ENGINEERING TRIPOS PART IB

Tuesday 6 June 2000 2 to 4

Paper 4

FLUID MECHANICS AND HEAT TRANSFER

Answer not more than ***four*** *questions.*

Answer at least **one** *question from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

*Answer at least **one** question from this section.*

1 (a) Show that, at equilibrium, for a column of fluid of density ρ (not necessarily constant), the rate of change of pressure P with depth z is

$$\frac{dP}{dz} = +\rho g \, . \qquad [6]$$

(b) A sea wall contains a sluice gate hinged along its top edge (A) which is located 3 m below the water level as shown in Fig. 1. The gate is 2 m high and 1.5 m wide. The density of the sea water varies with depth z below the surface according to $\rho = \rho_0(1 + bz)$, where $\rho_0 = 1010\,\text{kg}\,\text{m}^{-3}$ and b is a constant having the value $3.2 \times 10^{-3}\,\text{m}^{-1}$. Calculate:

- (i) the hydrostatic pressure at the depth corresponding to the bottom edge (B) of the gate; [4]
- (ii) the total force acting on the gate; [4]
- (iii) the force applied at the bottom edge (B) of the gate that is required to hold the gate closed. [6]

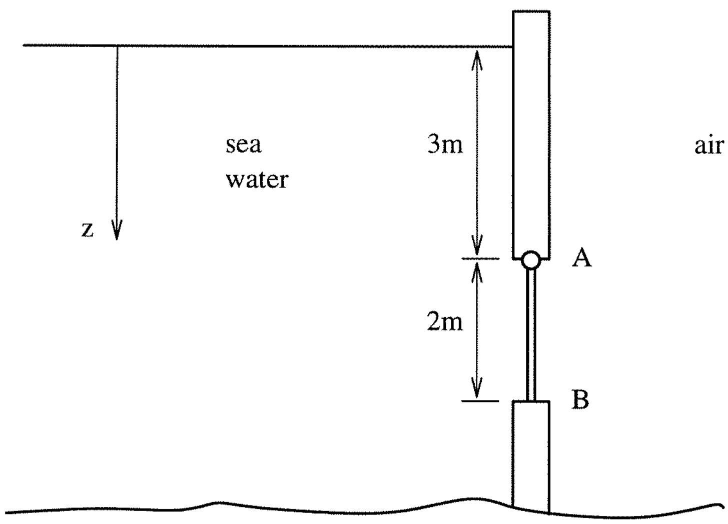

Fig. 1

2 An oil pipeline includes a side-branch located at a bend, as shown in Fig. 2. The diameter of the inlet pipe is 1 m while the diameter of the main exit pipe is 0.8 m and the diameter of the side-branch is 0.3 m. The density of the oil is $860\ \mathrm{kg\,m^{-3}}$ and the volumetric flow rate at the inlet is $6.28\ \mathrm{m^3\,s^{-1}}$. The pressure at the inlet is $150\ \mathrm{kN\,m^{-2}}$ above atmospheric pressure. The flow is incompressible and is free of significant losses, and the side-branch takes 20% of the inlet volumetric flow rate.

(a) Calculate the flow velocity in the main exit pipe and the side branch. [5]

(b) Show that the pressures in the main exit pipe and the side branch are $P_2 = 134.5\ \mathrm{kN\,m^{-2}}$ and $P_3 = 41.7\ \mathrm{kN\,m^{-2}}$ above atmospheric pressure. [5]

(c) Calculate the *x*-component of the force required to hold the assembly in place, given that there is no tension in the walls of the pipes. [10]

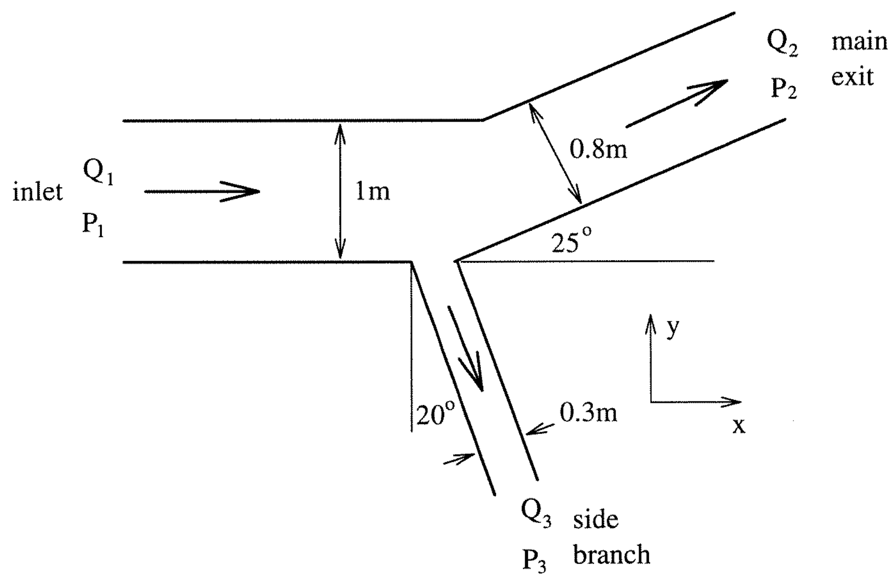

Fig. 2

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3 (a) Water flows in a long straight horizontal circular pipe of radius R. The flow is laminar and the pressure gradient is constant along the length of the pipe.

(i) Show that the velocity profile is given by

$$u(r) = U_0\left(1 - \left(\frac{r}{R}\right)^2\right),$$

where U_0 is the velocity of the fluid at the centre of the pipe and r is the radial distance from the centre of the pipe. Obtain an expression for U_0 in terms of the pressure gradient. [9]

(ii) Sketch the velocity profile and express the average velocity $\overline{u}$ in the cross-section in terms of the maximum velocity U_0. [4]

(b) A long straight pipe of radius 5 mm contains water flowing at $10^{-5}\,\mathrm{m^3 s^{-1}}$.

(i) Check that the regime of the flow is such that the results obtained in (a) can be applied. [4]

(ii) Calculate the maximum velocity of the flow. [3]

4 For small amplitudes, the velocity of propagation of waves on the surface of deep water depends only on the wavelength λ, the physical properties of water and gravity g.

(a) Identify the two physical properties that affect the wave propagation velocity. Note that viscosity is not relevant: it causes the waves to decay but does not affect the propagation velocity. [4]

(b) For very large wavelengths, the curvature of the surface is small and plays a negligible rôle. Use dimensional analysis to find an expression for the velocity of propagation of the waves. [5]

(c) For very small wavelengths, gravitational effects are negligible. Find an expression for the velocity of propagation in this case. [5]

(d) Using the results of (b) and (c), sketch the variation of wave velocity with wavelength. Find an estimate for the wavelength corresponding to the minimum velocity propagation and an estimate for that minimum velocity. [6]

SECTION B

*Answer at least **one** question from this section.*

5 A pipe carrying liquid sodium has outside diameter 0.2 m and outer surface temperature 1000°C. A 0.1 m layer of insulating material covers the pipe. Although Fourier's law of conduction is valid for this material, its thermal conductivity λ depends significantly on temperature.

(a) From the principle of heat conservation applied on any control volume within the insulating material, show that the local heat equation valid for this axisymmetric steady problem is

$$\frac{1}{r}\frac{d}{dr}\left[r\,\lambda(T)\frac{dT}{dr}\right] = 0,$$ [5]

where r is the radial distance from the centre of the pipe.

(b) The quantity F, which depends on temperature only, is defined as

$$F(T) = \int_{T_0}^{T} \lambda(v)\,dv,$$

where T_0 is an arbitrary reference temperature. Show that

$$\frac{dF}{dr} = \lambda(T)\frac{dT}{dr}.$$ [3]

(c) Using (b), give a general form for the solutions of the local equation in (a). [4]

(d) For the particular case of a linear dependence of λ, from $0.01\,\mathrm{W\,m^{-1}K^{-1}}$ at 20°C to $1\,\mathrm{W\,m^{-1}K^{-1}}$ at 1000°C, and for an outer surface heat transfer coefficient of $10\,\mathrm{W\,m^{-2}K^{-1}}$ due to air convection, find the rate of heat lost per metre length of pipe in a room at 20°C. [8]

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6 A mixture of equal mass of air and water (liquid or gaseous) is maintained at thermodynamic equilibrium at 2 bars and 100°C.

(a) Determine the partial pressures of air and steam. [4]

(b) Determine the mass fraction of steam, i.e. gaseous water. [4]

(c) Calculate the specific entropy per kilogram of the whole mixture. The datum used will be saturated liquid at 1 bar for air and saturated liquid at the triple point for water. Air can be treated as a perfect gas. [6]

(d) The mixture is expanded to 1 bar, adiabatically and reversibly. Using linear interpolation between the values of 0.45 bar and 0.5 bar for the partial pressure of steam, determine the temperature of the mixture and the steam mass fraction. [6]

END OF PAPER