ENGINEERING TRIPOS PART IB

Thursday 8 June 2000 9 to 11

Paper 5

ELECTRICAL ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of the question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer at least **one** *question from this section*

1 (a) The output signal from a transducer consists of a 1 mV, 5 kHz differential signal and an unwanted 100 mV, 50 Hz common-mode signal owing to mains interference. Explain why a differential amplifier is a good choice for amplifying this signal and determine the common-mode rejection ratio (CMRR) required if the 5 kHz component at the amplifier output is to be 60 dB greater than the corresponding 50 Hz component. [4]

(b) The differential amplifier of Fig. 1 is to be used in a different application. Draw small-signal circuits valid for differential and common-mode signals, and hence derive expressions for the gain (v_a / v_1 or v_b / v_2) for differential and common-mode signals. Hence show that the CMRR is

$$\text{CMMR} = \frac{h_{ie} + 2R_T\left(h_{fe} + 1\right)}{h_{ie}}$$

and determine values for R_C and R_T to give a differential gain of 40 dB and a CMRR of 60 dB.

Both transistors have identical small signal parameters: $h_{ie} = 1$ kΩ, $h_{fe} = 200$, h_{oe} and h_{re} are negligible and may be ignored. [8]

(c) Both transistors are biassed with $V_{CE} = 10$ V, $I_C = 20$ mA and $V_{BE} = 0.7$ V. Taking the quiescent voltage at their bases to be zero, determine the values for V_{CC} and V_{EE} required to attain this CMRR. [4]

(d) Explain how an additional bipolar transistor could be used in the circuit of Fig. 1 to provide the required CMRR without the use of an excessively large power supply voltage. [4]

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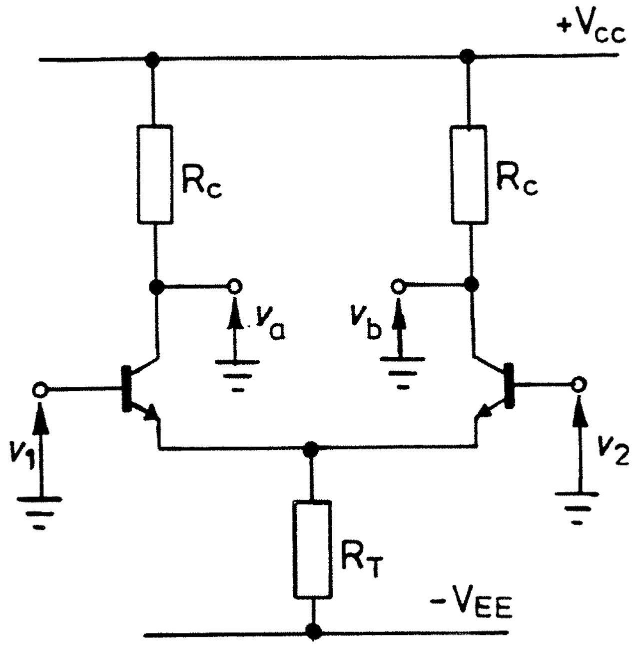

Fig. 1

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2 (a) The common-emitter amplifier shown in Fig. 2 is to be biassed so that $V_{BE} = 0.7$ V, $V_{CE} = 10$ V and $I_C = 20$ mA. At this operating point $h_{FE} = 100$. Explain why $V_{CE} = 10$ V is a suitable choice, and determine values for R_B and R_L. [4]

(b) Draw a small-signal circuit for the amplifier valid for mid-band frequencies, and determine the mid-band voltage gain, input resistance and output resistance. State and justify any assumptions. The small-signal parameters at the operating point are:

$h_{fe} = 100$, $h_{ie} = 1\text{k}\Omega$, h_{oe} and h_{re} are negligible and may be ignored. [6]

(c) Explain why the capacitor C_1 attenuates the gain at low frequencies and find its value if the frequency of the resulting -3 dB point is to be 20 Hz. [5]

(d) An improved model for the transistor at very high frequencies includes a parasitic capacitance between base and collector, of value 10 pF. Explain why this attenuates the gain at high frequencies, and find the frequency of the corresponding -3 dB point. State and justify any assumptions. [5]

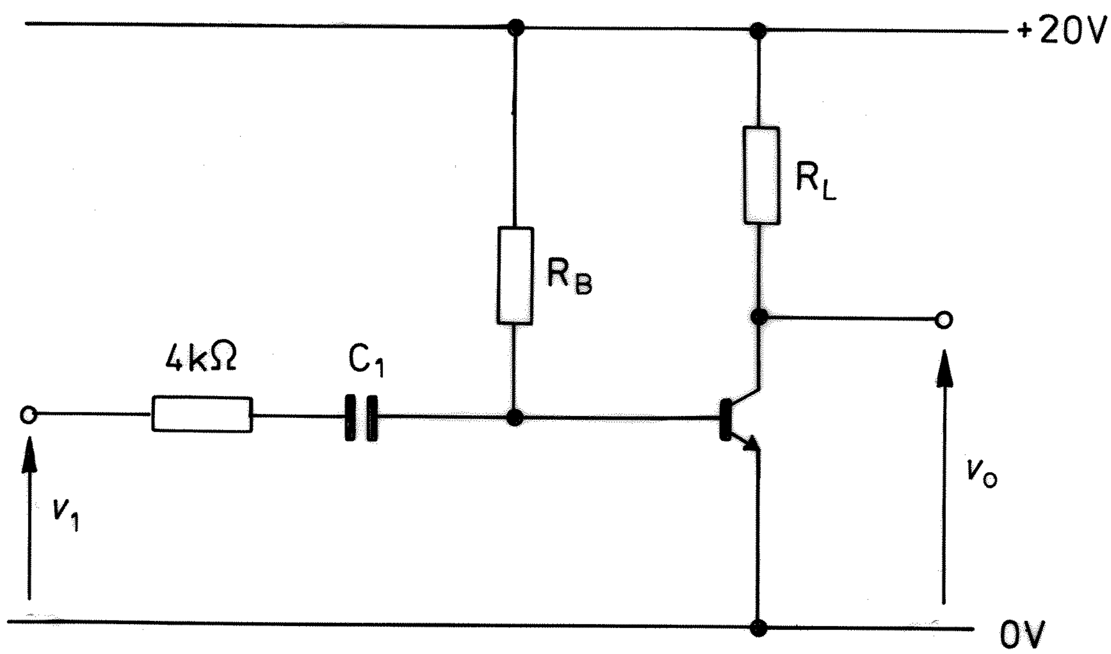

Fig. 2

SECTION B

Answer at least **one** *question from this section*

3 (a) Using phasor addition, show that if the three phases of a balanced three-phase supply are connected in series to form a single phase supply, the magnitude of the resulting single-phase voltage is twice that of the individual phase voltages. Hence explain why electrical power is generated as 3-phase ac. [5]

(b) Figure 3 shows a factory which consumes 1.4 MW at a power factor of 0.8 lagging from a 50 Hz three-phase power supply. The power supply is connected to the factory via lines of impedance (0.2+j1.0) Ω, and the voltage at the factory is fixed at 3.3 kV. Determine the line current, the total power dissipated in the lines, and the voltage at the source. [6]

(c) In order to improve the load power factor, balanced star-connected capacitors are connected in parallel with the load. Determine the value of the capacitors, which would correct the load power factor to 0.9 lagging, and the new line current. [4]

(d) The capacitors found in part (c) are mistakenly connected in delta. Find the overall load power factor and line current. Comment on the seriousness of this mistake. [5]

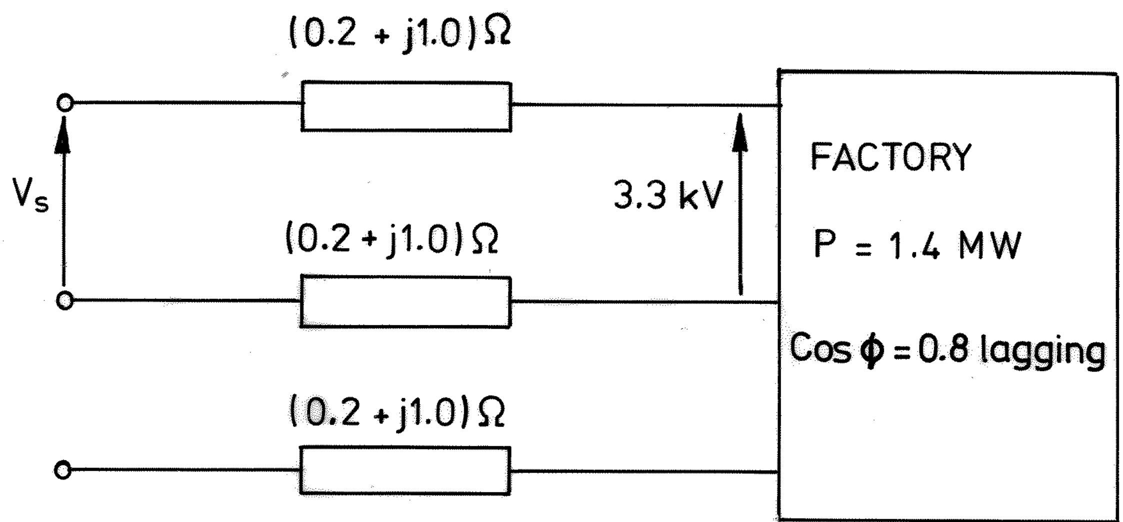

Fig. 3

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4 State two advantages of the per-unit system for the analysis of electrical power systems. [3]

Figure 4 shows a power system in which a 100 MVA, 11 kV synchronous generator of 0.2 pu reactance is connected via a 100 MVA, 11 kV/33 kV transformer of 0.12 pu reactance to a 33 kV bus-bar. The 33 kV bus-bar is connected to a 132 kV transmission line of impedance j10 Ω per phase via a 200 MVA, 33 kV/132 kV transformer of 0.3 pu reactance. The other end of the transmission line is connected to an 11 kV bus-bar via a 200 MVA, 132 kV/11 kV transformer of 0.25 pu reactance. The 11 kV bus-bar is connected to a load which takes 60 MW of real power and 45 MVAr of reactive power at a lagging power factor. The voltage at the 11 kV bus-bar is maintained at exactly 11 kV $\angle 0^0$.

(a) Determine the magnitude of the generator line-line emf, in volts, and the line current at the generator, in amps. [8]

(b) The load at the 11 kV bus-bar increases to 120 MW of real power and 90 MVAr of reactive power at a lagging power factor. In order to supply this increased load a 100 MVA, 33 kV generator of 0.15 pu reactance is connected directly to the 33 kV bus-bar in parallel with the original 100 MVA, 11 kV generator. The voltage at the 11 kV bus-bar continues to be maintained at exactly 11 kV $\angle 0^0$. Assuming that the generators operate with identical per-unit excitation emfs (***both in magnitude and phase***) find the excitation emfs (magnitude, in volts, and phase) of both generators. [6]

(c) Explain how the generators could be controlled so that they both contribute equal real and equal reactive power to the system. [3]

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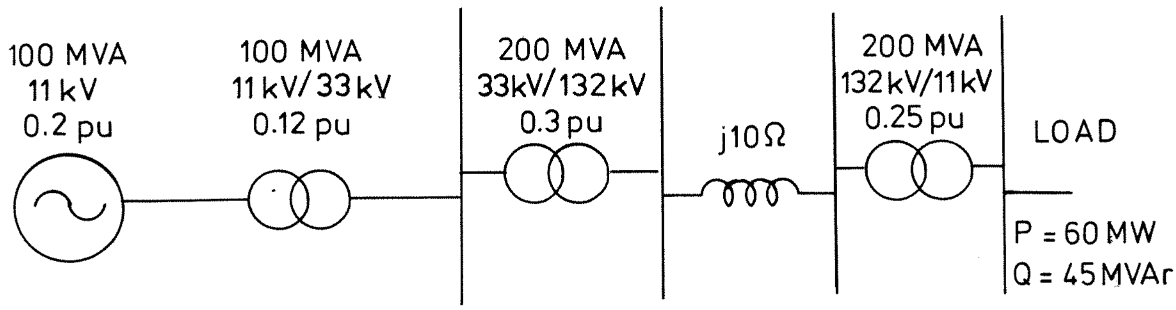

Fig. 4

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5 (a) Figure 5 shows the equivalent circuit for an induction motor in which the magnetising reactance and iron loss resistance are large enough to be ignored. Starting from the Electrical and Information Data Book expression for torque, show that the torque produced by such an induction motor is given by

$$T = \frac{3V^2 R_2'}{s\omega_s\left[(R_1 + R_2'/s)^2 + (X_1 + X_2')^2\right]}$$

[4]

(b) A 3-phase, 415 V, 50 Hz, 4 pole, star-connected slip-ring induction motor has the following per-phase equivalent circuit parameters:

$R_1 = 0.7\ \Omega$, $X_1 = 0.8\ \Omega$, $X_2' = 1.0\ \Omega$, $R_2' = 0.5\ \Omega$, R_0 and X_m both large enough to be ignored. Determine:

(i) The torque at zero speed.

(ii) The two speeds at which the motor can operate when used to drive a mechanical load which requires a constant torque of 160 Nm.

Using these results sketch the induction motor torque-speed curve. [8]

(c) Show that the efficiency of an induction motor represented by the equivalent circuit of Fig. 5 is given by

$$\eta = \frac{1-s}{1 + sR_1/R_2'}$$

and determine the induction motor efficiency at the two speeds found in (b) part (ii). Hence explain which speed is preferable. [5]

(d) Explain how the motor could be used to accelerate the 160 Nm load from standstill to the preferred speed found in part (c) above. [3]

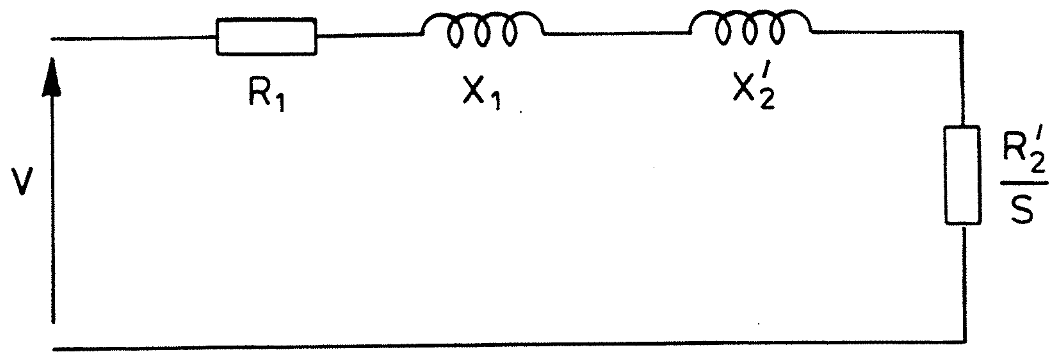

Fig. 5

SECTION C

Answer at least **one** *question from this section*

6 A serial data line is used to communicate digital signals between a micro-controller and a logic gate. Figure 6 shows a model suitable for representing the transition between logic '0' and '1' at the micro-controller output. In this model, the micro-controller output is represented by an ideal dc voltage source, V_{dc}, of value 5 V, in series with an output resistance R_0 of 200 Ω and a switch S. The data line is represented by a lossless transmission line of characteristic impedance Z_0, and the logic gate by an input resistance, R_{in}, of 1 kΩ. The length of the data line is 5 m, and its capacitance and inductance per unit length are 100 pFm^{-1} and 250 nHm^{-1} respectively.

(a) Determine Z_0 and the time, τ, taken for signals to propagate from one end of the line to the other. [4]

(b) Show from first principles that the voltage reflection coefficient of a transmission line of characteristic impedance Z_0 terminated by a load of impedance Z_L is given by

$$\rho_v = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Hence find its value at the two ends of the data line when switch S is closed. [5]

(c) A transition from logic '0' to logic '1' at the output of the micro-controller is modelled by the switch S closing. Following such a transition sketch a graph of the voltage across the input to the logic gate vs time for a period of 6τ, where τ is the propagation time found in part (a). [6]

(d) Derive an expression for the input voltage at the logic gate in terms of the number of reflections that have occurred. Hence determine the total communication delay if the input voltage to the logic gate must exceed a threshold value of 4 V to be recognised as a '1'. Find also the steady-state voltage at the logic gate. [5]

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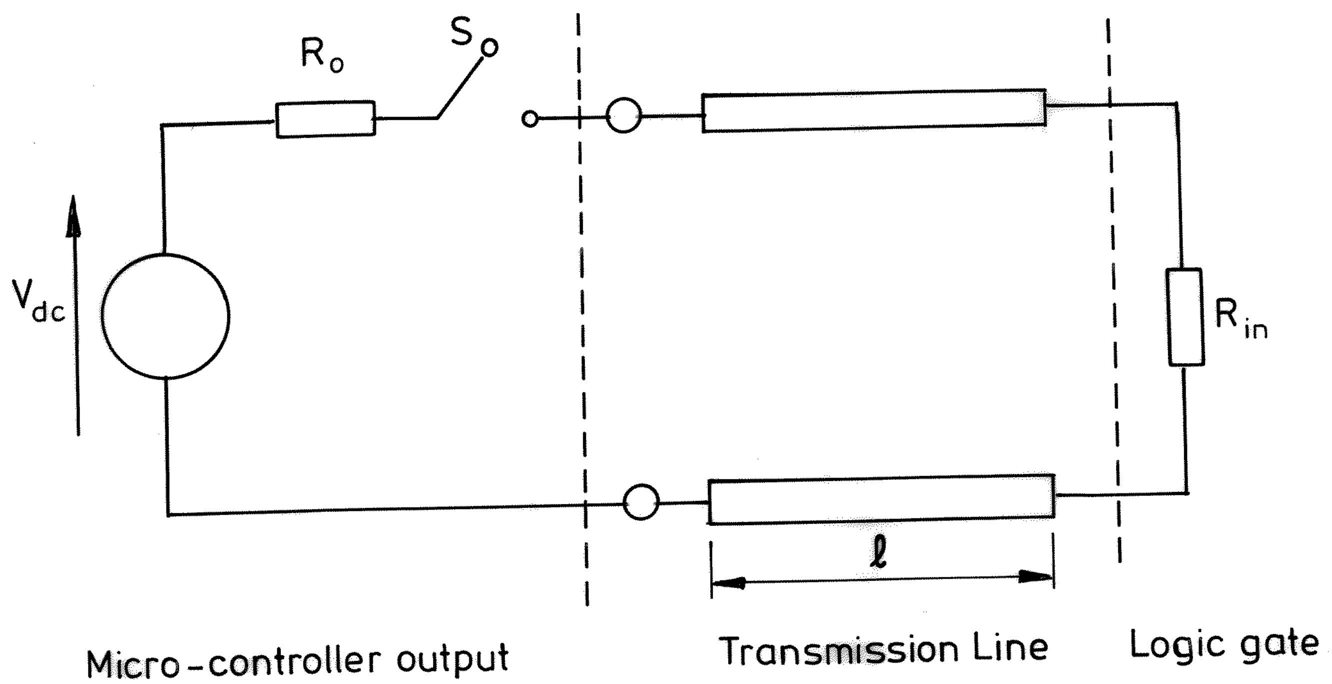

Fig. 6

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7 (a) Define and explain the physical significance of the term *radiation resistance* of an antenna. [3]

(b) The magnetic field intensity at a large distance r from a dipole antenna is given in spherical polar coordinates as

$$\tilde{\boldsymbol{H}} = \boldsymbol{e}_{\psi} \frac{\hat{I} l \beta}{4\pi r} \sin\theta \exp j(\omega t - \beta r)$$

where $\hat{I}$ is the peak antenna current and l is the dipole length. State the relationship between the magnetic field intensity vector, the electric field intensity vector and the Poynting vector for plane electromagnetic waves. Using these relationships, and by assuming that the electromagnetic wave at a large distance r from the dipole behaves as a plane wave, determine the electric field intensity vector. [5]

(c) Show that the power per unit area I_r radiating in the $\boldsymbol{e}_r$ direction is

$$I_r = \frac{\beta^2 \hat{I}^2 l^2 \eta_0 \sin^2\theta}{32\pi^2 r^2}$$

where η_0 is the impedance of free space. [4]

(d) Find the total power radiated, and show that the radiation resistance is

$$R_a = \frac{\omega^2 l^2 \eta_0}{6c^2 \pi}$$

where c is the velocity of electromagnetic waves in free space.
You may use the result $\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$ without proof. [5]

(e) Explain why high frequencies are desirable for efficient radiation from short antennae. [3]

END OF PAPER