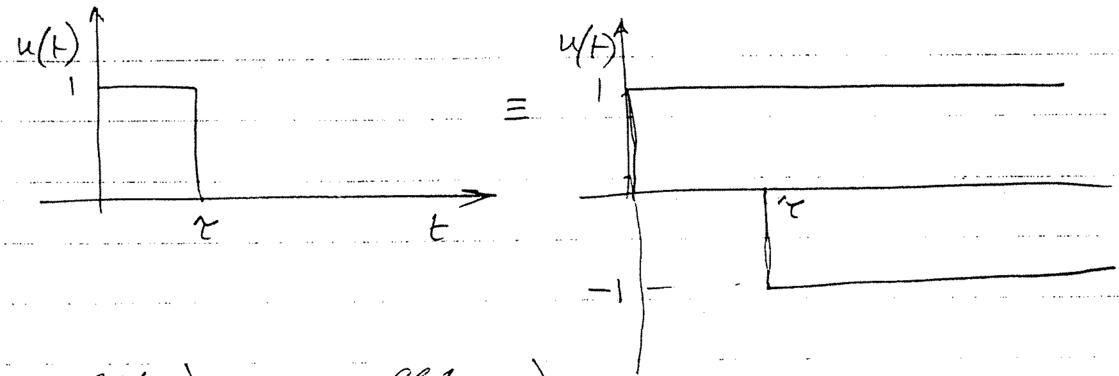
IB 2000 INFORMATION (CRIB)

Solution 1. (a) (i)



$$u(t) = H(t).1 - H(t-\tau).1$$

$$\text{L.T.} \rightarrow \quad u(s) = \frac{1}{s} - \frac{1}{s}e^{-s\tau} = \frac{1}{s}\left(1 - e^{-s\tau}\right)$$

(ii) As $\tau \rightarrow 0$, the pulse tends to an impulse of magnitude 10. i.e. $u(s) = 10$

(b) With the assumptions given & conservation equation.

Flow in of solute = Accumulation rate + Flow out.

$$Q.0 + m(t) = V.\frac{dc}{dt} + Qc$$

$$\frac{dc}{dt} + \frac{Qc}{V} = \frac{m(t)}{V}$$

(c)(i) With $m(t) = u(s)$ & $c(0) = 0$.

$$s\,c(s) + \frac{Q}{V}c(s) = \frac{1}{s}\left(1 - e^{-s\tau}\right)$$

& with given values

$$s\,c(s) + 0.01\,c(s) = \frac{1}{s}\left(1 - e^{-10s}\right)$$

$$c(s) = \frac{1 - e^{-10s}}{s(s+0.01)}$$

$$\mathcal{L}^{-1}\frac{1}{s(s+0.01)} = \mathcal{L}^{-1}\frac{1}{0.01}\left[\frac{1}{s} - \frac{1}{s+0.01}\right] = 100\left(1 - e^{-0.01t}\right)$$

$$\& \; c(t) = 100\left(1 - e^{-0.01t}\right) - H(t-10).100\left(1 - e^{-0.01(t-10)}\right)$$

For $t = 100$

$$c(t) = 100\left(1 - e^{-1}\right) - 100\left(1 - e^{-0.9}\right) = 100(-0.3679 + 0.4066)$$

$$= 3.87 \text{ kg m}^{-3}$$

cont (ii) Considering the second vessel, content of solute

$$= \int Q c \, dt = Q \int c(t) \, dt$$

& concentration $= \dfrac{Q \int c(t)\,dt}{\int Q\,dt} = \dfrac{\cancel{Q} \int c(t)\,dt}{\cancel{Q}.t}$

$$\int_0^t c(t)\,dt = 100 \left\{ t + \frac{e^{-0.01t}}{0.01} - \frac{1}{0.01} - H(t-10).\left[(t-10) + \frac{e^{-0.01(t}}{0.01} - \frac{1}{0.01}\right] \right.$$

For $t=100$; $c_2(t) = \dfrac{100}{100}\left\{ 100 + 36.788 - 100 - (90 + 40.657 - 100) \right\}$

$$= 100 + 36.788 - 90 - 40.657 = \underline{\underline{6.13 \text{ kg m}^{-3}}}$$

BUT

[Note also: Total solute = 10 kg.

Solids in first vessel = 3.87 kg

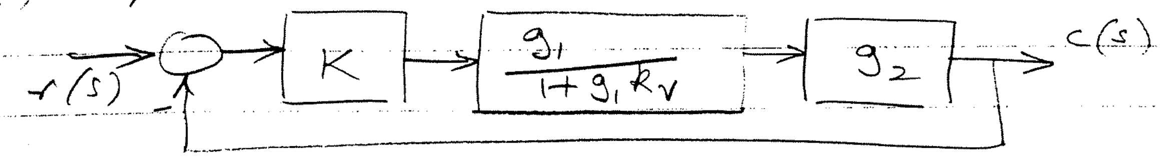
∴ Solids in second vessel = 6.13 kg.

Solvent to second vessel in this time $= Qt = 0.01 \times 100 = 1 \text{ m}$

Hence $c_2(t=100) = \underline{\underline{6.13 \text{ kg m}^{-3}}}$]

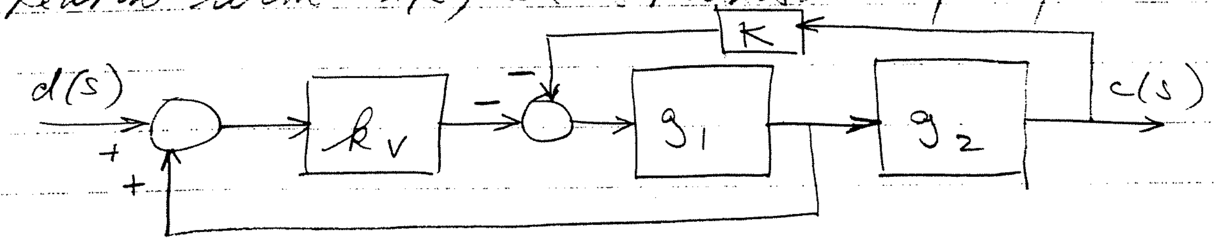
Solution 2 (a) Outer loop provides prime feedback, giving error & normal negative feedback from action. Velocity feedback gives additional damping to c.l. system but has no effect on any c.l. steady state errors.

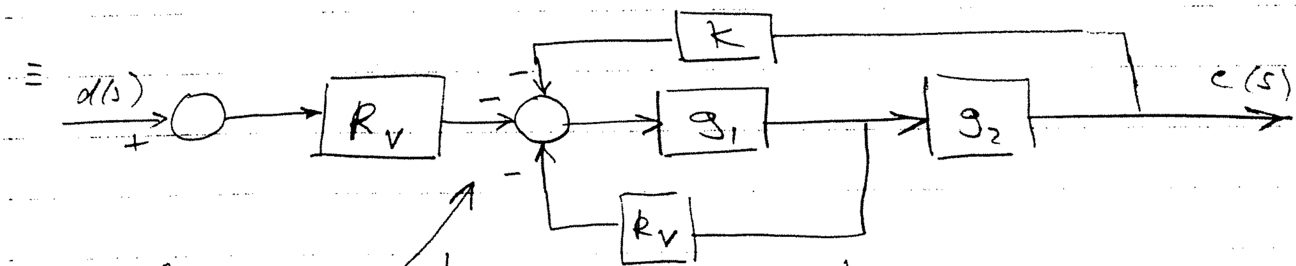
(b)(i) $c(s)/r(s)$ – i.e. consider $d(s) = 0$.

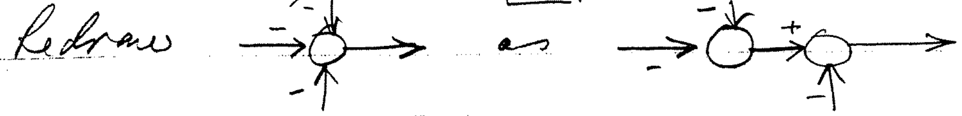


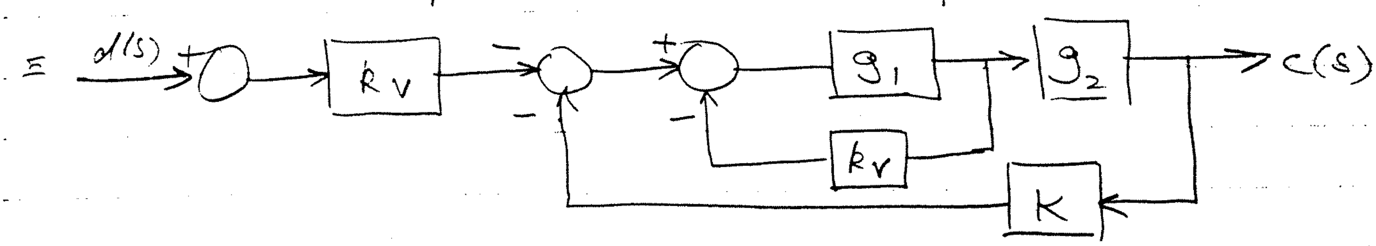
$$\rightarrow \quad r(s) \longrightarrow \boxed{\dfrac{K g_1 g_2}{1+g_1 k_v + K g_1 g_2}} \longrightarrow c(s)$$

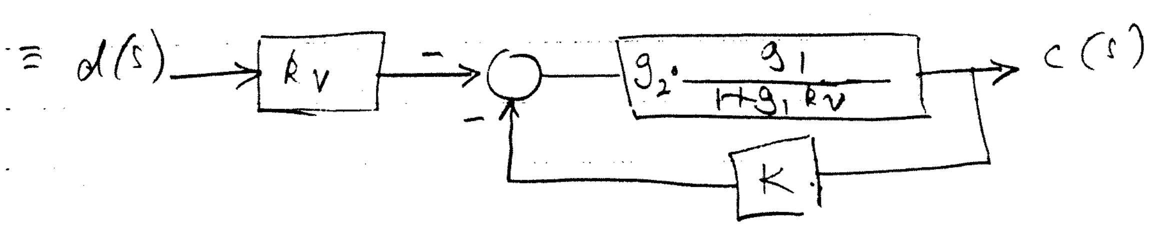
(ii) $c(s)/d(s)$ – this is more difficult because of 'overlapping' loops:
Redraw with $d(s)$ in standard input position:











Recall $= d(s) \longrightarrow \boxed{-k_v \cdot \dfrac{g_1 g_2}{1+g_1 k_v + K g_1 g_2}} \longrightarrow c(s)$

Note that the ch. equation in each case will be the same.

(c)(i) $c(s) = G(s)\, r(s)$ $\quad e(s) = r(s) - c(s) = r(s) - G(s) r$

i.e. $e(s) = (1 - G(s))\, r(s)$

$$\rightarrow \quad e(s) = \left\{ 1 - \frac{10}{s(s+1)\left(1 + \frac{0.1}{(1+s)}\right) + 10} \right\} r(s)$$

$$= \left\{ 1 - 10 / [\, s(1+s+0.1) + 10 \,] \right\} r(s)$$

$$= \left\{ 1 - \frac{10}{s^2 + 1.1s + 10} \right\} \cdot \frac{1}{s}$$

Step input

F.V.T. $\quad e(t \to \infty) = \lim_{s \to 0} s\, e(s) = 1 - 1 = \underline{\underline{0}}.$

(ii) Ramp input $r(t) = t$, $\quad r(s) = \frac{1}{s^2}$

$$e(t \to \infty) = \lim_{s \to 0} \left\{ \frac{s^2 + 1.1\cancel{s}}{s^2 + 1.1s + 10} \right\} \cdot \frac{\cancel{s}}{s^{\cancel{2}}} = \lim_{s \to 0} \left(\frac{s + 1.1}{s^2 + 1.1s + 10} \right)$$

$$= \underline{\underline{0.11}}$$

(iii) The error caused by $d(t)$ comes from the $c(s)/d(s)$ T.

$$c(s) = G_2(s)\, d(s) = \left\{ \frac{-0.1}{s^2 + 1.1s + 10} \right\}$$

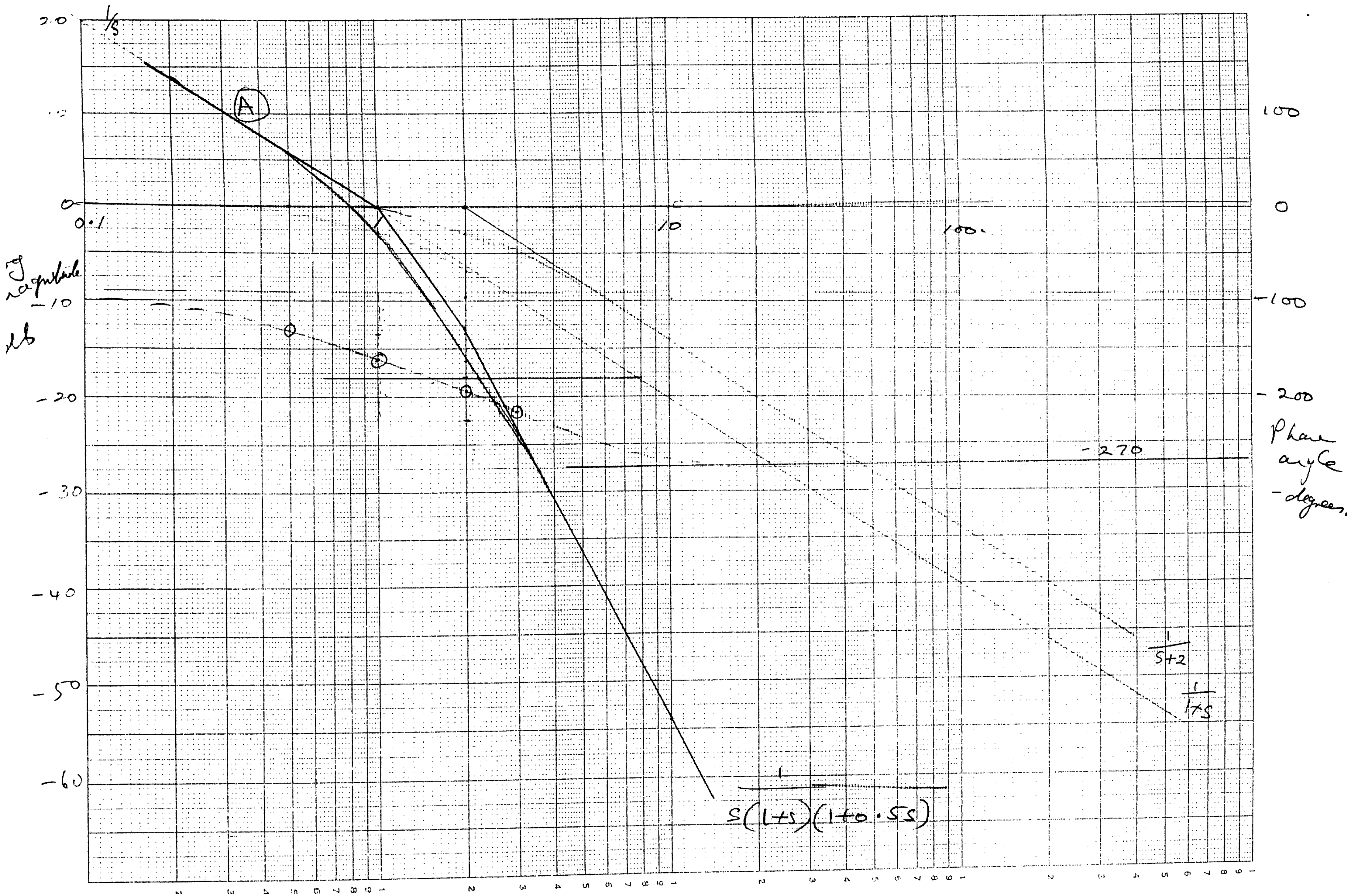
$$\& \quad c(t \to \infty) = \lim_{s \to 0} \left\{ \frac{-0.1}{s^2 + 1.1s + 10} \right\} \cdot \frac{0.1 \cdot s}{s}$$

$$= \underline{\underline{-0.001}}$$

(d) To remove the effect of disturbances entering the system the introduction of integral action is used.

Then $\quad K \rightarrow K\left(\frac{1 + T_i s}{T_i s}\right)$ & $c(t \to \infty) \rightarrow \underline{0}$.

Solution 3



1/S
A
20
10
0
0.1
10
100
Magnitude
-10
dB
-20
-30
-40
-50
-60
100
0
-100
-200
-270
Phase angle
-degrees
1/(S+2)
1/(1+S)
1/(S(1+S)(1+0.5S))

3 Solution. $G(s) = \dfrac{2}{s(s+1)(s+2)} = \dfrac{1}{s(s+1)(1+0.5s)}$

This gives plot (A), corner frequencies at 1 & 2 & note the effect of the 1/s term in particular.

Phase angle $= -90° - \tan^{-1}\omega - \tan^{-1} 0.5\omega$.
$\omega = 1$, $\angle = -161$; $\omega = 2$ $\angle = -198.4$
$\omega = 3$, $\angle = -218$. $\omega = .5$ $\angle = -131$

Gain crossover frequency : $= 0.8$
phase " " $= 1.55$
(iii) G.M. $= 10$ db, Hence $\underline{\underline{K_{max} = 3.2}}$.
(Calculated, e.g. R.-H, $K_{max} = 3.0$)

(ii) $\underline{\text{With } K = 1}$, G.M. $= \underline{\underline{10 \text{ db}}}$.
P.M. $= \underline{\underline{30°}}$.

*

(b) Phase compensation to show high & low frequency effects, position of corner frequencies, e.g. to raise phase angle near crossover region etc.

×

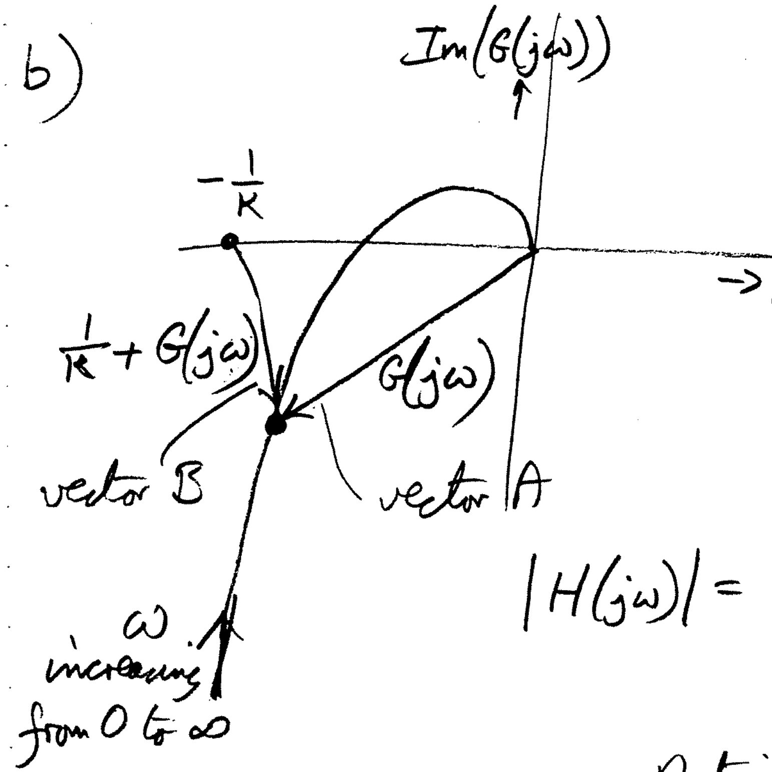
4. a) From diagram:

$$\bar{y}(s) = \left(\bar{r}(s) - \bar{y}(s)\right) . KG(s)$$

$$\therefore \bar{y}(s)\left(1 + KG(s)\right) = \bar{r}(s) . KG(s)$$

$$\therefore H(s) = \frac{\bar{y}(s)}{\bar{r}(s)} = \frac{KG(s)}{1+KG(s)} = \frac{G(s)}{\frac{1}{K}+G(s)}$$

b)



Nyquist: Let $s = j\omega$

$$|H(j\omega)| = \frac{|G(j\omega)|}{\left|\frac{1}{K} + G(j\omega)\right|}$$

$$= \text{Ratio of } \frac{|\text{vector } A|}{|\text{vector } B|}$$

c) Phase margin is when $|KG(j\omega)| = 1$

& $\angle G(j\omega) = \cancel{45} -135°$

But $\angle G(j\omega) = -90° - \tan^{-1}\left(\frac{3\omega}{4-\omega^2}\right) = -135°$

$$\therefore \tan^{-1}\left(\frac{3\omega}{4-\omega^2}\right) = 45°$$

$$\therefore 3\omega = 4 - \omega^2 \qquad \therefore \omega^2 + 3\omega - 4 = 0$$

4. (cont) $\therefore (\omega+4)(\omega-1)=0$

$\therefore \omega = 1$ at phase margin freq.

~~$\therefore G(j\omega) = \frac{1}{j\omega \ldots}$~~

When $\omega=1$:

$$|G(j\omega)|^2 = \frac{1}{1.((4-1)^2+3^2)} = \frac{1}{2.3^2}$$

$$\therefore K = \frac{1}{|G(j\omega)|} = \underline{\underline{\sqrt{2}.3}}$$

Magn of C.L. freq response at $\omega=1$

$$= \frac{1}{2\sin\frac{45^\circ}{2}} = \frac{1}{2\sin\frac{\pi}{8}} = 1.3066$$

The response is unlikely to go much above this value.

(eg at the Gain margin freq, $\omega=2$

& $|G(j\omega)| = \frac{1}{2.3.2} = \frac{1}{12}$

so $|K\,G(j\omega)| = \frac{\sqrt{2}.3}{12} = \frac{1}{2\sqrt{2}}$ & the CL response will have dropped well below unity.)

d) At low frequencies the C.L. freq resp is approx unity. At high frequencies it tends to zero, due to system inertia. If it rises significantly above zero in between, then this implies a rather oscillatory step response, which is undesirable.

5. (a) The first term in the equation for H(s) is a frequency-independent gain. The second is a high-pass filter because, by substitution of $s = j\omega$, its frequency response has magnitude 0 at $\omega=0$ and magnitude 1 as $\omega \to \infty$. The third term is a low-pass filter because its frequency response has magnitude 1 at $\omega=0$ and magnitude 0 as $\omega \to \infty$.

It is the high-pass filter which defines the lower channel bandedge frequency, and its –3dB frequency is given by $\omega_c T_1 = 1$. Hence $T_1 = 1/(2\pi 300)$ = **0.53 msec**.
The low-pass filter defines the upper channel bandedge frequency, and its –3dB frequency is given by $\omega_c T_2 = 1$. Hence $T_2 = 1/(2\pi 3400)$ = **47 μsec**.

T_1 is typically defined by the series resistance of the wires together with the inductance of parallel coils and transformers.

T_2 is typically defined by the series resistance of the wires together with the capacitance between the conductors.

(b) by Partial Fraction Expansion, the step response of the channel is given by

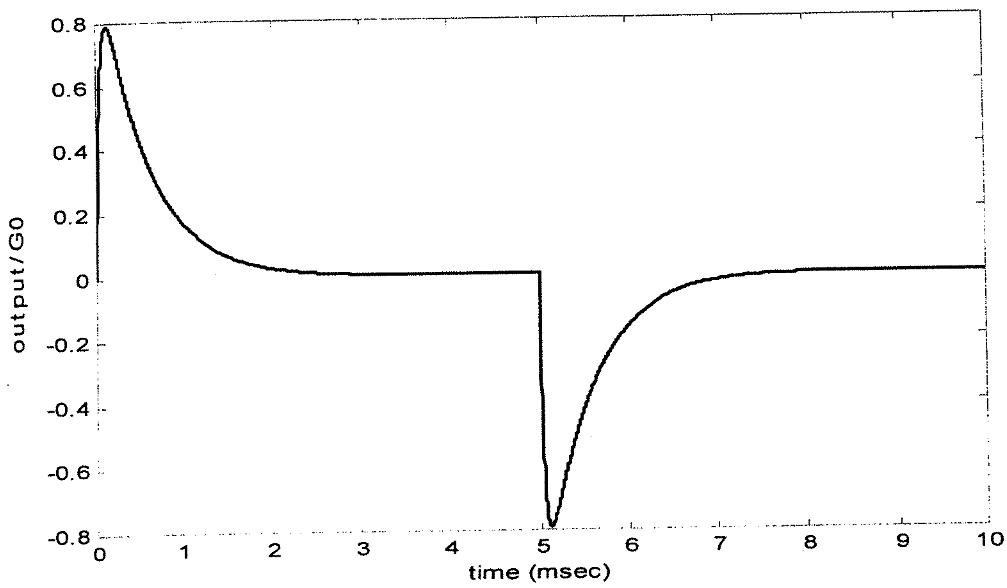
$$(1/s)\, H(s) = A/(s + T_1^{-1}) + B/(s + T_2^{-1})$$

Where $$A = -B = G_0\, T_1/(T_1 - T_2) = -1.0968\, G_0$$

Hence by inverse Laplace Transform the step response is

$$\mathbf{h(t) = A\,(\exp(-t/T_1) - \exp(-t/T_2))}$$

and by treating a rectangular pulse as the linear superposition of a step at t=0 and a negative step at t = 5msec, the output is :



(c) The 5 msec rectangular pulses of a direct 200 bit /s signal will each be converted to the shape shown above. A run of positive/negative pulses will become a single positive/negative 'blip' at the start and then a zero signal. Hence simple thresholding cannot be used to recover the transmitted signal.

A solution is <u>modulation</u>, which produces a signal having a bandwidth matched to that of the channel, so that it is much less distorted by being passed through the channel. (Modulation can be AM, PM or FM, giving ASK, PSK, FSK signals.)

6. (a) The output of a quantiser is different from the input V_{in}. The quantiser output integer k represents one of the quantised levels, $k\delta V$. We can express the error as

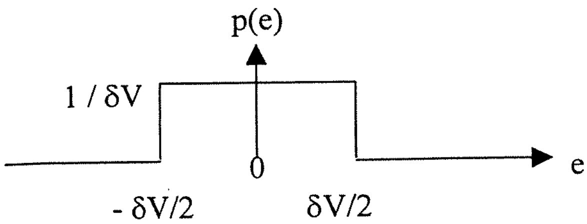
$$e = k\delta V - V_{in}$$

Thus we can treat the error as random noise added to the input signal:

$$k\delta V = V_{in} + e$$

If the error e can be assumed to be statistically independent of the input V_{in}, then we describe e as additive random noise. This assumption is reasonable for sufficiently complicated signals such as speech or musical audio (especially if their amplitude is also large compared to δV).

For a quantiser which selects the output level nearest to the input voltage, the error lies between - $\delta V/2$ and $\delta V/2$ with uniform probability density. The area of the pdf is made 1 by giving it value $1 / \delta V$ over this range:



(b) The mean-squared value of e is defined as

$$\text{mse} = \int p(e)\, e^2\, de = (1/\delta V) \int_{-\delta V/2}^{\delta V/2} e^2\, de = (1/\delta V) \left[e^3/3 \right]_{-\delta V/2}^{\delta V/2} = \delta V^2 / 12$$

so the rms value, defined as $\text{mse}^{0.5}$, is **<u>$\delta V / \sqrt{12}$</u>**.

(c) The maximum signal-to-noise (SNR) ratio is achieved by having the smallest quantising steps. Hence the range of the ADC should be set equal to the range of the input signal, i.e. –5V to 5V. If the quantiser produces 15 bit values then its step size is

$$\delta V = 10 / 2^{15} = 0.3052 \text{ mV}.$$

Hence the r.m.s. quantising noise is

$$\delta V / \sqrt{12} = 88.1 \ \mu V.$$

and (since the r.m.s. signal voltage is given as $5 / \sqrt{3}$) the SNR in dB is

$$SNR = 20 \log_{10} (5\ 2^{15} \sqrt{12} / 10 \sqrt{3}) = 20 \log_{10} (2^{15}) = \underline{\mathbf{90.3\ dB}}.$$

(d) The quantisation noise has a power density spectrum which is approximately 'flat' (constant) from zero frequency to half the sampling frequency.

The output filter is a low-pass filter. If its cutoff frequency is reduced, then it filters out the high frequency components of the quantising noise (which is an improvement, since it reduces the r.m.s. value of the quantising noise). However, it also filters out the high frequency harmonics of the triangle wave (which distorts the triangle wave, and so is undesirable). Hence the choice of cutoff frequency is a tradeoff between these two factors.

A first class answer might mention further details: the distortion of the triangle wave is predominantly a 'rounding off' of its sharp corners. A triangle wave (from the data book) has only odd harmonics, and their amplitude is inversely proportional to the square of their frequency, so for example the harmonic at 11 times the fundamental frequency has amplitude 1/121 that of the fundamental. Thus placing the cutoff frequency at about 10kHz would only alter the power of the triangle wave by about $1/121^2$ or less than 0.01%, while it would reduce the quantising noise power by about a factor of 10 (10dB).