10/2/2000

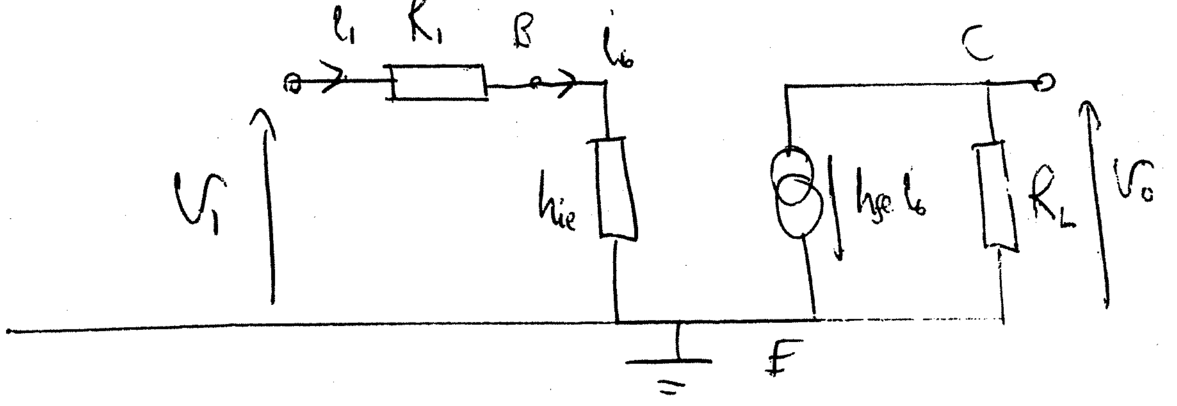
IB Paper 5 June 2000 CRIB.

1/ a) Choose $V_{CE} = 10V$ since it is midband between 0V and 20V, and therefore allows equal output voltage swings in the positive and negative directions.

$$\frac{20 - V_{CE}}{R_L} = I_c \Rightarrow R_L = \frac{20-10}{20\,mA} = \underline{\underline{500\,\Omega}}$$

$$\frac{20 - V_{BE}}{R_B} = I_B = \frac{I_c}{h_{FE}} \Rightarrow R_B = \frac{20-0.7}{20\times10^{-3}/100} = \underline{\underline{96.5\,k\Omega}}$$

b) Mid-band frequencies $\Rightarrow$ assume $1/j\omega C_1 \simeq 0$. Also $R_B \gg h_{ie}$, so ignore R_B.



$$i_b = \frac{V_1}{R_1 + h_{ie}} \qquad V_o = -h_{fe} i_b R_L = -\frac{h_{fe} V_1 R_L}{R_1 + h_{ie}}$$

$$\Rightarrow \text{Gain} = \frac{V_o}{V_1} = \frac{-h_{fe} R_L}{R_1 + h_{ie}} = \frac{-100\times 500}{5000} = \underline{\underline{-10}}$$

$$R_{in} = \frac{V_1}{i_1} = R_1 + h_{ie} = \underline{\underline{5\,k\Omega}}$$

$$R_o = \frac{V_o}{i_o}\Big/_{V_i=0} = R_L = \underline{\underline{500\,\Omega}}$$

c) At low frequencies capacitive reactance of C_1 is large, so i_b is reduced, thus reducing $h_{fe} i_b$ and therefore V_o.

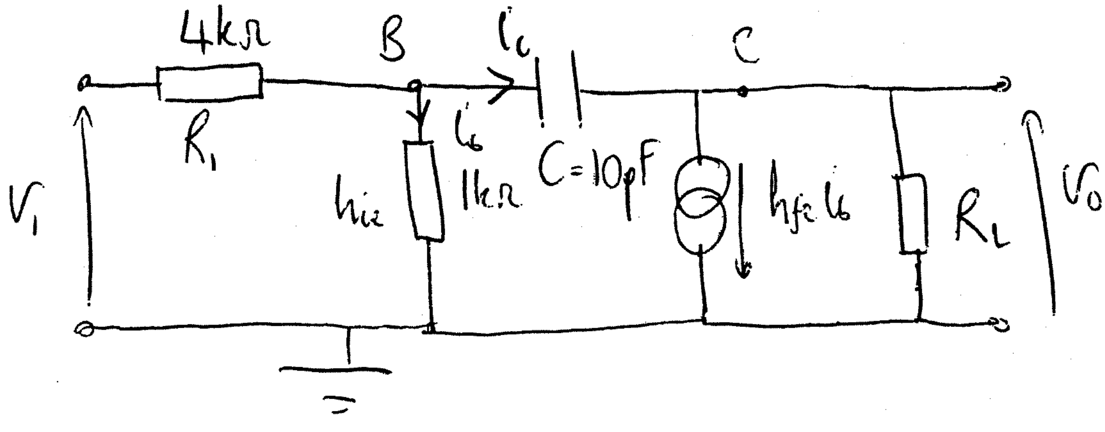
$$i_b = \frac{V_i}{R_1 + h_{ie} + 1/j\omega C_1}$$

$\frac{V_o}{V_i}$ −3dB point is also when i_b is −3dB compared to mid-band value

$$\therefore \; R_1 + h_{ie} = \frac{1}{\omega C}$$

$$5\times10^{-3} = \frac{1}{2\pi \times 20 C} \Rightarrow C = 1.59\,\mu F.$$

d)



At high frequencies $1/j\omega C$ is small, and so i_b is reduced owing to shunting effect of C. In turn, $V_o \propto i_b$, and so gain is reduced.

Ignore capacitor current i_c in terms of its effect on the output, since $i_c \ll h_{fe} i_b$
Ignore R_B as before since $\gg h_{ie}$.

Kirchoff's current law at B

$$\frac{v_1 - h_{ie} i_b}{R_1} = i_b + i_c$$

$$i_c = \frac{h_{ie} i_b - (-h_{fe} i_b R_L)}{1/j\omega C} = j\omega C (h_{ie} + h_{fe} R_L) i_b$$

$$\Rightarrow v_1 = (R_1 + h_{ie}) i_b + j\omega C R_1 (h_{ie} + h_{fe} R_L) i_b$$

$$i_b = \frac{v_1}{R_1 + h_{ie} + j\omega C R_1 (h_{ie} + h_{fe} R_L)}$$

-3dB point for v_o/v_i corresponds to -3dB point for i_b, since $v_o = -h_{fe} i_b R_L$

$$\Rightarrow R_1 + h_{ie} = \omega_{3dB} C R_1 (h_{ie} + h_{fe} R_L)$$

$$5 \times 10^3 = 2\pi f_{3dB} \times 10 \times 10^{-12} \times 4000 (1000 + 100 \times 500)$$

$$f_{3dB} = 390 \text{ kHz.}$$

2/ a) Differential amplifier has much greater gain for differential input signals than common-mode signals. ∴ Only the desired 5kHz signal will be amplified significantly, whilst the 50Hz unwanted component will not be.

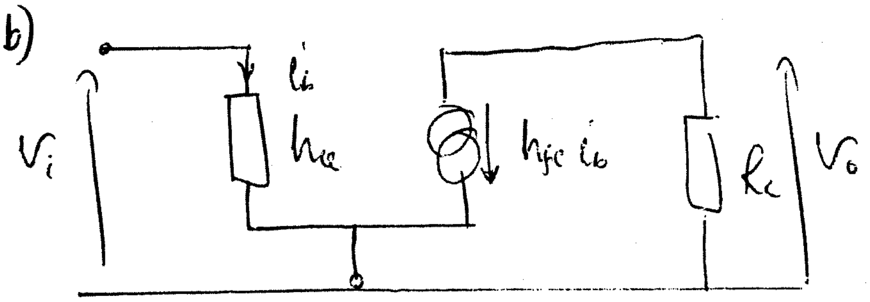
$$V_{o_{diff}} = A_{diff} \times 1mV$$

$$V_{o_{com}} = A_{com} \times 100mV$$

$$\frac{V_{o_{diff}}}{V_{o_{com}}} = 1000 = \frac{A_{diff}}{A_{com}} \times \frac{1}{100} = \frac{CMRR}{100}$$

$$\therefore CMRR = 1000 \times 100 = 10^5 \text{ or } 100\,dB.$$

b)

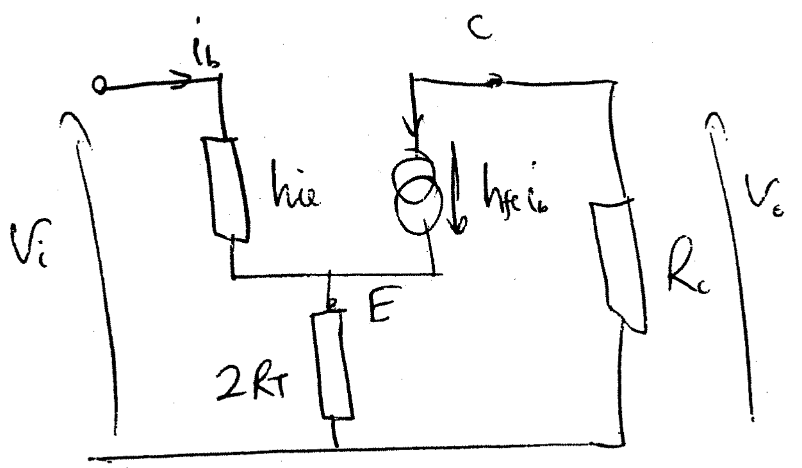


Differential mode

Use half-circuit for differential input, with R_T a short-circuit since there is no small-signal voltage across it.

$$i_b = \frac{V_i}{h_{ie}} \qquad V_o = -h_{fe}\, i_b R_c = -h_{fe} \frac{V_i}{h_{ie}} R_c$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-h_{fe} R_c}{h_{ie}}$$



Common-mode
N.B. R_T appears as $2R_T$ in half-circuit.

$$V_i = i_b h_{ie} + 2R_T(h_{fe}+1)\, i_b$$

$$V_o = -R_c h_{fe} i_b$$

$$\frac{V_o}{V_i} = \frac{-h_{fe} R_c}{h_{ie} + 2R_T(h_{fe}+1)}$$

$$\text{CMRR} = \frac{A_{diff}}{A_{common}} = -\frac{h_{fe} R_c}{h_{ie}} \Bigg/ \frac{-h_{fe} R_c}{h_{ie} + 2R_T(h_{fe}+1)}$$

$$= \frac{h_{ie} + 2R_T(h_{fe}+1)}{h_{ie}}$$

For $A_{diff} = 40\,\text{dB} = 100 \Rightarrow \dfrac{h_{fe} R_c}{h_{ie}} = 100$

$$\frac{200\, R_c}{1000} = 100 \qquad \underline{\underline{R_c = 500\,\Omega}}$$

$$\text{CMRR} = 60\,\text{dB} = 1000 = \frac{1000 + 2R_T(201)}{1000}$$

$$\Rightarrow \underline{\underline{R_T = 2.49\,\text{k}\Omega}}$$

c) $V_{CC} = 0 - V_{BE} + V_{CE} + I_C R_C$

$= -0.7 + 10 + 20\text{mA} \times 0.5\text{k}\Omega$

$= \underline{\underline{19.3\text{V}}}$

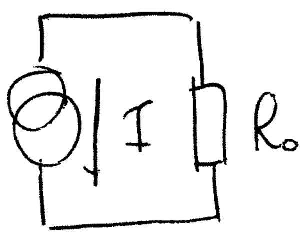
$-V_{EE} = -V_{BE} - 2 I_E R_T$

$\simeq -0.7 - 2 \times 20\text{mA} \times 2.49\text{k}\Omega$

$= 100.3$

$\Rightarrow \underline{\underline{V_{EE} = 100.3\text{V}}}$

d) Replace R_T with

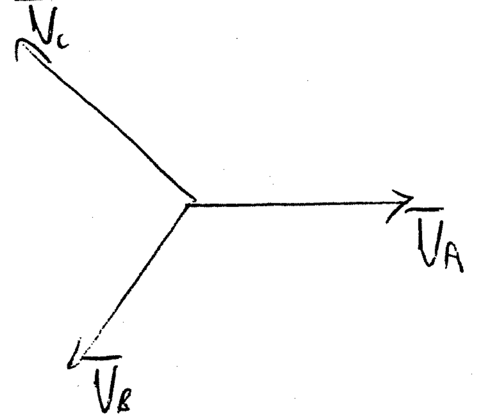


avoids excessive V_{EE} whilst retaining high effective R_T and hence CMRR.

$I \approx 2 I_C = 40\text{mA}$

$\underline{\underline{R_o = R_T = 2.49\text{k}\Omega.}}$

3/a)



Reverse $\bar{V}_B$ and sum :- $\bar{V}_{1\phi} = \bar{V}_A - \bar{V}_B + \bar{V}_C$

$$= Ve^{j0} - Ve^{-j2\pi/3} + Ve^{j2\pi/3}$$

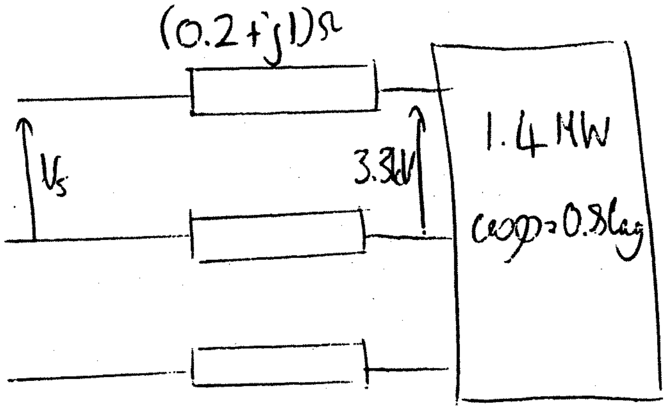
$$= V\left(1 - \left(0.5 - j\frac{\sqrt{3}}{2}\right) + 0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$= V\left(1 + j\sqrt{3}\right)$$

$$\therefore |\bar{V}_{1\phi}| = V\,(1+3)^{1/2} = \underline{\underline{2V}}$$

Maximum o/p power of 1ϕ generator is $2VI_{max}$ cf $3VI_{max}$ for 3ϕ. Increasing phases beyond 3 brings negligible benefits, but increases cost & complexity.

b)



$P = 1.4$ MW, $Q = P\tan\phi = +1.05$ MVAr

$S = (P^2 + Q^2)^{1/2} = 1.75$ MVA $= \sqrt{3}\,V_L I_L$

$\rightarrow I_L = 306.2$ A.

$P_{line} = 3I_l^2 R_l = 3 \times 306.2^2 \times 0.2 = \underline{\underline{56.2\ kW}}$

$Q_{line} = 3I_l^2 X_l = 3 \times 306.2^2 \times 1 = 281.2\ kVAr$

$P_{source} = 1.4\ MW + 0.0562\ MW = 1.456\ MW$

$Q_{source} = 1.05\ MVAr + 0.281\ MVAr = 1.331\ MVAr$

$S_{source} = (P^2 + Q^2)^{1/2} = 1.973\ MVA = \sqrt{3}\, V_s I_l$

$\Rightarrow \underline{\underline{V_s = 3.72\ kV}}$

c) P_{load} unaffected, find new Q

$\tan\phi = \frac{Q}{P}$, $Q = P\tan\phi = 1.4 \tan(\cos^{-1} 0.9)$

$= 0.678\ MVAr$

Old $Q = 1.05\ MVAr$

$\therefore$ Capacitor must generate $0.372\ MVAr$

$3 V_{ph}^2 / X_c = 372 \times 10^3$

$\frac{V_l^2}{X_c} = 372 \times 10^3 \Rightarrow X_c = \frac{(3.3 \times 10^3)^2}{372 \times 10^3}$

$\frac{1}{\omega C} = X_c = 29.3\ \Omega$ $\quad \underline{\underline{C = 109\ \mu F}}$

New load $S = \frac{P}{\cos\phi} = \frac{1.4}{0.9} = 1.56\ MVA = \sqrt{3}\, V_l I_l$

$\underline{\underline{I_l = 272.2\ A}}$

d) $Q_{cap} = -3\frac{V_i^2}{X_c} = \frac{-3\times(3.3\times10^3)^2}{29.3} = -1.12$ MVAr

$P = 1.4$ MW, $Q = 1.05 - 1.12$ MVAr $= -65$ kVAr

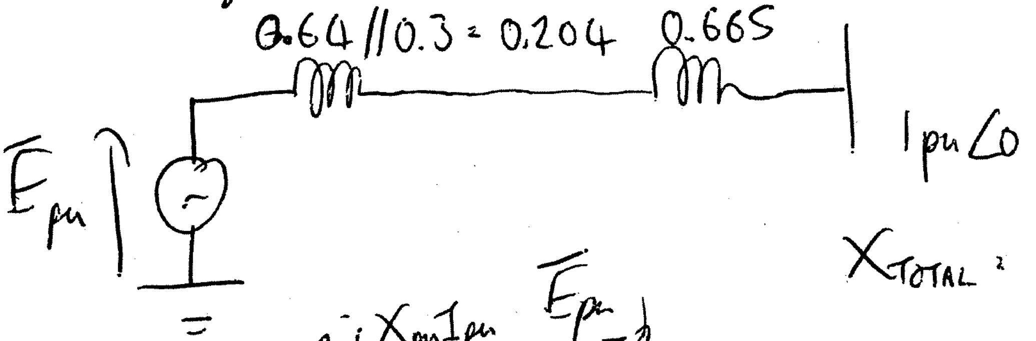
~~cos φ =~~ $S = (1.4^2 + (0.065)^2)^{1/2} = 1.402$ MVA

$\cos\phi = \frac{P}{S} = 0.999$ leading.

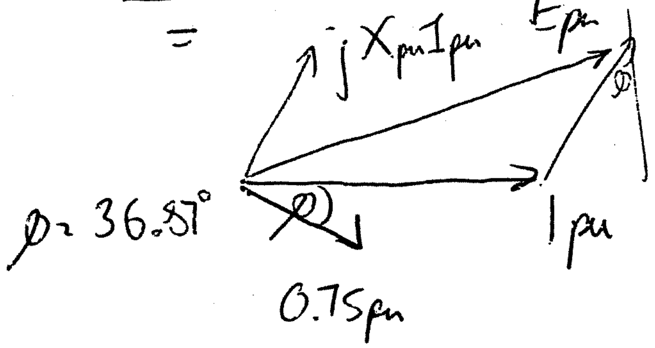
$\therefore \quad I_{line} = S/\sqrt{3}V_i = 245$ A.

Power factor improved even more, resulting in an even lower line current ⇒ good. However, capacitor may not be adequately rated.

Since generator excitations are identical, can connect together :-



$X_{TOTAL} = 0.869\,pu$



$$Re(\bar{E}_{pu}) = 1 + XI\sin\phi$$
$$= 1 + 0.75 \times 0.869 \times 0.6 = 1.391$$

$$Im(\bar{E}_{pu}) = XI\cos\phi = 0.75 \times 0.869 \times 0.8 = 0.521$$

$$\bar{E}_{pu} = 1.49 \angle 20.5°$$

$\therefore$ For 11kV generator, $E = 16.39\,kV \angle 20.5°$

For 33kV generator, $E = 49.17\,kV \angle 20.5°$

iii) Control prime-mover input power to control output electrical power

Control rotor field current to control output VARs.

1 a) $$T = \frac{3 I_2'^2 R_2'}{s\omega_s}$$

$$I_2'^2 = \frac{V^2}{(X_1 + X_2')^2 + (R_1 + R_2'/s)^2}$$

$$\therefore T = \frac{3V^2 R_2'}{s\omega_s\left((R_1 + R_2'/s)^2 + (X_1 + X_2')^2\right)}$$

b) i) At $\omega_r = 0$, $s = 1$ $\quad \omega_s = \omega/p = 2\pi \times 50/2 = 157.1\ \text{rad s}^{-1}$

$$\therefore T = \frac{415^2 \times 0.5}{1 \times 157.1\left((0.7+0.5)^2 + 1.8^2\right)} = \underline{\underline{117\ \text{Nm.}}}$$

ii) $$160 = \frac{415^2 \times 0.5}{157.1\left((0.7 + 0.5/s)^2 + 1.8^2\right)s}$$

$$= \cancel{86112.5}\ \frac{548.1 s}{0.49s^2 + 0.7s + 0.25 + 3.24s^2}$$

$$3.73s^2 + 0.7s + 0.25 = 3.43s$$

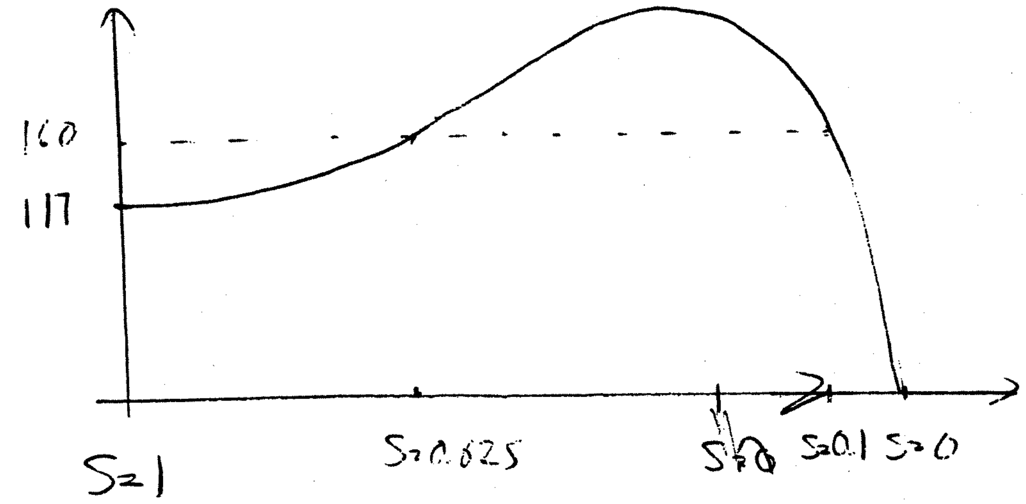
$$0.304s^2 + 0.7s + 0.25 = 0$$

$$s = \frac{-0.7 \pm \sqrt{0.7^2 - 4 \times 0.304 \times 0.25}}{2 \times 0.304} = -0.7 \pm$$

$$3.73s^2 - 2.73s + 0.25 = 0$$

$$s = \frac{2.73 \pm \sqrt{2.73^2 - 4 \times 3.73 \times 0.25}}{2 \times 3.73} = \frac{2.73 \pm 1.93}{7.46}$$

$S = 0.625$ or $s = 0.108 \Rightarrow \omega$ 562.5 rpm or 1338 rpm



d) $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{P_{in} - P_{losses}}{P_{in}}$

$$P_{in} = 3I^2\left(R_1 + \frac{R_2'}{S}\right) \qquad P_{loss} = 3I^2(R_1 + R_2')$$

$$I = \frac{V}{\left((R_1 + R_2'/S)^2 + (X_1 + X_2')^2\right)^{1/2}}$$

$$\eta = \frac{3I^2(R_1 + R_2'/s) - 3I^2(R_1 + R_2')}{3I^2(R_1 + R_2'/s)}$$

$$= \frac{R_1 + R_2'/s - (R_1 + R_2')}{R_1 + R_2'/s} = \frac{R_2'/s - R_2'}{R_1 + R_2'/s} = \frac{R_2'(1-s)}{R_2' + sR_1}$$

$$= \frac{(1-s)}{1 + sR_1/R_2'}$$

$$\therefore \eta = \frac{1-s}{1 + S \times 1.4} \qquad \eta_1 = \frac{1 - 0.108}{1 + 0.108 \times 1.4} = 77.5\%$$

$$\eta_2 = \frac{1 - 0.625}{1 + 0.625 \times 1.4} = 20\%$$

Clearly the low slip condition is best at 1338 rpm.

d) 117 Nm starting torque means motor cannot accelerate load unless additional rotor resistance is used. This can be done since the motor is a slip-ring machine, and enables the peak motor torque to be available at all speeds up to the speed corresponding to peak torque with no external resistance added.

6/ a) $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{250\times10^{-9}}{100\times10^{-12}}} = \underline{\underline{50\Omega}}$

$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{250\times10^{-9}\times100\times10^{-12}}} = 2\times10^{8}\ ms^{-1}$

$\therefore T = \frac{5}{2\times10^{8}} = \underline{\underline{25\ ns}}$

b) Incident voltage $= V_+$, reflected $= V_-$

$I_+ = \frac{V_+}{Z_0}$, $I_- = -\frac{V_-}{Z_0}$

Current in $R_L = I_+ + I_- = \frac{V_+ + V_-}{R_L}$

$\therefore \frac{V_+}{Z_0} - \frac{V_-}{Z_0} = \frac{V_+ + V_-}{R_L}$

$R_L(V_+ - V_-) = Z_0(V_+ + V_-)$

$V_-(R_L + Z_0) = V_+(R_L - Z_0)$

$\frac{V_-}{V_+} = \rho_V = \underline{\underline{\frac{R_L - Z_0}{R_L + Z_0}}}$

At source end, $\rho_{V_S} = \frac{200-50}{200+50} = \underline{\underline{0.6}}$

At logic gate end, $\rho_{V_L} = \frac{1000-50}{1000+50} = \underline{\underline{0.905}}$

c) $$\frac{V_{dc} - V_+}{\cancel{200\Omega}\ R_o} = \frac{V_+}{Z_o}$$ (Continuity of current at start of transmission line)

$$V_+ = \frac{Z_o}{Z_o + R_o} V_{dc}$$

$\therefore$ Initial $V_+ = \frac{50}{250} V_{dc} = 1V$

Reflected voltage $= \rho_{V_L} \times V_+ = 0.905\,V$

$1 \times \tau$

$\tau = 25$ ns, $V = V_+ + V_- = 1.905\,V$

Voltage reflected from source $= 0.6 \times 0.905 = 0.543V$

Reflected voltage at gate $= 0.905 \times 0.543 = 0.491$

3τ

$\tau = 75$ ns

$V = 2.94V$

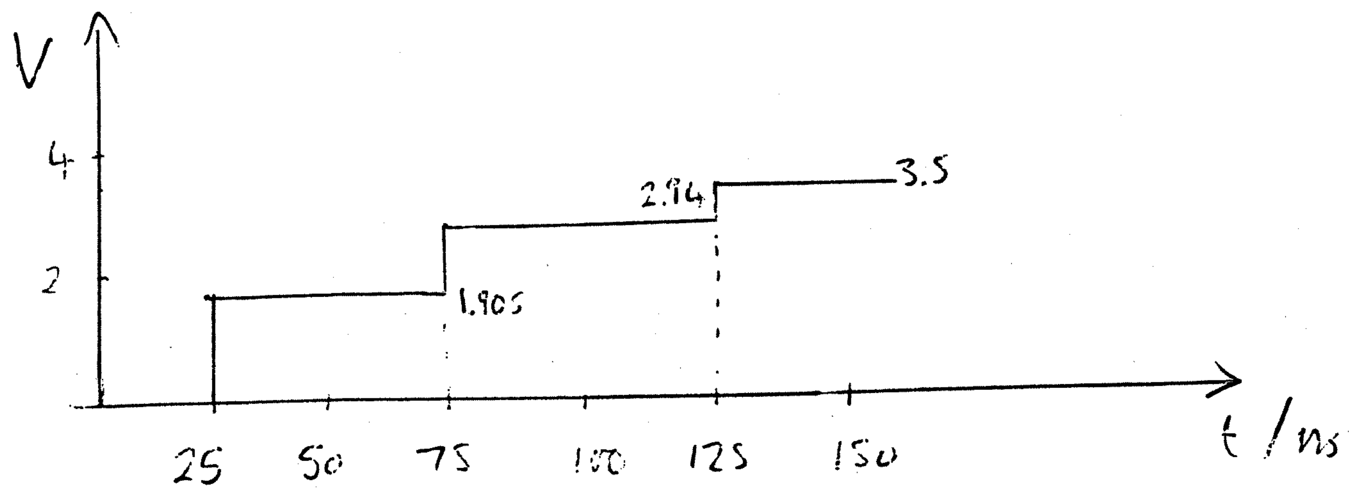
" " $= 0.6 \times 0.491 = 0.295V$

" " $= 0.905 \times 0.295 = 0.267V$

5τ

$\tau = 125$ ns

$V = 3.5V$



d) $V = V_{+_0} + \rho_{V_L} V_{+_0} + \rho_{V_S}\rho_{V_L} V_{+_0} + \rho_{V_S}\rho_{V_L}^2 V_{+_0} + \rho_{V_S}^2\rho_{V_L}^2 V_{+_0} + \rho_{V_S}^2\rho_{V_L}^3 V_{+_0}$

$+ \ldots$

$= V_{+_0}(1+\rho_{V_L})\underbrace{\left(1 + \rho_{V_L}\rho_{V_S} + (\rho_{V_L}\rho_{V_S})^2 + \ldots\right)}_{G.P.}$

$= V_{dc} \dfrac{Z_0}{Z_0 + R_0}\left(1 + \dfrac{R_L - Z_0}{R_L + Z_0}\right)\left(\dfrac{1-(\rho_{V_L}\rho_{V_S})^n}{1-\rho_{V_L}\rho_{V_S}}\right)$

$V = 1(1+0.905)\left(\dfrac{1-(0.6\times0.905)^n}{1-0.6\times0.905}\right)$

$4 = 4.168(1 - 0.543^n)$

$0.543^n = 0.0403$

$n = 5.25 \Rightarrow$ round up to 6.

$\therefore$ Total communication delay $= (2n+1)\tau = 0.325\,\mu s$.

Steady state voltage $= \lim_{n\to\infty} V = \dfrac{1.905}{0.457} = \underline{\underline{4.168\,V}}$

(Physical approach – at steady-state transmission line connects load directly to source, since capacitors all o.c. & inductors all s.c. in equivalent circuit.

$\therefore V_\infty = \dfrac{1k}{1k+200} \times 5V = \dfrac{5}{1.2} = \underline{\underline{4.167\,V}}$

7/ a) R_{rad} given by $\frac{\hat{I}^2}{2} R_{rad} = P_{radiated}$.

radiation resistance.

It is a measure of how efficiently an antenna radiates electromagnetic waves.

b) $\underset{\sim}{E} \times \underset{\sim}{H} = \underset{\sim}{S}$

so $\underset{\sim}{E}$, $\underset{\sim}{H}$ and $\underset{\sim}{S}$ form a set of mutually orthogonal vectors

$\underset{\sim}{e}_\theta \times \underset{\sim}{e}_\varphi = \underset{\sim}{e}_r$ in Spherical polars

Direction of $\underset{\sim}{E}$ is $\underset{\sim}{e}_\theta$, since power is radiated in $\underset{\sim}{e}_r$ direction

Also $\frac{|\underset{\sim}{E}|}{|\underset{\sim}{H}|} = \eta_0$

$$\therefore \underset{\sim}{E} = \underset{\sim}{e}_\theta \frac{\hat{I} l \beta \eta_0}{4\pi r} \exp j(\omega t - \beta r) \sin\theta$$

c) $$\underset{\sim}{S} = \frac{1}{2} \underset{\sim}{E} \times \underset{\sim}{H}^* = \frac{1}{2} \frac{\hat{I} l \beta \eta_0}{4\pi r} \sin\theta \, e^{j(\omega t - \beta r)} \times \frac{\hat{I} l \beta}{4\pi r} \sin\theta \, e^{-j(\omega t - \beta r)}$$

$$= \frac{\hat{I}^2 l^2 \beta^2 \eta_0 \sin^2\theta}{32\pi^2 r^2}$$

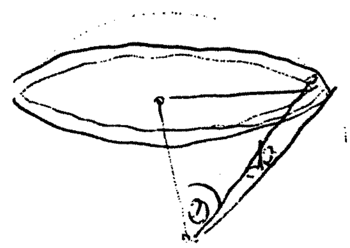
d) Total power radiated $= \int_{\text{closed surface}} \underset{\sim}{S} \cdot d\underset{\sim}{S}$

$= \int_{\text{spherical surface}} S \, dS$ (since $\underset{\sim}{S}$ and $d\underset{\sim}{S}$ are parallel if the surface is a sphere centred on $r=0$)

~~$dS = r^2 \sin\theta$~~ $\underset{\sim}{S}$ is constant w.r.t. r and ψ (at const. r)

so $dS = 2\pi r \sin\theta \, r \, d\theta$

$= 2\pi r^2 \sin\theta \, d\theta$



$$P = \frac{\beta^2 \hat{I}^2 l^2 \eta_0}{32\pi^2 r^2} \cdot 2\pi r^2 \int_0^\pi \sin^3\theta \, d\theta$$

$$= \frac{\beta^2 \hat{I}^2 l^2 \eta_0}{16\pi} \cdot \frac{4}{3}$$

$$= \underline{\underline{\frac{\beta^2 \hat{I}^2 l^2 \eta_0}{12\pi}}}$$

$$P = \frac{\hat{I}^2}{2} R_{rad} = \frac{\beta^2 \hat{I}^2 l^2 \eta_0}{12\pi}$$

$$\therefore R_{rad} = \underline{\frac{\beta^2 l^2 \eta_0}{6\pi}}$$

Using $c = \omega/\beta$ to replace β

$$R_{rad} = \underline{\underline{\frac{\omega^2 l^2 \eta_0}{6\pi c^2}}}$$

e) For efficient radiation, need $\hat{I}$ to be small i.e. want large radiation resistance $\therefore$ $\frac{\omega l}{c}$ large $\Rightarrow$ either large ω or large l

$\therefore$ To keep antenna length reasonable, high frequencies are desirable.

2000 Part 1B - PAPER 5 - ANSWERS

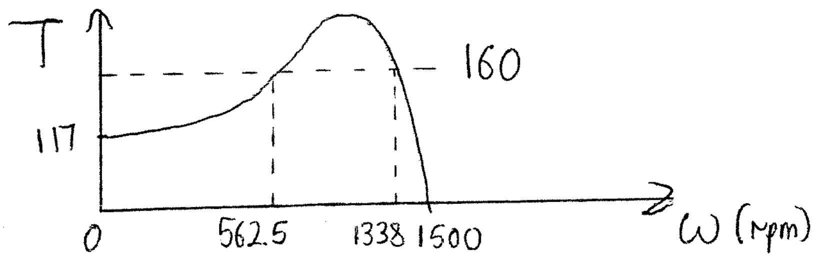
1. a) CMRR=100 dB b) $R_C = 500\ \Omega$; $R_T = 2.49\ \text{k}\Omega$ c) $V_{CC} = 19.3$ V, $V_{EE} = 100.3$ V
d) Use a 40 mA constant current source in place of R_T.

2. a) $R_B = 96.5\ \text{k}\Omega$; $R_L = 500\ \Omega$ b) Gain = -10; $R_{in} = 5\ \text{k}\Omega$; $R_{out} = 500\ \Omega$
c) $C_1 = 1.59\ \mu\text{F}$ d) $f_{3dB} = 390$ kHz

3. b) $I_{line} = 306.2$ A; $P_{lines} = 56.2$ kW; $V_S = 3.72$ kV c) $C = 109\ \mu\text{F}$; $I_{line} = 272.2$ A
d) $\cos\varphi = 0.999$ leading; $I_{line} = 245$ A. Even better power factor, but capacitors may not be adequately rated.

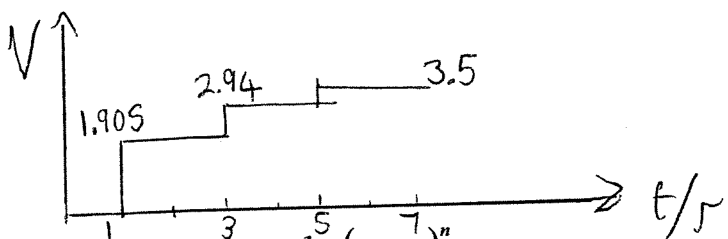
4. a) E = 14.87 kV; I = 3.94 kA b) For 11 kV generator, $E = 16.39\ \text{kV}\ \angle 20.5^0$;
For 33 kV generator, $E = 49.17\ \text{kV}\ \angle 20.5^0$ c) Prime-mover input power controls generator output real power, rotor field current controls generator output reactive power.

5. b) i) Torque = 117 Nm ii) Speed = 562.5 rpm or 1338 rpm



c) $\eta = 77.5$ % at 1338 rpm, 20 % at 562.5 rpm. Therefore, operating at 1338 rpm is preferable. d) Connect additional rotor resistance via slip-rings.

6. a) $Z_0 = 50\ \Omega$; $\tau = 25$ ns b) $\rho_S = 0.6$; $\rho_L = 0.905$
c)



d) $V = V_{dc}\dfrac{Z_0}{Z_0 + R_0}(1+\rho_L)\dfrac{1-(\rho_L\rho_S)^n}{1-\rho_L\rho_S}$; No. of reflections = 6; Total delay = 325 ns;
$V_\infty = 4.17$ V

7. b) $\boldsymbol{E}$, $\boldsymbol{H}$ and Poynting vector ($\boldsymbol{E} \times \boldsymbol{H}^*/2$) are mutually orthogonal, $\eta_0 = \dfrac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$;

$\boldsymbol{E} = \boldsymbol{e}_\theta \dfrac{\hat{I} l \beta \eta_0}{4\pi r}\sin\theta \exp(\omega t - \beta r)$ d) $P = \dfrac{\beta^2 \hat{I}^2 l^2 \eta_0}{12\pi}$ e) Need large R_a for efficient radiation otherwise current is large for given radiated power $\Rightarrow$ large ω for small l.