ENGINEERING TRIPOS PART IB

PAPER 3: MATERIALS
(JUNE 2000)

Note: Examiner's comments are given at the end of the crib

Question 1

(a)   Hardenability is a measure of the ability of a steel to form martensite on quenching from the austenite field of the phase diagram. It primarily relates to the size of component which is able to form a given % of martensite through-thickness - the larger the component the higher the hardenability.

Critical cooling rate means the minimum cooling rate required to form 100% martensite on quenching from the austenite field of the phase diagram (and thereby avoid the diffusion-controlled transformations to ferrite, pearlite or bainite).

(b)   In tests for hardenability, the cooling rate of interest is defined at a temperature around 500°C, since this is typically the temperature of fastest transformation to phases other than martensite (i.e. the "knee" of the TTT or CCT diagram). The cooling rate past this temperature is therefore most relevant to obtaining 100% martensite.

From Figure 1, the equation of the straight line is:

$$\phi = \frac{100}{40} x \quad \text{so} \quad x = 0.4\phi \quad \text{where } \phi \text{ is the diameter of oil-quenched bar, and } x \text{ is}$$

the distance from the quenched end of the Jominy bar.

Substituting into the equation relating cooling rate to x:

$$\frac{\partial T}{\partial t} = \frac{1.75 \times 10^{-3}}{x^2} = \frac{1.75 \times 10^{-3}}{(0.4\phi)^2} = \frac{1.09 \times 10^{-2}}{\phi^2} \quad (\phi \text{ in metres})$$
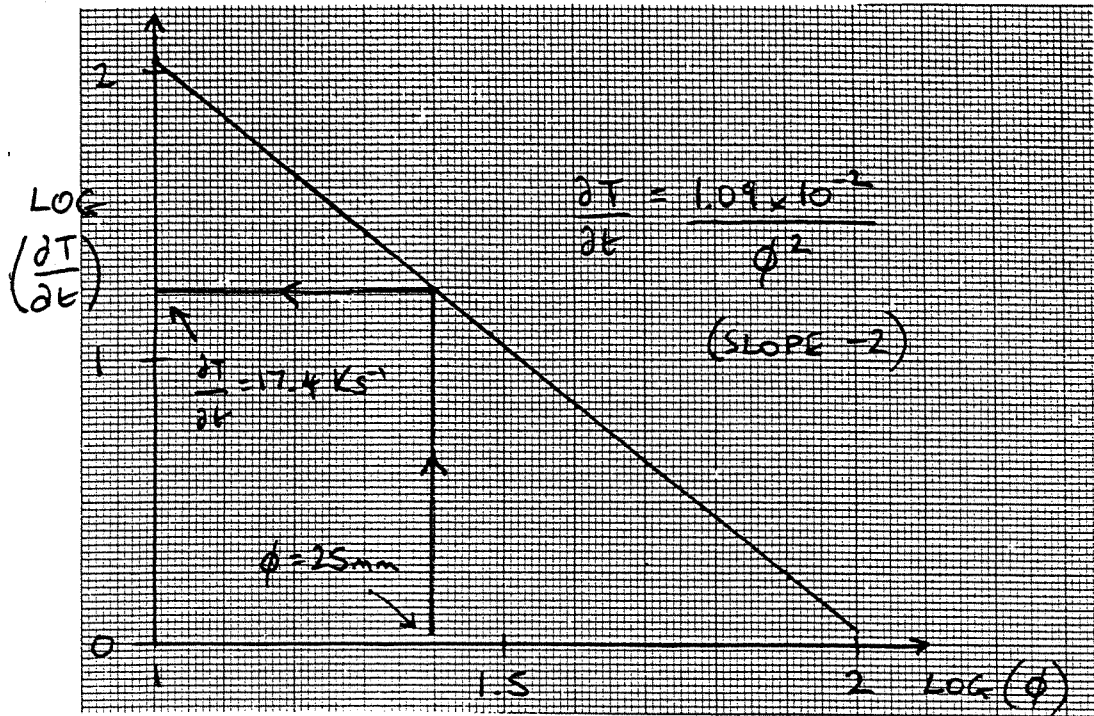
Plot a log-log graph:

$$\log\left(\frac{\partial T}{\partial t}\right) = \log(1.09 \times 10^{-2}) - 2\log\phi$$

Straight line of slope = -2

Locate one point on graph, e.g. $\phi = 0.01$m gives $\log \dfrac{\partial T}{\partial t} = 2.04$

$$\frac{\partial T}{\partial k} = \frac{1.09 \times 10^{-2}}{\phi^2}$$

(SLOPE −2)

$$\frac{\partial T}{\partial k} = 17.4 \ K \ s^{-1}$$

$\phi = 25 \ mm$

(Graph axes: vertical $LOG\left(\frac{\partial T}{\partial k}\right)$; horizontal $LOG(\phi)$)

(c)   From graph, or by substitution of ϕ = 25 mm, cooling rate at centre of 25 mm bar is 17.4 K s^{-1}. Chosen steel must therefore have critical cooling rate lower than this to form 100% martensite.

Substituting composition for different steels to find CCR:

Steel	Weight percentages					CCR (K s^{-1})
	C	Mn	Cr	Mo	Ni	
A	0.30	0.80	0.50	0.20	0.55	110
B	0.36	0.80	1.50	0.25	1.50	3.9
C	0.41	0.80	0.50	0.25	0.55	44.7
D	0.40	0.80	0.65	0.55	2.55	1.4

Hence steels B or D can form 100% martensite - choose the cheaper of the two: steel D.

A bulk heat treatment is subsequently conducted to temper the martensite, which is too brittle. This restores acceptable toughness (important for a camshaft), with a moderate loss of yield strength.

A surface heat treatment is conducted to raise the hardness of a thin surface layer to a high value, to provide wear resistance (which will also be important for a camshaft). One possibility would be to transformation harden the surface back to martensite (using a laser or other heat source).

Question 2

(a)   Design-limiting properties of polymers:

- low Young's modulus, so unsuitable for deflection-limited design (and cannot be shaped efficiently to overcome this problem)
- low strength, so not good for strength-limited design
- strong dependence of Young's modulus and strength on temperature (in most polymers)
- low fracture toughness, so susceptible to fracture from small defects
- low melting point (most limited to use below 100°C, at best around 200°C)
- can be susceptible to degradation in UV light
- difficult to separate for recycling

Advantages of polymers over other classes of materials:
- mostly cheap raw material, easy to shape by injection moulding and other processes, and easy to colour, so suitable for mass-produced household goods, toys etc
- low density, so lightweight products
- mostly good corrosion resistance
- low thermal and electrical conductivity - suitable for insulation
- molecular microstructure can be tailored to give customised properties

(b)   Amorphous thermoplastic: long-chain molecules are arranged randomly, and held together only by entanglement and weak van der Waals bonds.

Semi-crystalline thermoplastics: parts of the molecular structure consist of chains which are aligned and packed in an organised manner (e.g. as a spherulite), giving much stronger van der Waals bonding between chains due to the closer packing of the side-groups. The remainder of the structure is amorphous.

Above the glass transition, Young's modulus and strength are both strongly influenced by the degree of crystallinity. Semi-crystalline polymers retain stiffness and strength until the crystalline regions melt, while amorphous polymers become viscous liquids. Amorphous thermoplastics (without fillers) are also transparent, while semi-crystalline polymers are not.

Example thermoplastics, with applications:

Amorphous:
LDPE - carrier bags, packaging, dustbins
PVC - plastic wallets/ring-binders, window frames

Semi-crystalline:
HDPE - hip replacement sockets
PET - drink bottles
Nylon - gears, clothing

Degree of crystallinity is affected by:
Molecular factors: side-group chemistry and arrangement along the chain, and chain branching, determine how closely the molecules can pack together
Processing factor: cooling rate - the faster the cooling rate the more difficult it is for the molecules to arrange into crystalline regions; drawing also causes alignment and promotes crystallinity.

(c)   Cost equation:

C_L is the "labour cost", though this includes all running costs which are costed on an hourly basis (e.g. overheads, power etc).
C_C is the "capital cost" of equipment, but would include all one-off costs associated with the manufacturing process (such as dedicated tooling)

	C_m (£)	$C_L / \dot{n}$ (£)	n	C_C / n (£)	C (£)
Injection moulding	0.25	0.17	1000 50,000	5 0.1	5.42 0.52
Rotational moulding	0.20	0.5	1,000 50,000	1 0.02	1.7 0.72

Hence rotational moulding is cheapest for a batch size of 1,000, and injection moulding is cheapest for 50,000.

Injection moulding may use more material than rotational moulding because: (a) the wall thickness of the bucket may need to be greater for this process, or (b) there is more material wasted in the feeding system to the mould.

Question 3

(a)   Objective: a quantity which is to be maximised or minimised in a design, e.g. mass, cost, energy stored per unit volume or mass, heat conducted.

Constraint: a limit imposed on the design, which may be a geometrical limit (e.g. specified depth of beam, width of panel, thickness of tube etc) or may be a functional limit (e.g. specified deflection per unit load, maximum stress must not exceed failure etc).

Merit index: a group of material properties to be maximised, in order to optimise selection for the target objective (e.g. for minimum mass beams of given stiffness, merit index could be E/ρ or $E^{1/2}/\rho$ depending on the geometric constraint).

(b)   (i)   Objective is heat conducted per unit area, q. Need to eliminate free variable t in equation for q. The constraint is that the maximum stress in the tube wall must be below the failure stress. For given pressure difference Δp, hoop stress in tube

$$\sigma = \frac{\Delta p.r}{t}$$, where r is the radius and t is the wall thickness.

At failure, $\sigma = \sigma_y$ or $\sigma_y = \dfrac{\Delta p.r}{t}$

Radius r is fixed, but thickness t may vary, so $t = \dfrac{\Delta p.r}{\sigma_y}$

Substituting for t in expression for q: $q = \lambda \dfrac{\Delta T}{t} = \lambda \dfrac{\Delta T.\sigma_y}{\Delta p.r} = \dfrac{\Delta T}{\Delta p.r}(\lambda \sigma_y)$
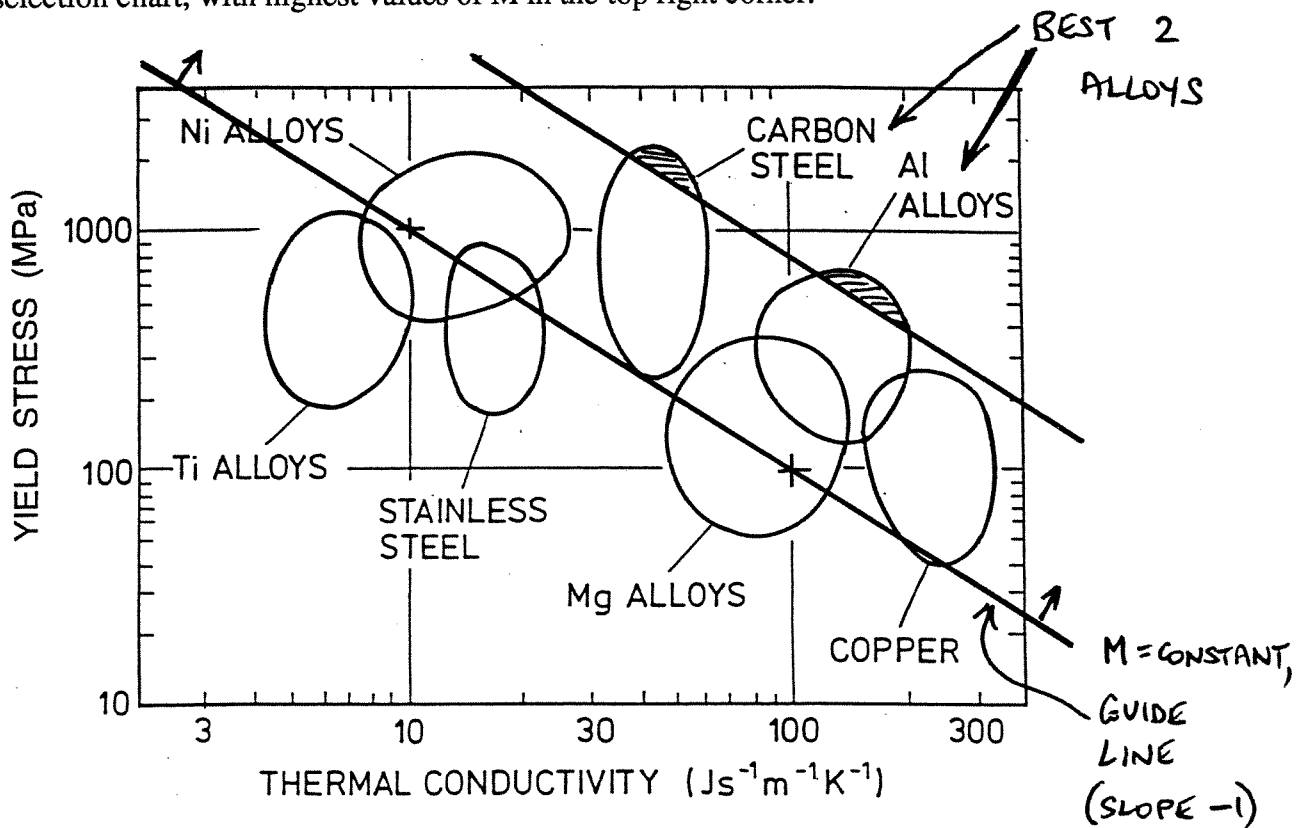
Hence to maximise heat conducted per unit area, maximise merit index $M = \lambda \sigma_y$.

(ii) For $M = \lambda\sigma_y = C$ (constant):

$$\log\lambda + \log\sigma_y = \log C$$

or $\log\sigma_y = -\log\lambda + \log C$

Hence lines of constant values of M have slope -1 on the
selection chart, with highest values of M in the top right corner.

Best two classes of alloy: carbon steel, Al alloys

(iii)   Other property requirements for this application:

Young's modulus (to resist buckling)
Corrosion (particularly at elevated temperature)
Ductility (must be suitable for drawing or extruding into tubes)
Fracture toughness (to avoid fast fracture)
Cost
Density (if heat exchanger is to be transportable)
Melting point (depending on operating temperature of fluids)
Thermal expansion (must be compatible with container, to avoid thermal stresses)

(iv)   Welding of carbon steel might present problems of embrittlement if the cooling
rate is sufficient to lead to the formation of brittle microstructures. For tubes it is likely
that drawn low carbon steels would be chosen, which are readily weldable.

Welding of aluminium alloys may lead to softening and loss of strength. In
extruded and formed non-heat-treatable alloys, softening would be by recrystallisation,
while in an age hardened alloy, softening is by dissolution of the hardening precipitates.

Question 4

(a)   Work hardening: massive increase in dislocation density by deformation, reducing the spacing between dislocations and thereby increasing stress needed to box dislocations between one another. Dislocation intersections also lead to jogs, which reduce the mobility of the dislocations.

Recovery: reduction in stored dislocation energy by either dislocation annihilation (dislocations of opposite sign being attracted to one another) or by rearrangement of dislocations into lower energy configurations (such as low angle subgrain boundaries). The driving force comes from the dislocation line energy. This process may start during the deformation (dynamic recovery) and continue if the temperature is sufficient after deformation (static recovery).

Recrystallisation: New grains of low dislocation density nucleate and grow, by boundaries sweeping through the old deformed grains until they impinge on one another. Driving force comes from the dislocation line energy in the deformed grains.

Final grain size may be controlled by changing the strain applied in the deformation stage. Beyond the critical strain needed to start recrystallisation, the higher the strain the greater the stored energy. The number of new grains nucleating increases (and they also grow quicker), so the recrystallised grain size falls. A further factor affecting the final grain size is the initial grain size before deformation, since many of the new grains nucleate on old grain boundaries, so finer grains provide more nucleation sites per unit volume.

(b)   In heat-treatable aluminium alloys, dislocations may interact with precipitates by shearing or bypassing.

Shearing occurs when the precipitates are small and coherent (or semi-coherent) so that the slip planes between the aluminium lattice and the precipitate are continuous.

Bypassing takes place when the particles are large and incoherent, so the dislocations bow out and pass between the obstacles instead.

(NOT SAME SCALE!)
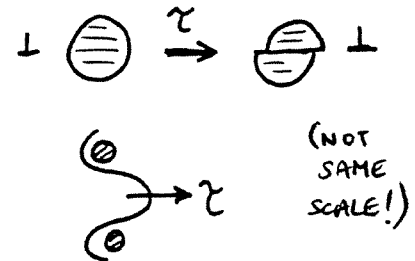
During ageing there is a continuous increase in particle size (by coarsening) and a steady loss in coherency with the aluminium matrix.

For the shearing mechanism, the strength to shear particles increases with size so the ageing curve initially increases with time.

For the bypassing mechanism, the particle spacing controls the yield stress, so as particle size (and spacing) increase the yield stress falls with time.

Hence the ageing curve goes through a peak when there is a change in the dominant dislocation mechanism.

σ_y SHEARING   BYPASSING   RADIUS $(\propto TIME^{1/3})$
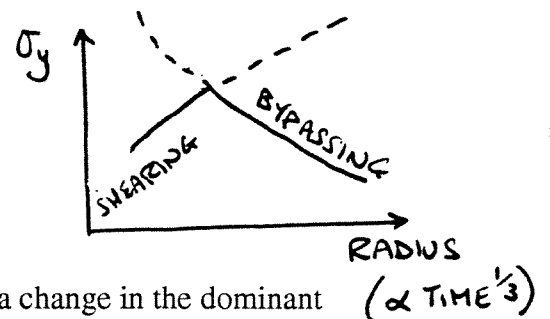
(c)   Factors driving the development of lower weight cars:
      - improved fuel efficiency and reduced emissions
      - added safety features (e.g. air bags, which add weight, so car itself must be lighter to maintain same fuel efficiency)

Aluminium alloys currently used for: (a) large cast items such as cylinder blocks and heads; (b) reciprocating parts where strength is not the primary concern, such as pistons (giving the added benefit of secondary weight savings due to the lower inertial loads in accelerating lighter components); (c) structural parts of bumpers.

Future uses: whole aluminum cars are being developed, where the chassis and panelling structure (currently made in steel) are replaced by an aluminium equivalent, or with a "spaceframe" of aluminium extrusions welded together. All body panels may be replaced with stamped aluminium alloy sheet.

Question 5

(a)   Ceramics contain a statistical distribution of flaws, which cause failure by fracture in tension (or crushing in compression). A single worst flaw (in the worst orientation) in the specimen leads to catastrophic failure, so a weakest link theory is required. The basis of the theory is that the probability that a given volume of material will survive at a certain tensile stress is controlled by the probability that that volume of ceramic contains a flaw above the critical size for the applied stress. Critical size depends on material fracture toughness.

(b)   In the Weibull equation:

   - σ_0 is a reference strength, being the stress which a volume V_0 can survive with a probability equal to (1/e).
   - m is the Weibull modulus, which is a measure of the variability of the failure stress (a material with a low value of m has a significant probability of failure at sresses well below σ_0, while a high value gives a sharper transition from survival to failure around this stress level).

Weakest link theory states that the probability that n samples of volume V_0 all survive a stress σ is $\left[P_s(V_0) \right]^n$ (i.e. must *multiply* the individual probabilities that each survive). If these n samples were "glued together" into a single specimen of volume $V = n V_0$ the survival probability is the same:

$$P_s(V) = \left[P_s(V_0) \right]^n = \left[P_s(V_0) \right]^{V/V_o}$$

Substituting from the Weibull expression for $P_s(V_0) = \exp\left\{ -\left(\sigma / \sigma_0 \right)^m \right\}$ gives:

$$P_s(V) = \left[\exp\left\{ -\left[\sigma / \sigma_0 \right]^m \right\} \right]^{V/V_o} = \exp\left\{ -(V / V_0) \left[\sigma / \sigma_0 \right]^m \right\}$$

Failure stresses for two volumes V_1 and V_2 at constant probability of survival:

$$P_s(V_1) = P_s(V_2) \Rightarrow \exp\left\{ -(V_1 / V_0) \left[\sigma_1 / \sigma_0 \right]^m \right\} = \exp\left\{ -(V_2 / V_0) \left[\sigma_2 / \sigma_0 \right]^m \right\}$$

Hence $V_1 \sigma_1^m = V_2 \sigma_2^m$ or $\sigma_1 / \sigma_2 = \left(V_2 / V_1 \right)^{1/m}$

(c)   Volume of test specimens: $V_2 = \left(\pi d^2 / 4 \right) \times \ell = \frac{\pi}{4}(5)^2 \times 20$

Fibre volume exposed between crack faces:

$$V_1 = \left(\pi d^2 / 4 \right) \times \ell = \frac{\pi}{4}(25 \times 10^{-3})^2 \times (75 \times 10^{-3})$$

Hence using ratio of failure stresses at constant probability from (b):

$$\sigma_1 = \sigma_2 \times (V_2 / V_1)^{1/m} = 270 \times \left(\frac{5^2 \times 20}{(25 \times 10^{-3})^2 \times (75 \times 10^{-3})} \right)^{1/8} = 2041 \text{ MPa}$$

Alternatively (slower), solve for σ_0 using $P_s = 1/e = 0.37$, and then substitute back into Weibull equation.

This analysis assumes that the fibres and the test specimens have an identical flaw distribution (i.e. as if they come from the same batch of material). The fibres will be processed directly, and in practice are likely to have rather different flaws than the bulk text pieces, so the analysis is very approximate.

The analysis shows however that individual fibres may have failure strengths (at 50% probability of failure) of the order of 2 GPa. This is a very high stress, so incorporating SiC fibres in a brittle matrix could give significant toughening, since when a crack crosses the bulk matrix material, there is a good probability that most of the fibres bridging the crack can carry the load, forcing further cracks to form before failure.

Question 6

(a)   A eutectic reaction is when a single liquid phase transforms during cooling to 2 solid phases. This occurs at a constant temperature.

Eutectic alloys are suitable for soldering because they have the lowest possible melting point for the alloy system used, so joints can be made with the minimum risk of thermal damage to the surrounding electronic components.
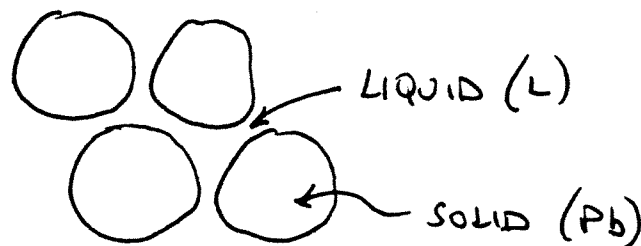
(b)   For 190°C:

(i)   Phases present are liquid (L) and Pb-rich solid solution (Pb)
Using the lever rule, but reading the compositions from the top scale of the diagram (wt%):

wt% liquid = $\frac{82-72}{82-40}$ = 24%, so wt% (Pb) = 76%

(ii)   Microstructure, isolated grains of (Pb) surrounded by liquid:
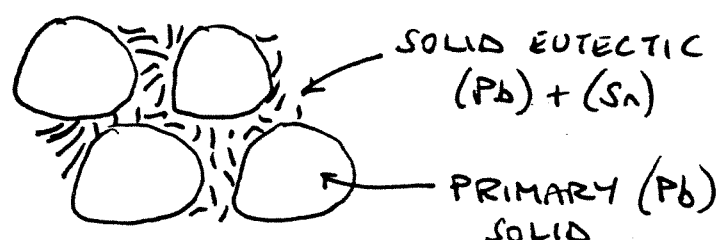
For 175°C:

     (i)   Phases present are Sn-rich solid solution (Sn) and Pb-rich solid solution (Pb)
Using the lever rule, reading the compositions from the top scale of the diagram (wt%):

wt% (Sn) = $\frac{84-72}{84-4}$ = 10%, so wt% (Pb) = 90%

     (ii)   Microstructure, isolated grains of primary (Pb) surrounded by two-phase
eutectic microstructure, made up of more (Pb) and all the (Sn):

The only two-phase region is the eutectic microstructure at 175°C. This part of the
structure formed from liquid of eutectic composition, so the proportions of the phases is
found by applying the lever rule at the eutectic composition, just below the eutectic point:

wt% (Sn) in eutectic = $\frac{82-36}{82-5}$ = 60%, so wt% (Pb) = 40%

On further cooling, the Pb-rich solid solution becomes supersaturated as its equilibrium
solubility for Sn falls. Sn-rich solid may therefore form precipitates within the primary
(Pb) grains.
In the eutectic microstructure, the two phases may become purer on further cooling by
inter-diffusion between the phases.

(c)   Segregation means that a gradient in concentration is formed across a solidifying
region. This is because the first solid to form from a melt generally has a lower
equilibrium solute level than the liquid from which it forms. As the fraction of solid
grows, this purer solid becomes enclosed by more solid of steadily lower purity. As real
cooling rates are too fast to maintain equilibrium, there is insufficient time for the solute
to become uniform and the concentration gradient remains.

Segregation is a particular problem in metals processing, since impurities in the melt are
not taken into the solid which is forming and may reach damaging levels in the final
region of liquid to solidify. This can lead to weaker material or porosity near the centre of
castings, or on the centre-line of a weld.

Examiner's Comments

268 candidates took this paper.

Question 1 (123 attempts, average marks)
Few considered a log-log graph to plot the inverse square equation. Common error was to choose steels with higher critical cooling rate rather than lower, as compared with the actual cooling rate.

Question 2 (113 attempts, above average marks)
Many students incorrectly trotted out failure *mechanisms* (e.g. crazing, shear yielding) as design-limiting *properties*.

Question 3 (221 attempts, above average marks)
Most students could derive the merit index and idenitfy that the slope of a selection line should be -1 on the log-log plot. A large proportion were then unable to plot this line, or to identify the classes using the correct line.

Question 4 (215 attempts, below average marks)
Most candidates knew about annealing, recovery and the shape of the ageing curve. A few confused steels with aluminium alloys and talked about quenching rates and martensite. Most were vague about the uses of aluminium in cars and quoted parts at random, such as crankshafts.

Question 5 (157 attempts, average marks)
The analysis was well done; most marks were lost due to poor explanation of weakest link theory.

Question 6 (243 attempts, below average marks)
The main error was to apply the lever rule by measuring from the diagram, not appreciating that the weight % scale at the top is non-linear. Others missed this scale and wrote pages turning atomic percent into weight percent.

Hugh Shercliff
June 2000

1(c)   Cooling rate $= 17.4\,\mathrm{Ks^{-1}}$. Choose steel D.

2(c)   Injection moulding: £5.42 for 1000, £0.52 for 50000
        Rotational moulding: £1.70 for 1000, £0.72 for 50000
        Choose RM for 1000, IM for 50000.

3(b)   Carbon steel, Al alloys

5(c)   2041 MPa

6(b)   190°C: 76% (Pb), 24% L
        175°C: 90% (Pb), 10% (Sn)
        In eutectic region: 60% (Sn), 40% (Pb)