ENGINEERING TRIPOS PART IB

Thursday 8 June 2000 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer at least ***one*** *question from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

(TURN OVER

SECTION A

*Answer at least **one** question from this section.*

1 (a) The function $p(t)$ is a pulse of unit magnitude, commencing at $t = 0$ and of duration τ seconds. By considering $p(t)$ as a combination of two step inputs, find its Laplace transform $\bar{p}(s)$. [4]

(b) Figure 1 shows a simple mixer vessel in which the contents are perfectly mixed at all times. Solvent flows in at a constant rate Q m^3 sec^{-1} . The solution leaves the vessel at the same rate so that the volume V m^3 of liquid in the vessel remains constant. A soluble solid is added at a varying rate $m(t)$ kg sec^{-1} and dissolves rapidly without any significant increase in the solution volume.

If $c(t)$ kg m^{-3} is the concentration of the dissolved solids in the vessel at time t , show that the differential equation relating $c(t)$ to $m(t)$ is

$$\frac{dc}{dt} + \frac{Qc}{V} = \frac{m}{V} \quad .$$ [4]

(c) If $m(t) = m_0\, p(t)$ from part (a) with $\tau = 10$ seconds and $m_0 = 0.01$ kg sec^{-1} , and if $c(0) = 0$, $V = 2$ m^3 and $Q = 0.02$ m^3 sec^{-1} , obtain an expression for the concentration $c(t)$. [8]

(d) Sketch $c(t)$ and calculate the concentration after 5 seconds, and also after 100 seconds. [4]

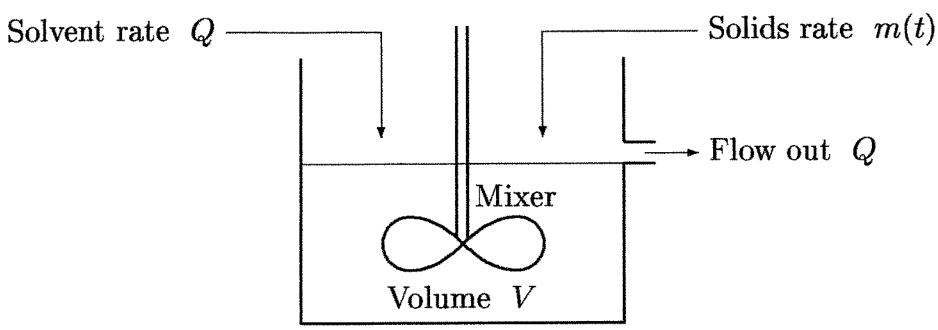

Fig. 1

2 Figure 2 represents a mechanical system to control the position $c(t)$ of an object, in response to a reference input $r(t)$. There is a disturbance $d(t)$ in the measurement in the velocity feedback loop. The Laplace transforms of these variables are $\bar{c}(s)$, $\bar{r}(s)$ and $\bar{d}(s)$ respectively.

(a) Suggest reasons why each of the two feedback loops would have been included in this design. [3]

(b) Establish the two transfer functions relating $\bar{c}(s)$ to $\bar{r}(s)$, and $\bar{c}(s)$ to $\bar{d}(s)$ in terms of K_c , $G_1(s)$, $G_2(s)$ and K_v . [6]

(c) The error of this system is defined as $\epsilon(t) = r(t) - c(t)$. Using the values given in Fig. 2, find the steady state error for the following three cases:

(i) a unit step input $r(t) = u(t)$;

(ii) a ramp input $r(t) = t\ u(t)$;

(iii) a step disturbance $d(t) = 0.1\ u(t)$. [8]

(d) Suggest how the controller transfer function K_c could be modified to remove the steady state error due to a step disturbance $d(t)$. [3]

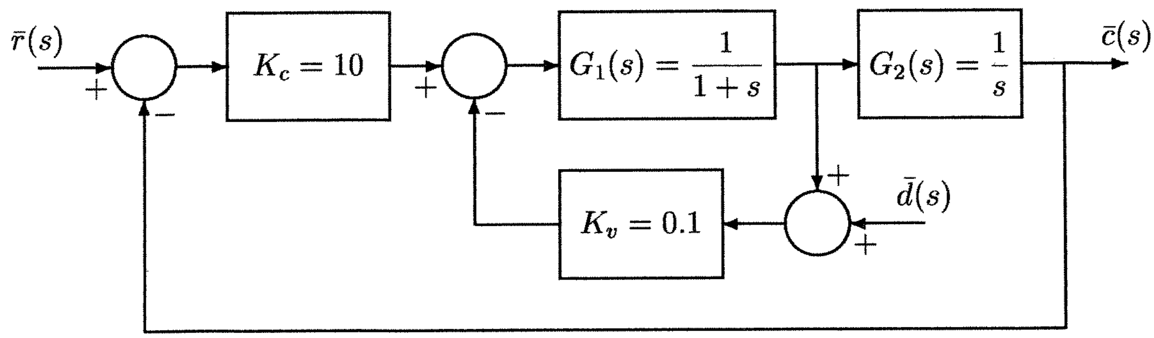

Fig. 2

3 A system has the transfer function

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

and controller $K(s)$, connected as shown in Fig. 3.

(a) Using the semi-log paper provided, sketch clearly the Bode plots for $G(s)$. (A few spot calculations will probably be required to obtain reasonable accuracy for the next part of this question.) [6]

(b) From your plots, estimate the gain and phase margins, if proportional control is used with $K(s) = 1$. [6]

(c) A phase compensator is used to replace the original controller, such that

$$K(s) = \frac{8(s+0.5)}{s+4} \quad .$$

Modify your Bode plots to take account of this controller and determine the new gain and phase margins, showing your constructions clearly. [8]

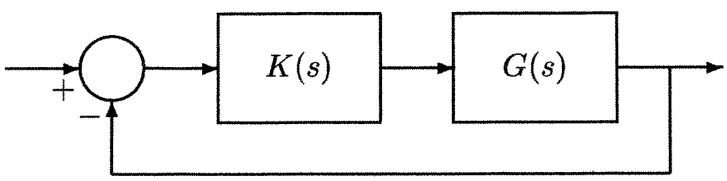

Fig. 3

4 In the simple control system shown in Fig. 4, the plant has a transfer function $G(s)$ and the controller transfer function is a pure positive gain K. The Laplace transforms of the reference input and the output are $\bar{r}(s)$ and $\bar{y}(s)$ respectively.

(a) Show that the closed-loop transfer function relating $\bar{y}(s)$ to $\bar{r}(s)$ is given by

$$H(s) = \frac{G(s)}{\frac{1}{K} + G(s)} \quad .$$

[3]

(b) If the plant transfer function is

$$G(s) = \frac{1}{s(s^2 + 3s + 4)} \quad ,$$

sketch the Nyquist plot for the plant over positive values of angular frequency ω, and show clearly how the magnitude of the closed-loop frequency response, relating the output to the reference input, may be measured graphically from the plot. [7]

(c) If the phase margin of the system is $45°$, calculate K, and estimate the peak magnitude of the closed-loop frequency response. [7]

(d) Briefly suggest why, for typical control systems, it is usually considered undesirable for the peak magnitude of the closed-loop frequency response to be significantly greater than unity. [3]

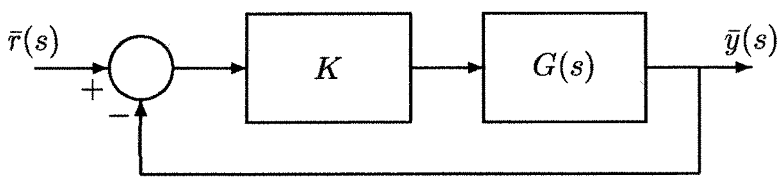

Fig. 4

SECTION B

*Answer at least **one** question from this section.*

5 (a) A telephone channel has a transfer function

$$H(s) = G_0 \left(\frac{sT_1}{1 + sT_1} \right) \left(\frac{1}{1 + sT_2} \right)$$

where G_0 is the approximate mid-band gain. If the channel pass band (between its -3dB frequencies) extends from 300 Hz to 3400 Hz, calculate the time constants T_1 and T_2 . What physical elements of the channel typically define the values of T_1 and T_2 ? [7]

(b) Calculate the step response of this channel, and hence sketch its response to a rectangular pulse input of amplitude 1 volt and duration 5 ms. [7]

(c) Why does this result show that 200 bit/s binary data signals should *not* be transmitted directly over this channel? Discuss what further processing would allow satisfactory transmission of such signals. [6]

6 (a) When a signal is digitised, the quantisation process introduces distortion. Explain why this distortion may usually be modelled as additive random noise; and sketch the probability density function of the noise, assuming an ideal uniform quantiser with small step size δV . [4]

(b) Show that the rms noise voltage introduced by such a quantiser is proportional to δV , and derive the constant of proportionality. [6]

(c) A periodic symmetrical triangular wave of zero mean and peak amplitude 5 volts is defined by $v(t) = 5 - (20|t|/T)$ for $-T/2 \leq t \leq T/2$, where T is the period of the wave. This wave is quantised by a 15-bit analogue-to-digital conversion system at a sampling rate of 100 kHz. The fundamental frequency of the triangular wave is approximately 1 kHz. Calculate the maximum signal-to-noise power ratio (in dB) that could be achieved with this system, assuming ideal anti-aliasing lowpass filters at input and output. [6]

(d) If the bandwidth of the output filter could be varied, comment on how this would affect the output signal wave-shape and the output noise level. Hence comment on the trade-offs which might then be made. [4]

Note:

For a zero-mean triangular wave, rms amplitude = (peak amplitude) $/\sqrt{3}$.

END OF PAPER