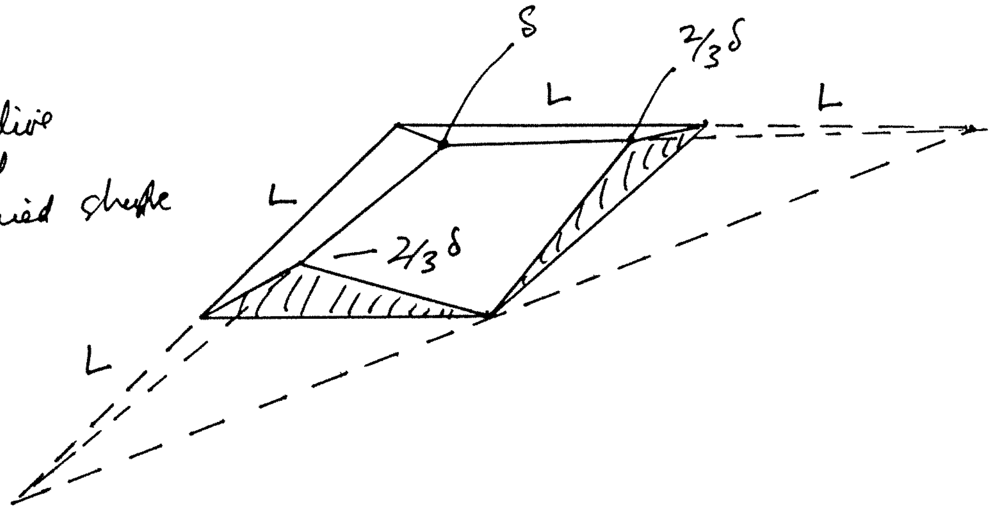
$= (0.707L \,.\, 1.414\,\theta + 2 \,.\, 0.527L \,.\, 1.581\,\theta)\,m$

$= \underline{2.67\, mL\theta}$

(4 marks)

δ = central deflection = $\theta \cdot \frac{L}{2}$

Perspective view of displaced shape



To calculate the swept volume

Volume of complete pyramid less the two dotted regions

$$\frac{\delta}{3} \cdot \frac{(2L)^2}{2} - 2 \cdot \frac{2}{3}\frac{\delta}{3}\frac{L^2}{2} = \frac{4}{9}\delta L^2$$

two regions

$$= \frac{2}{9}\theta L^3$$

Equate to hinge work $2\text{-}67 m \cancel{L}\cancel{\phi} = \frac{2}{9}\cancel{\phi} L^{\cancel{3}\,2} \cdot w$

$$\underline{\underline{w = 12\frac{m}{L^2}}}$$

(c) If extra work is done in AB ~~...~~ of

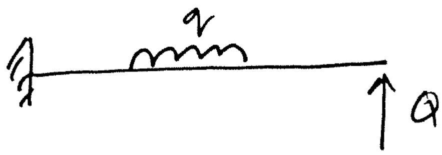
$\frac{M}{2} . L . \theta$

$\therefore$ New collapse load $= \frac{(2.67 + 0.5)}{0.222} \frac{M}{L^2}$

$= \underline{\underline{14.25\ M/L^2}}$

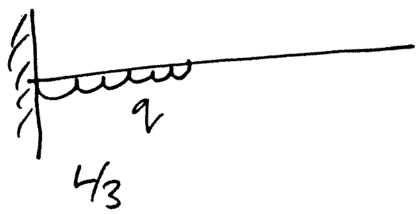
(3 marks)

Most popular question and well done by most. Only a handful got the mechanism wrong. Some sketched the rotation hodograph but then wrote down answers that were incompatible with it. They had been encouraged to scale from drawings for dimensions, but quite a number seemed to assume that this meant that answers to 1 significant figure were acceptable. It should be possible to get dimensions accurate to 1%, even under examination conditions. The biggest error was the assumption that the displaced shape was a pyramid, therefore the work done by the loads could be taken as a third of the area times the peak displacement.

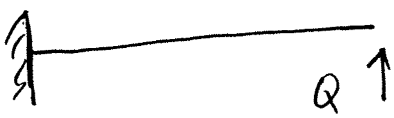
5. (a) 

can be split into | Tip deflection

$$\frac{qL^4}{EI}\left(\frac{16}{81}\cdot\frac{1}{8}+\frac{8}{162}\cdot\frac{1}{3}\right) = .04115\,\frac{qL^4}{EI}$$

$+$ 

$$-\frac{qL^4}{EI}\left(\frac{1}{81}\cdot\frac{1}{8}+\frac{1}{162}\cdot\frac{2}{3}\right) = -.00566\,\frac{qL^4}{EI}$$

$+$ 

$$-\frac{QL^3}{3EI}$$

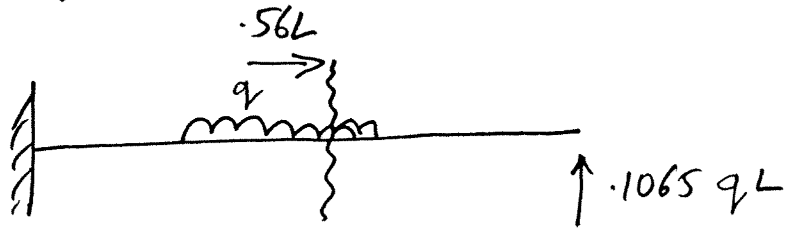
Tip deflection is zero when

$$\frac{QL^3}{3EI} = (.04115 - .00566)\frac{qL^4}{EI}$$

or $\underline{Q = .1065\,qL}$ (6 marks)

(b) Consider section cut at $x = 0.56L$

If deflection is a maximum, slope will be zero.



At the cut, the shear force will be

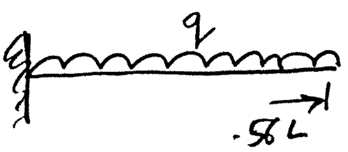
$$.1065\,qL - q(2/3 - .56)L = -.0002\,qL$$

The bending moment will be

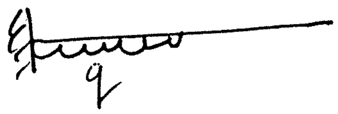
$$.1065\,qL \cdot 0.44L - q\frac{(2/3 - .56)^2 L^2}{2} = .0412\,qL^2$$

So the slope will be

End slope

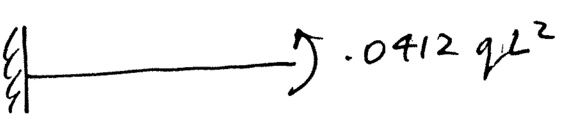


$$\frac{q \cdot 0.56^3}{6EI} = 0.0293$$



$$-\frac{qL^3}{162EI} = -0.0062$$

$$+\frac{0.0002qL \cdot .56^2}{2EI} \approx 0$$



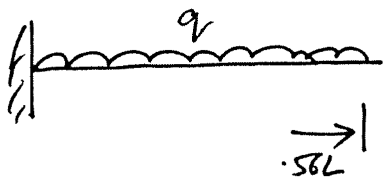
$$-.0412\,qL^2 \cdot .56 = -0.02307$$

$$\underline{\underline{0}}$$

(8 marks)

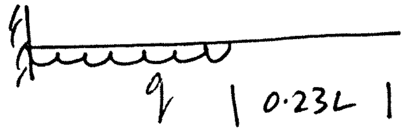
Q.E.D.

(c)



Tip deflection

$$\frac{q.(.56)^4 L^4}{8EI} = 0.01229 \frac{qL^4}{EI}$$



$$-\frac{qL^3}{EI}\left(\frac{1}{81}\cdot\frac{1}{8} + \frac{.23 \times 1}{162}\right)L = -.00294\frac{qL^4}{EI}$$

$$\approx 0$$

$$-\frac{.0412qL^4 \cdot .56^2}{2EI} = -.006459\frac{qL^4}{EI}$$

Total deflection

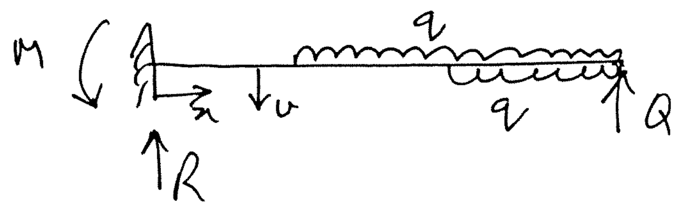
$$= 0.002906\frac{qL^4}{EI}.$$

(6 marks)

Generally not well done. Most students who tried the question had some idea of the appropriate methods, but not how to apply them. Most students tried a data-book coefficient approach to find the value of the indeterminate support reaction, but then virtually all the students who tried part (b) and (c) reverted to a Macaulay approach and effectively did part (a) again. In part (b), approximately half the students stated that the maximum deflection position corresponded to the point of maximum moment and proceeded to find out where the shear force was zero.

Alternative solution using Macaulay's method

(a)



Bending moment $EI\dfrac{d^2v}{dx^2} = M - Rx + \dfrac{q}{2}\{x - \tfrac{L}{3}\}^2 - \dfrac{q}{2}\{x - \tfrac{2L}{3}\}^2$

$$EI\frac{dv}{dx} = Mx - \frac{Rx^2}{2} + \frac{q}{6}\{x - \tfrac{L}{3}\}^3 - \frac{q}{6}\{x - \tfrac{2L}{3}\}^3 + A$$

(B.C. $\dfrac{dv}{dx} = 0$ at $x = 0$ $\Rightarrow A = 0$)

$$EIv = \frac{Mx^2}{2} - \frac{Rx^3}{6} + \frac{q}{24}\{x - \tfrac{L}{3}\}^4 - \frac{q}{24}\{x - \tfrac{2L}{3}\}^4 + B$$

(B.C. $v = 0$ at $x = 0$ $\Rightarrow B = 0$)

B.C. $v = 0$ at $x = L$

$$0 = \frac{ML^2}{2} - \frac{RL^3}{6} + \frac{q}{24}.\frac{16}{81}.L^4 - \frac{q}{24}\frac{L^4}{81}$$

$$= \frac{ML^2}{2} - \frac{RL^3}{6} + \frac{5}{684}qL^4$$

Moments about R.H end will involve the same variables

$$q\frac{L^2}{6} + M = RL$$

Substitute into previous equation

$$\frac{RL^3}{6} = \frac{ML^2}{2} + \frac{5qL^4}{648} = \frac{ML^2}{6} + \frac{qL^4}{36}$$

$\Rightarrow M = \dfrac{13}{216}qL^2$ $\qquad \Rightarrow R = \dfrac{49}{216}qL$

$R + Q = \dfrac{qL}{3}$ $\Rightarrow$ $Q = \dfrac{23}{216}qL^2$ (Vertical equilibrium)

Everything now known, so (b) and (c) simply by substitution

(b) Slope

$$EI\frac{dv}{dx}\cdot\frac{1}{qL^3} = \frac{13}{216}\cdot 0.56 - \frac{49}{216}\cdot 0.56^2 + \frac{0.227^3}{6}$$

(at $x = 0.56$)

$$= .0337 - .0355 + .0020 = 0$$

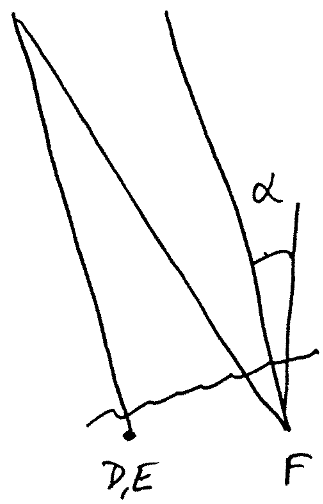
Q.E.D

(c) Deflection at $x = 0.56$

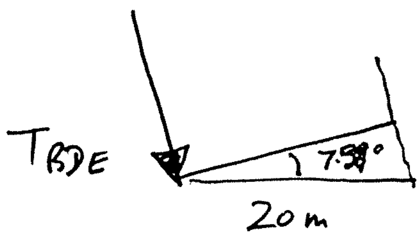
$$\frac{EIv}{qL^4} = \frac{13}{216}\cdot\frac{0.56^2}{2} - \frac{49}{216}\cdot\frac{0.56^3}{3} + \frac{(0.227)^4}{24}$$

$$= 0.00291$$

6. (a) First find the force in the frame BDE.



Angle $\alpha = \tan^{-1}\frac{20}{150} = \underline{7.59°}$



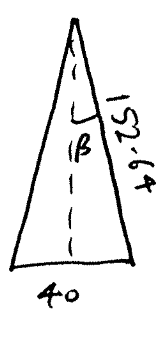
Lever arm of T_{BDE} about F is 19.82 m

∴ Take moments about F

$45 \times 10 . 10^6 = T_{BDE} \times 19.82$

$\Rightarrow T_{BDE} = 22.7 . 10^6$ N (compression)

But this is split into two members (true view)



Length BD $= \sqrt{20^2 + 20^2 + 150^2}$
$= 152.64$ m

Angle $\beta =$
$= \sin^{-1}\frac{20}{152.64} = 7.53°$

(c) Imperfection grows as

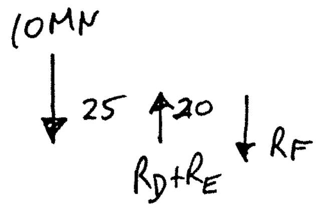
$$e_0 \frac{1}{(1 - P/P_{cr})}$$

When $P/P_{cr} = 1/2$, the imperfection will double to $\underline{\underline{\frac{L}{500}}}$. (4 marks)

The least popular question. Most of the students who tried this question made reasonable progress, but nearly half started by considering the equilibrium of the axle, and although this can lead to the correct answer, it is much more complicated than taking moments about one of the support points, in particular point F. About a fifth of the students applied the correct angle calculation to get the compression in the strut, although many got the answer to part (a) very nearly correct by taking effectively approximate angles. In part (b) and (c) most of the students who proceeded to this part of the question knew the necessary buckling and deflection growth equations, but the calculation of I for the thin tube was much more problematic. Well over half the students took the radius of gyration to be r and, so got an I value twice as large as that required.

Alternative solution for part (a)

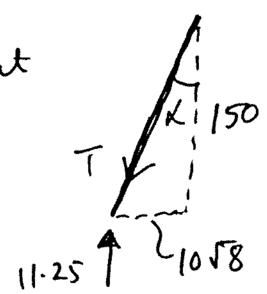
First find vertical reactions at D, E



Moments about F. $20(R_D+R_E) = 45.10$

$\Rightarrow R_D = R_E = \underline{11.25 \text{ MN}}$

True view of strut



$\therefore \alpha = \tan^{-1} \frac{10\sqrt{8}}{150}$

$T.\cos\alpha = 11.25$ MN

$\therefore T = \underline{\underline{11.45 \text{ MN}}}$