IB ANSWERS

ENGINEERING TRIPOS Part IB 2001

Paper 1 – MECHANICS

Q1 (b) Relative sliding velocity at A = 190 mms^{-1}

Relative sliding velocity at B = 550 mms^{-1}

Angular velocity of link ABC = 2.0 $rads^{-1}$, clockwise

Angular velocity of link OB = 7.4 $rads^{-1}$, clockwise

(c) Relative sliding acceleration at B = 2700 mms^{-2}

Angular acceleration of link ABC = 11.75 $rads^{-2}$, clockwise

Angular acceleration of link OB = 28.0 $rads^{-2}$, clockwise

Q2 (b) $\mathbf{v}_B = \dot{\theta}(R + a\sin\phi)\,\mathbf{e}_\theta$

$\mathbf{a}_B = \ddot{\theta}(R + a\sin\phi)\,\mathbf{e}_\theta - \dot{\theta}^2(R + a\sin\phi)\,\mathbf{e}_r$

(c) $\omega_{DISC} = \dot{\phi}\,\mathbf{e}_\theta + \dot{\theta}\,\mathbf{k}$

$\dot{\omega}_{DISC} = -\dot{\phi}\dot{\theta}\,\mathbf{e}_r + \ddot{\phi}\,\mathbf{e}_\theta + \ddot{\theta}\,\mathbf{k}$

(d) $\mathbf{v}_B = \dot{\phi}a\cos\phi\,\mathbf{e}_r + \dot{\theta}(R + a\sin\phi)\,\mathbf{e}_\theta - \dot{\phi}a\sin\phi\,\mathbf{k}$

$$\mathbf{a}_B = [-\dot{\theta}^2(R + a\sin\phi) + \ddot{\phi}a\cos\phi - \dot{\phi}^2 a\sin\phi]\,\mathbf{e}_r$$
$$+ [\ddot{\theta}(R + a\sin\phi) + 2\dot{\theta}\dot{\phi}a\cos\phi]\,\mathbf{e}_\theta$$
$$- [\ddot{\phi}a\sin\phi + \dot{\phi}^2 a\cos\phi]\,\mathbf{k}$$

Q3 (a) $\omega_{BC} = \dfrac{\omega}{2}$, anticlockwise

$\omega_{DC} = \omega$, anticlockwise

$v_E = \dfrac{l\omega}{2}\mathbf{i} + \dfrac{l\omega}{2}\mathbf{j}$

$a_E = -\dfrac{l\omega^2}{4}\mathbf{i} + \dfrac{3l\omega^2}{4}\mathbf{j}$

(b) $T = \dfrac{mgl}{2} + \dfrac{ml^2\omega^2}{4}$

(c) $T_i = \dfrac{mgl}{2} + \dfrac{5ml^2\omega^2}{12}$

(d) $T_{ai} = 2Q$

Q4 (b) $\beta = \frac{3g}{5l}$; tension, $T = \frac{\sqrt{2}}{5} mg$

(c) $S_B = \frac{-mg}{5}$; $S_c = M_B = M_c = 0$

(d) $F = \frac{mg}{10l}(x - \frac{3x^2}{2l} + \frac{l}{2})$; $M = \frac{mg}{20l}(-l^2 + lx + x^2 - \frac{x^3}{l})$

Q5 (b) ω_f= 171 rpm; Energy lost = 0.9J.

(c) No.

(d) ω_D = 343 rpm

Q6 (b) Relative to pulley A, B at 90° clockwise and C at 233° clockwise

(c) $F_Q = F_P =$ 76N

(d) Radius = 0.23m; relative to pulley A, mass at C added at 38° clockwise and mass at A added at 218° clockwise

2001 Part IB Paper 2

Answers

1 a) 23.8×10^9 N m^2

b) M_A = 19860 kNm, M_C = 8715 kNm, S_A = 824 kN, S_C = 662 kN,
T_A = 5000 kNm, T_C = 5000 kNm

c) 176 N/mm^2

d) 20.8 N/mm^2

e) 0.33 degs, 255 mm downwards

2 b) $5.66m$

c) $5.66m$

e) 49.5 kN

3 a) 33.2 kNm

b) $1.5M_p$

c) 24.9 kN

d) $1.5M_p$, same

e) No change

4 a) 3

b) (ii) $wL^2/3$

b).(iii) $wL^4/12EI$

5 b) 52×10^{-6} , -384×10^{-6} , 22.5 degs

c) 69.5 kNm

6 a) (i) 1 (ii) 1 (iii) 1

b) $t_I = -0.41F$, $t_{II} = +0.41F$, $t_{III} = +0.41F$

c) $t_{SP} = F/2$, $t_{PQ} = +1.71F$

Engineering Tripos Part 1B 2001

Paper 3, Materials

Answers

1. (a) 2.9h

2 (a) (i) Two-phase mixture 87% α, 13% β
α has composition 97%Pb, 3% Sn.
β precipitates are almost pure Sn.

(ii) 28% primary α, composition 19%Sn, 81%Pb
72% Eutectic mixture: 46% α and 54% β, in alternating plates.
α: same composition as the primary precipitates; β: composition 97.5%Sn, 2.5%Pb

(iii) At 610°C: 64% δ solid: single-phase solid solution containing 24%Cu, 76%Zn
36% Liquid solution, composition 13% Cu, 87%Zn

At 500°C: 100% single-phase ε : solid solution composition 80%Cu, 20%Zn.

4 (i) Increase all cylinder linear dimensions by 1.098.
(ii) α =0.145

5 (a) Merit index: Maximise $\sigma_F / \rho = M$
(b) Al_2O_3, GFRP uniply, GFRP laminates, high-strength steel, tungsten alloys (W alloys), lead alloys.

2001 Part IB Paper 4

Answers

Fluid Mechanics and Heat Transfer

1.b	(i) 256.5 ms^{-1}
	(iii) 0.982
	(iv) 7.25 *kJ/kgK*, 8.20 *kJ/kgK*
2.b	(i) 243.9°C
	(ii) 8.58 *kg/s*
	(iii) 556.5°C
	(iv) 7.87 *kJ/Ks*
2.c	710 m^2
3.b	0.34 *kg/s* , 8.97 *kW*
c	PER = 5.58 , PER (IDEAL) = 8.65
d	PER = 6.03
4.b	$u = \frac{-1}{2\mu}\frac{dp}{dz}\left(Wx - x^2\right) - V\frac{x}{W}$
c	$u = \frac{V}{W^2}\left(8Wx - 9x^2\right)$
5.c	0.1°C
6.a	$\dot{Q} = \lambda \Delta \mathrm{T}\, f\left(Vd/\upsilon, \upsilon/\alpha\right)$
c	$\dot{Q} \sim \alpha^{-0.4}$

Cambridge University Engineering Department 2001 **Part IB Paper 5** ANSWERS

1. a) approximate that $I_C = I_E$; collector current = 1.33 mA
   b) emitter current = 1.23 mA
   c) large h_{fe} gives the maximum gain = $-R_C / R_E$ = -20
   d) add a capacitor to the circuit to bypass R_E
2. b) CMRR = differential mode gain / common mode gain
   c) $R_3 = R_4$; common mode gain = $-g_m R_3 / (2 g_m R_s + 1)$;
   differential mode gain = $-g_m R_3$; CMRR = $2 g_m R_s + 1$
   d) differential gain = -14; CMRR = 210 (to two significant figures)
   e) take either V_3 or V_4 as the output; CMRR = 106
3. a) line voltage = 1.732 x phase voltage for a star connected load
   b) line current per phase = 1.09 kA
   c) maximum length = 140 km (to two significant figures) and line voltage = 55 kV
   d) add capacitance to load to bring the load factor closer to unity;
   use transformer to step up the ac voltage over the long distance
4. a) adjust the field winding current to adjust the reactive power
   b) (line current = 12.5 kA) excitation voltage = 15.2 kV;
   load angle = 17^o (to two significant figures)
   c) 90^o corresponds to maximum torque which is 5.5 MNm
5. a) at B the slip is zero and the motor produces no torque
   c) (i) 0.02; (ii) 122 + j 32 (according to the approximation chosen);
   (iii) 5.7 A; (iv) 44 Nm; the machine is behaving as a generator
6. b) maximum length = 16 mm c) (i) the first wave is a -1 V step and
   the voltage at the load drops from 3 V to 2 V;
   (ii) after multiple reflections the voltage decreases to approximately zero
7. a) speed of propagation is $1 / (\varepsilon_o \mu_o)^{0.5}$ c) 2 pW

PART IB - Paper 6, June 2001, Answers:

1. (b) (i) 0.25 (ii) 0.767sin(t+0.0768) (iii) 1.25+0.767sin(t+π/4+0.0768)
2. (a) Phase margin is approximately 8 degrees, hence system is oscillatory. (b) New PM is approximately 22 degrees.
3. (c)(i) GM=2.8, PM=27 degrees. (ii)0.22 (iii) 2.6 (iv) w>0.34 rad/s.
4. (c) Amplitude of y = $2.01.10^{-4} A$, amplitude of z is 1.002A, amplitude of x=1.002A.
5. (c) (i) 4.9km (ii) 0.344 km.
6. (c) passband frequency range = twice the bandwidth of x. $a_0 = 2ba$, $a_0 m_A = 2a$, hence $m_A = 1/b$

SG/July 2001

Part IB 2001

Paper 7: Mathematical Methods

Answers

1. (c) $\rho_e = \rho_0 \exp(-t/\tau)$, $\tau = \epsilon/\sigma$.

2. (b) (i) $2\mathbf{k}$, 0, (iii) $2\pi a^2$, 0.

3. (a) $\omega_n = cn\pi/L$.
   (b) B_n are the coefficients of a Fourier sine series for $f(x)$.

4. (a) (ii) $O(h)$.
   (b) (i) 42.82, (ii) 51.7 (predicted), 50.4 (actual), difference due to $O(h^3)$ terms.

5. (a) $x_1 = 0.9956$, $x_2 = 1.001$.
   (b) $x_1 = 1.000$, $x_2 = 1.000$.
   (c) Reorder the equations to make the coefficient matrix diagonally dominant.

6. (c) $4T/3$.
   (d) $c_n = X(n\omega_0)/2T$.

7. (a) (i) 0 (0 Hz), 2 (1 Hz), 0 (2 Hz), 2 (3 Hz).
   (b) (ii) 0.

8. (b) (i) 0.0547, (ii) No.
   (c) (i) Central limit theorem, (ii) Under the null hypothesis, 97.3% of samples would have a lower mean, so the observation does support the examiner's claim.
   (d) From the test in part (c).

Paper 8

Q1 (a) 5.4 kPa, 10 kPa, 111.6 kPa, 167 kPa and 219 kPa at surface, 1m, 9m, 9m (clay) and 12 m below ground level.
(b) S = 123.5 kPa

Q2 (b) Stability ratio = 2 (<5) so open faced tunnelling can be used.

Q3 (a) H_A = 420 kN ; H_B = 665 kN
(b) Max BM = 961.2 kNm @ 3.881 m from the top of the wall
(c) d = 400 mm and A_s = 8004 mm^2

Q4 (b) 1.95 kg 1.22 kNm2
(c) 37 mm; no

Q5 (b) 889 Nm2
(c) 12.5 Hz 3.72 kN/m
(d) 124 N/m

Q7 (b) 51.15 kPa 285.8 K
(c) 370 m/s
(d) 1037 K 258.6 kPa
(e) 630 K 36.9 kPa
(f) 452 m/s

Q8 (c) 43.66 deg -26.25 deg
(d) 517.6 kJ/kg 1337.5 K
(e) 266.3 kPa
(f) 1310 K 248 kPa 0.36 m^2

Q9 (a) 216.7 K 19.3 kPa 250.8 m/s
(b) 3.31 kg/s
(c) 1220.5 K 179.2 kPa 639.3 m/s 94.7 kPa
(d) 3.32 kN
(e) 10.2 kN

Q10 d) Area = 0.0063m^2, Time taken = 17.4 hours

Q11 a) $V_g = Neh^2/[2\varepsilon]$ b) 5×10^{20} donor/m^3 c) $L^2 = \mu V_d t$, L = 2μm
d) R = 625Ω.m e) 160μF/m^2

Q12 c) U = 2.85×10^{-3}eV d) $V_h = 5.77\times10^4$m/s e) L = 1.5μm f) t = 15psec
g) J = 3.2×10^7 A/m^2