ENGINEERING TRIPOS PART IB

Tuesday 6 June 2000 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

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1 (a) Explain briefly the principle of conservation of linear momentum and the principle of conservation of moment of momentum. [4]

(b) The uniform rigid disc shown in elevation in Fig. 1, which has a mass m and radius r, is initially spun up to an angular speed of ω_0 while supported just clear of the horizontal plane. The disc is then dropped on to the plane such that it remains in contact with the plane and does not bounce. The coefficient of friction between the disc and the plane is μ. Show that the velocity of the disc when skidding stops is $v_1 = \frac{r\omega_0}{3}$, and that the distance travelled is $d = \frac{1}{18}\frac{r^2\omega_0^2}{\mu g}$, where g is the gravitational acceleration. [8]

(c) The disc then rolls along the plane without energy loss until it hits a step of height $\frac{r}{2}$. Assuming the disc remains in contact with the corner and does not slip, determine an expression for the speed of the disc v_2 as it rolls along the higher horizontal plane in terms of ω_0, r and g. Determine the value of ω_0 if v_2 is zero. [8]

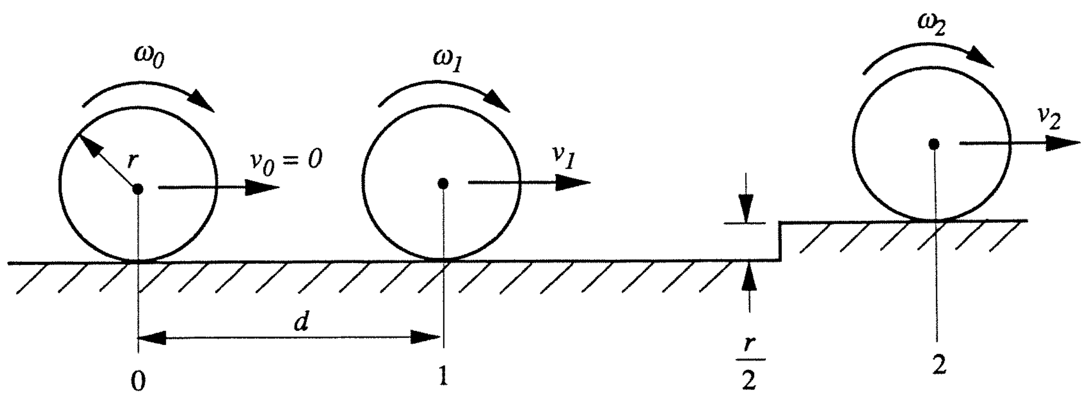

Fig. 1

2 (a) For a uniform rectangular block of mass m with dimensions $a \times b \times h$, show from first principles and the application of the perpendicular axis theorem that the moment of inertia about an axis parallel to the edges of length h and passing through the centre of gravity is given by

$$I_G = \frac{m}{12}\left(a^2 + b^2\right). \qquad [4]$$

(b) The two rigid uniform blocks A and B shown in Fig. 2 model a collision between two cars of the same size at a crossroads. The blocks are both of length $2a$, width a and mass m. Before the collision they are travelling perpendicular to each other at the same speed u. Assume that the two colliding corners remain in contact with each other after the collision.

(i) Without calculation, what can you say about the angular velocities of the two blocks immediately after the collision? [2]

(ii) Determine an expression for the velocity of the centre of gravity of block B immediately after the collision in terms of u. [14]

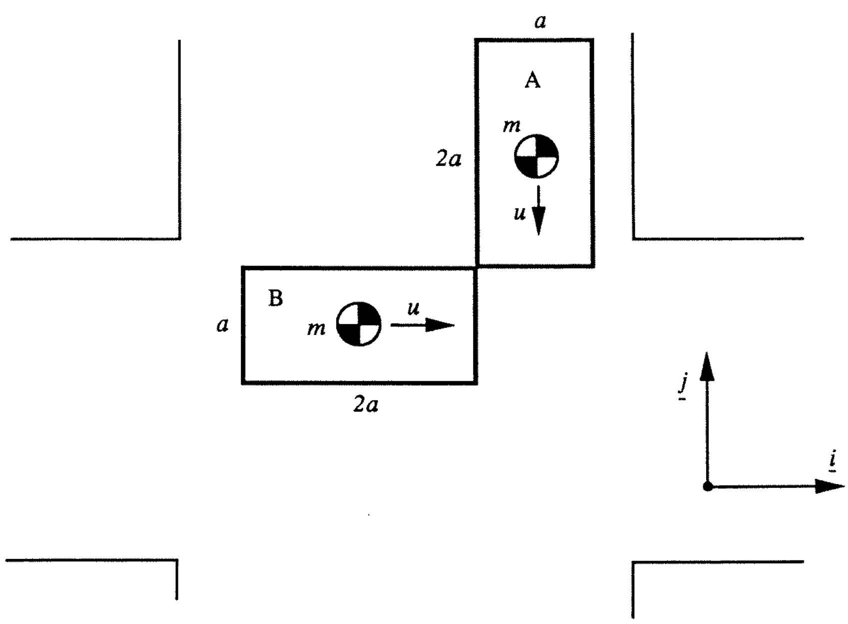

Fig. 2

3 (a) Figure 3 shows a *fixed* frame of reference F with axes defined by fixed unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. A two-dimensional *rotating* body R has an absolute angular velocity of $2\boldsymbol{k}$ rads^{-1} and an absolute angular acceleration of $5\boldsymbol{k}$ rads^{-2}. A point Q is fixed in the body and has an absolute velocity of $6\boldsymbol{i}$ ms^{-1} and an absolute acceleration of $23\boldsymbol{i}$ ms^{-2}. A second point P is moving from A to B along a circular path on the body of radius 2 m. At the instant shown in Fig. 3, the position of P relative to Q is $3\boldsymbol{j}$ m, its speed along the path is 4 ms^{-1}, and this speed is increasing at a rate of 12 ms^{-2}.

(i) Draw a velocity diagram to illustrate the absolute velocity of P. A suitable scale is 10 mm to represent 1 ms^{-1}. On the diagram identify each of the vectors in the expression for $\boldsymbol{v}_P$ given on page 2 of the Mechanics Data Book. [4]

(ii) Draw an acceleration diagram to illustrate the absolute acceleration of P. A suitable scale is 5 mm to represent 1 ms^{-2}. On the diagram identify each of the vectors in the expression for $\boldsymbol{a}_P$ given on page 2 of the Mechanics Data Book. [6]

(b) The industrial robot manipulator shown diagrammatically in Fig. 4 is rotating about the z axis with constant absolute angular velocity $5\boldsymbol{k}$ rads^{-1}. At the instant shown the arm AB is held fixed at an angle of 30° relative to the body of the robot OA. The arm BC is rotating about joint B with angular velocity $2\boldsymbol{j}$ rads^{-1} and angular acceleration of $3\boldsymbol{j}$ rads^{-2}, both relative to AB. The centrelines of the manipulator arms AB and BC lie in the x-z plane. Determine the velocity and acceleration of the grip located at C at the instant shown in Fig. 4. [10]

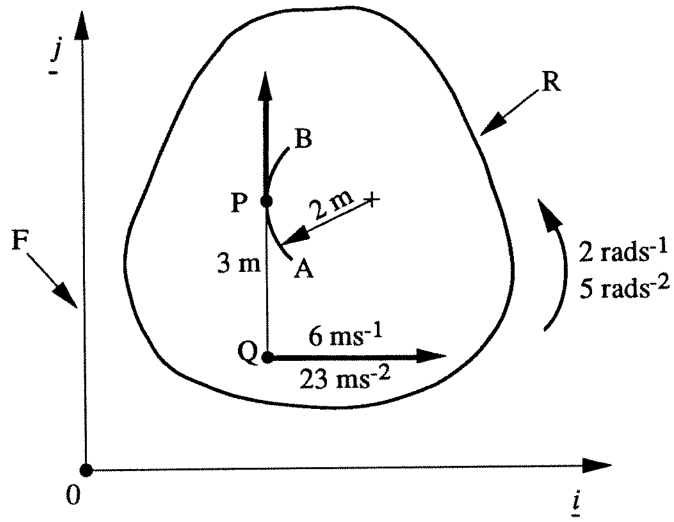

Fig. 3

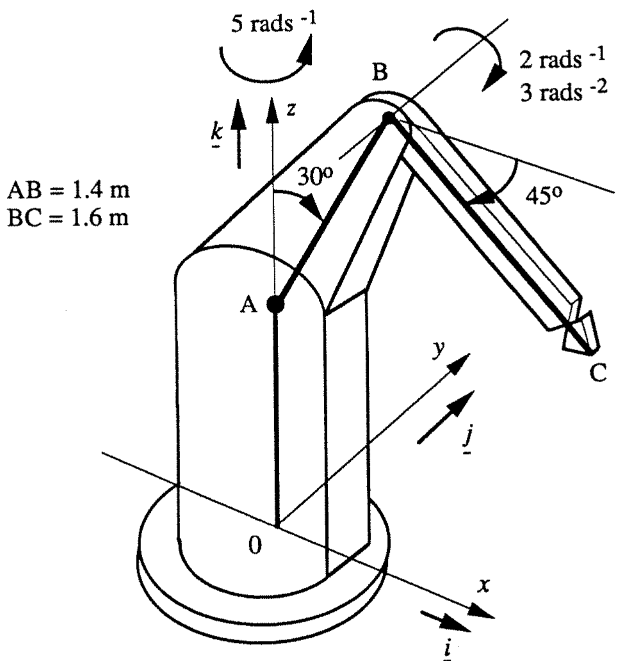

Fig. 4

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4 (a) Explain D'Alembert's principle. Illustrate the application of D'Alembert's principle using the motion of a satellite in a circular orbit around the Earth. [6]

(b) A uniform rod of mass m and length L lies at rest on a smooth horizontal plane. A force F is applied perpendicular to the rod at one end. Determine the initial acceleration of the centre of gravity of the rod and the initial angular acceleration of the rod in terms of m, L and F. [4]

(c) Two similar uniform rods each of length $2\sqrt{2}$ m and mass 3 kg lie at rest at right angles to each other on a smooth horizontal plane as shown in Fig. 5. The rods are connected together by a frictionless pivot at B. A force of 40 N is now applied at A in the direction shown. Determine the initial angular acceleration of rod BC and the initial acceleration of the pivot at B. [10]

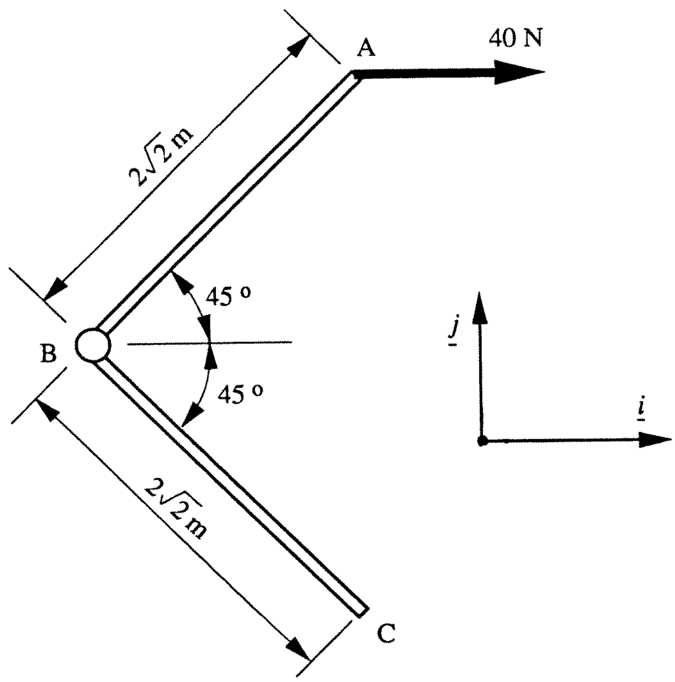

Fig. 5

5 (a) In a ball game a player strikes a ball with a bat. The bat is modelled as a uniform rod AB of length L and mass M smoothly pivoted at A, and the ball as a uniform sphere of mass m . At the moment of strike both the bat and the ball lie in a horizontal plane with the bat rotating at an angular speed of ω and the ball travelling at a speed of u in the directions shown in Fig. 6. At what distance x from A along the bat should the ball be hit if the player is to feel no impulsive reaction? Explain if the analysis would change if the plane containing the bat and ball was not horizontal. [10]

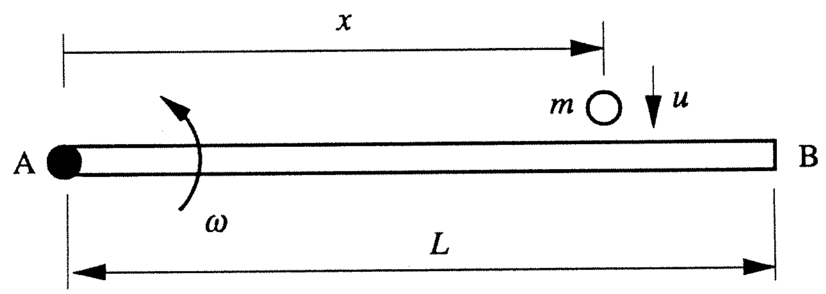

Fig. 6

(b) A racing car is travelling at a constant high speed round a uniform left-hand bend of radius R . The outside rear wheel is modelled as a uniform disc of mass m and radius r . The wheel is rotating without slipping and the speed of the centre of the wheel is v . Select an appropriate set of coordinate axes and unit vectors and determine an expression for the gyroscopic couple *on* the wheel in terms of m , R , r and v . Take particular care to describe the direction of the couple on the wheel. Explain the reason for this gyroscopic couple in terms of the accelerations of the particles around the perimeter of the wheel. [10]

6 The mechanism shown in Fig. 7 is driven by the crank AB . A slider E is constrained to move in a straight line. In the position shown in Fig. 7, the crank is being driven at an angular speed of 0.4 $rads^{-1}$ and an angular acceleration of 0.1 $rads^{-2}$ in the directions shown. Your answers are to be **drawn on the sheet provided and attached to your script**.

(a) Locate the instantaneous centre of CE on the space diagram on side 1 of the sheet provided. [4]

(b) Draw a velocity diagram on side 1 of the sheet provided and by taking reasonably accurate scale measurements from the diagram determine the angular velocities of rods DF and CE , and the relative sliding velocity at B . A suitable scale for the velocity diagram is 1 mm to represent 1 mms^{-1}. [6]

(c) Draw an acceleration diagram on side 2 of the sheet provided and by taking reasonably accurate scale measurements from the diagram determine the angular acceleration of CE and the acceleration of E . A suitable scale for the acceleration diagram is 3 mm to represent 1 mms^{-2}. [10]

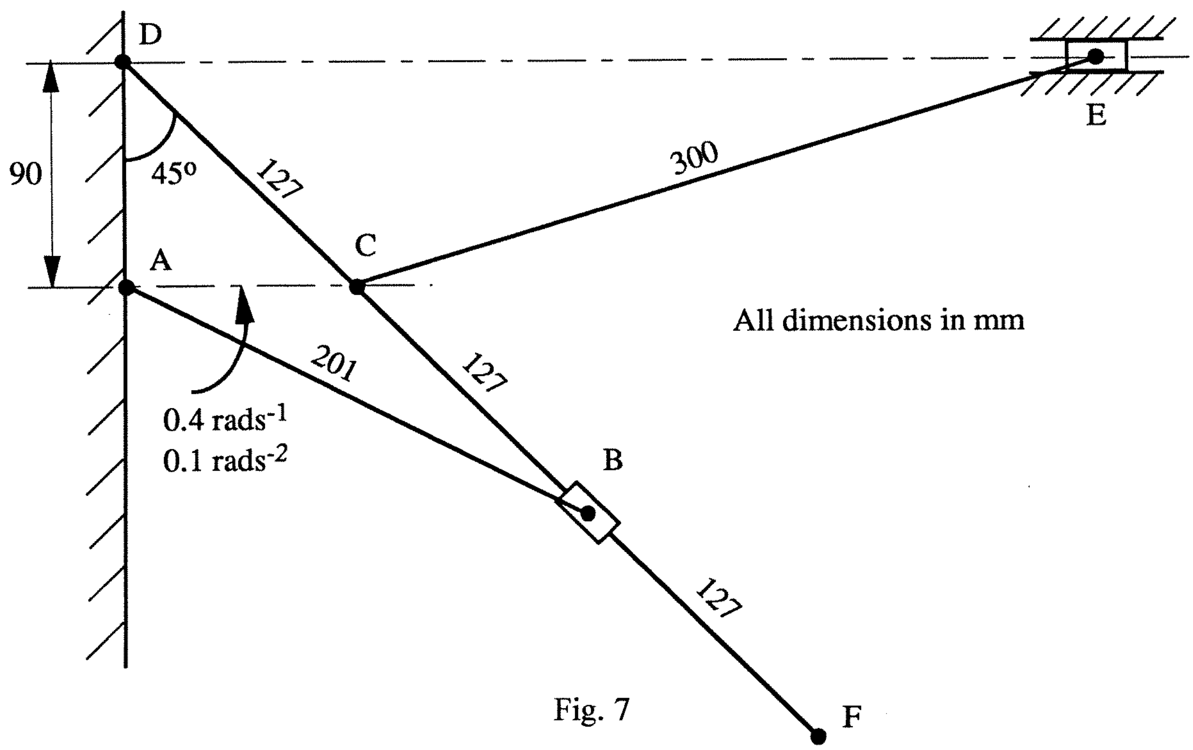

Fig. 7

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ENGINEERING TRIPOS PART IB Paper 1 MECHANICS Question 6

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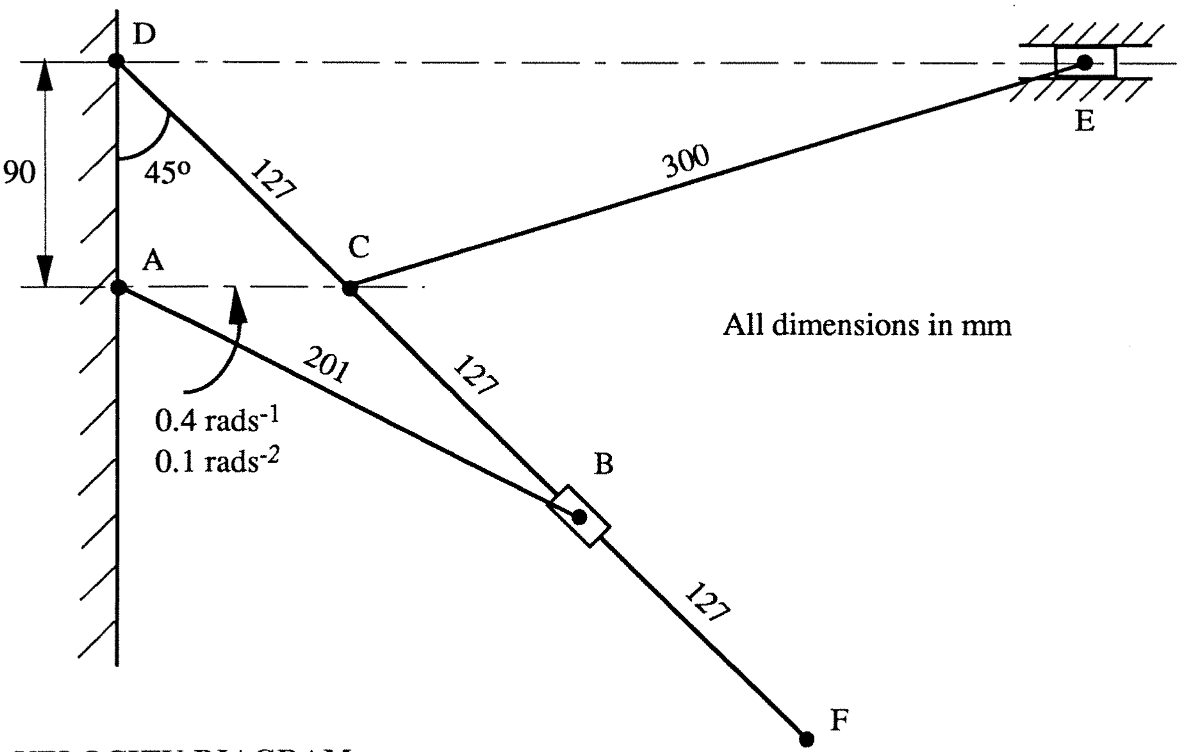

VELOCITY DIAGRAM

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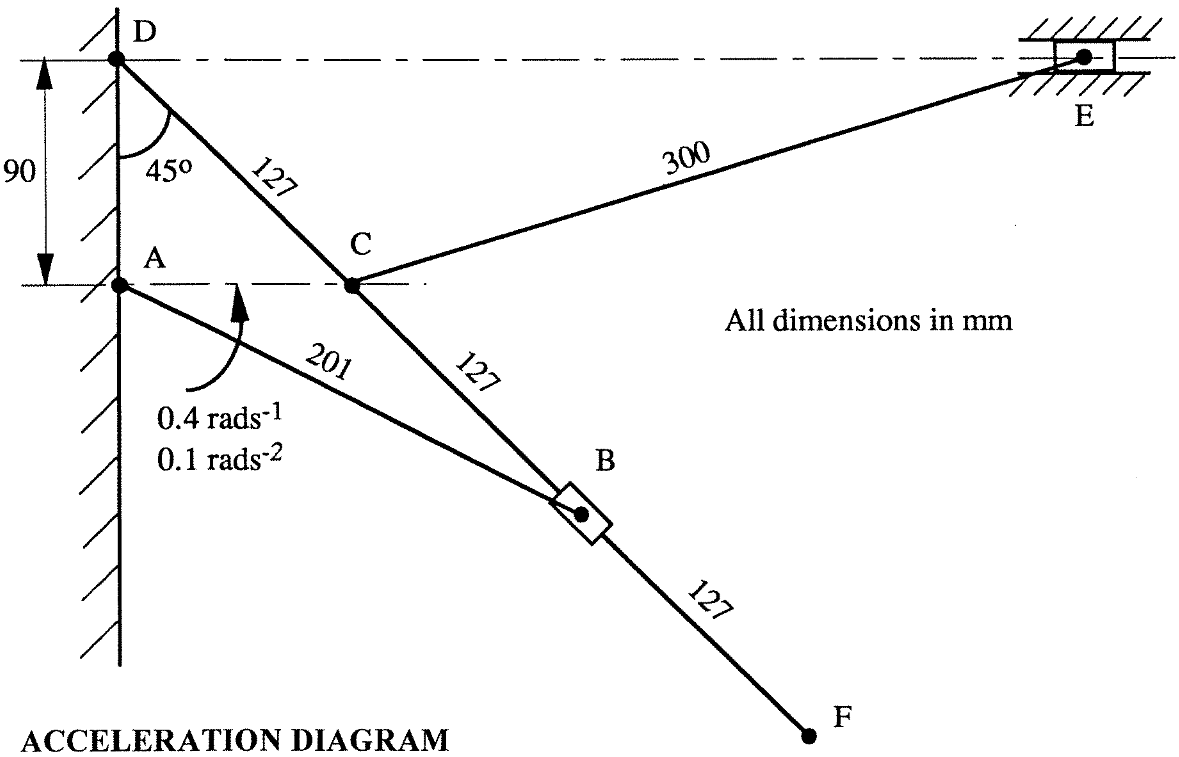

ACCELERATION DIAGRAM