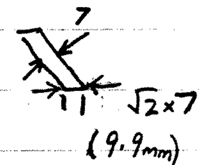
Question 1

(a) $A_{section} = 5 \times 0.010 \times 2 + \sqrt{2} \times 0.007 \times 4$

$= 0.1 + 0.04$

$= \underline{0.14\ m^2}$

$\bar{y} = 1.0m$ (Symmetric about axis XX)



$I_{xx} = \Sigma(I_{old} + \text{Area} \times \text{Shift}^2)$

Here we can neglect the contribution of I_{old} for the two flanges.

$\simeq \underbrace{\left[0 + (5 \times 0.010) \times 1^2\right] \times 2}_{\text{Top \& bottom flanges}} + \underbrace{\left(\frac{\sqrt{2} \times 0.007 \times 2^3}{12} \times 2\right)}_{\text{LH \& RH webs}}$

$= 0.1 + 0.013$

$= \underline{0.113\ m^4}$ $(1.13 \times 10^{11}\ mm^4)$

$E = 210\ GPa$ (Databook)

$EI = 210 \times 10^9 \times 0.113 = \underline{23.8 \times 10^9\ Nm^2}$

---

c.f. Rigorous calculation of Ixx

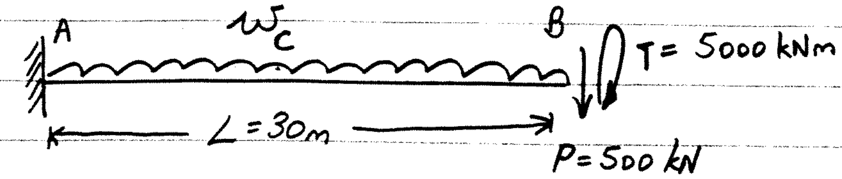
$I_{xx} = \left(\frac{5 \times 0.010^3}{12} + 5 \times 0.010 \times 1^2\right) \times 2 + \left[\frac{\sqrt{2} \times 0.007 \times 1^3}{12} + 1 \times (\sqrt{2} \times 0.007) \times 0.5^2\right] 4$

$= (\underbrace{\cancel{4.16 \times 10^{-7}}}_{\text{Negligible}} + 0.05) \times 2 + 0.0132$

(A)

[3] $= 0.1 + 0.013 = 0.113$ as above.

1 (b) $\rho_{steel} = 7840 \text{ kg/m}^3$ (Databook.)



$$w_{SW} = \rho_{steel} \cdot g \cdot A_{section} = 7840 \times 9.81 \times 0.14 = 10.8 \times 10^3 \text{ N/m}$$

(10.8 kN/m)

Moment

At A: $M_{A\,(SW)} = w \cdot L \cdot \frac{L}{2} = 10.8 \times 30 \times 15 = 4860 \text{ kNm}.$

$M_{A\,(P)} = P.L = 500 \times 30 = \underline{15000 \text{ kNm}}$

$\underline{\underline{M_A = 19860 \text{ kNm}}}$

Midspan

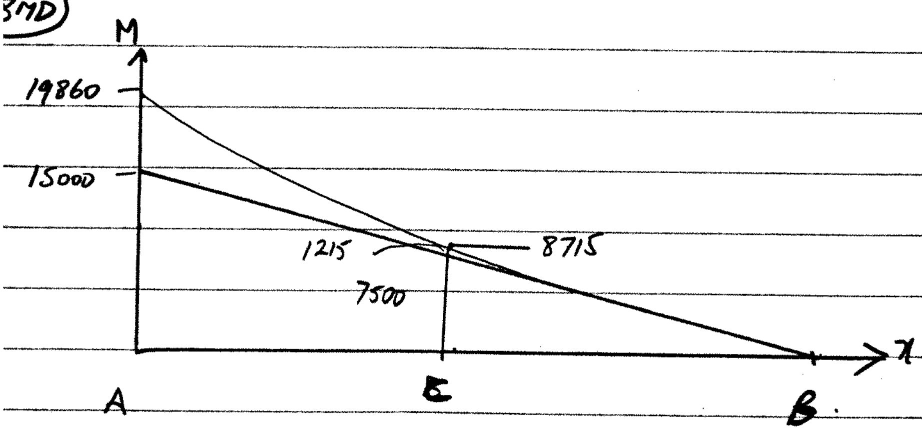
At C: $M_{C\,(SW)} = 10.8 \times 15 \times \frac{15}{2} = 1215 \text{ kNm}.$

$M_{C\,(P)} = 500 \times 15 = \underline{7500 \text{ kNm}}.$

$M_{C\,TOTAL}$ $\underline{\underline{8715}}$ kNm

$\boxed{+}$

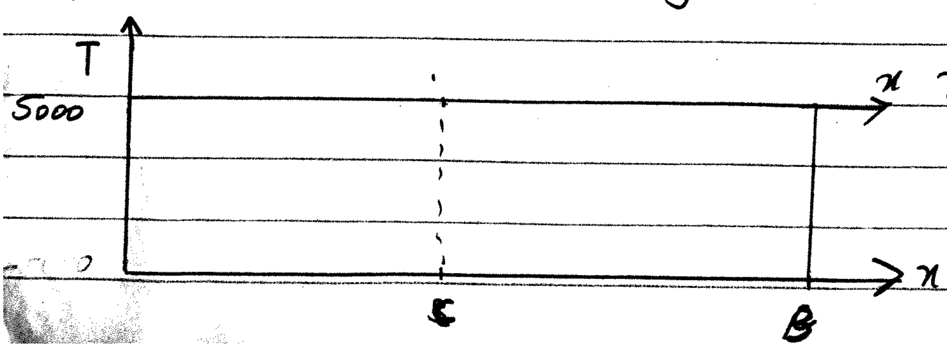
BMD



Torque $T = 5000$ kNm Constant along beam.

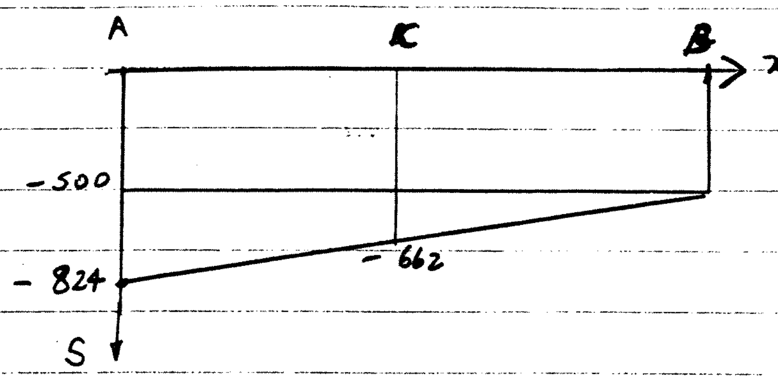
$T_A = +5000$ kNm

$T_C = +5000$ kNm



Shear

$$S_A = w \times L + P = 10.8 \times 30 + 500 = 824 \text{ kN}$$
$$S_C = w \times \frac{L}{2} + P = 10.8 \times 15 + 500 = 662 \text{ kN}.$$



NB -ve shear using this sign convention

1 (c) Max BM at A will be at the level of the top fibre in tension & bottom fibre in compression.

$$\sigma = \frac{My}{I} = \frac{19860 \times 1}{0.113} = 176 \times 10^3 \text{ kN/m}^2 \quad (176 \text{ MPa})$$

1 (d) Shear Stress at R

i) Due to applied shear force $\tau = \frac{VA\bar{y}}{I_{xx} b}$ $A = 0 \Rightarrow \tau = 0$. (Symmetry)

ii) Due to applied torque $\tau = \frac{T}{2A_e t}$ $\left(q = \frac{T}{2A_e}\right.$ in databook

$$= \frac{5000 \times 10^3}{2 \times 12 \times 0.010}$$
$$= 20.8 \times 10^6 \text{ N/m}^2 \quad (21 \text{ MPa})$$

1 (e) $T = G \cdot \dfrac{4A_e^2 \phi}{\oint \frac{ds}{t}}$ (Databook)

$G = 81 \times 10^9 \ N/m^2$ (Databook)

$A_e = 5 \times 2 + 1 \times 2 = 12 m$

$\therefore \phi = \dfrac{T}{G \cdot 4A_e^2} \oint \dfrac{ds}{t} = \dfrac{5000 \times 10^3 \times 1808}{81 \times 10^9 \times 4 \times 12^2}$

$= 1.938 \times 10^{-4}$ rads (A)

$\equiv 0.011$ degs.

$\left[\oint \dfrac{ds}{t} = \dfrac{5}{0.010} \times 2 + \dfrac{\sqrt{2}}{0.007} \times 4 = 1000 + 808 = 1808 \right]$

$= 30 \times 1.94E\text{-}4 = 0.006$ rads

$\therefore \theta = \phi L = 30 \times 0.011 = 0.33$ degs.

π rads = 180 degs.

$x = \dfrac{\text{Rads}}{\pi} \times 180$

$\therefore \delta_s = 3.5 \times 0.006 = 0.020$ (20 mm)

due to Torque.

$\delta_{s\,(P)} = \dfrac{PL^3}{3EI}$ (databook) $= \dfrac{500 \times 10^3 \times 30^3}{3 \times 23.8 \times 10^9} = 0.189 m$ (189 mm)

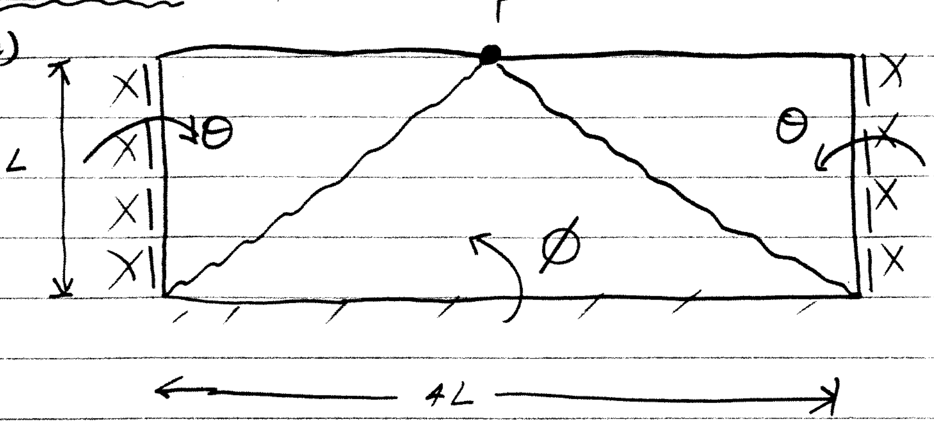
$\delta_{s\,(SW)} = \dfrac{wL^4}{8EI}$ (databook) $= \dfrac{10.8 \times 10^3 \times 30^4}{8 \times 23.8 \times 10^9} = 0.046 m$ (46 mm)

WL^3

$\delta_{TOTAL} = 20 + 189 + 46 = 255 mm$ downwards ; $\delta_{HORIZ} = 0$.

Question 2

2(a)



$\theta = \frac{\delta}{2L}$

$\phi = \frac{\delta}{L}$

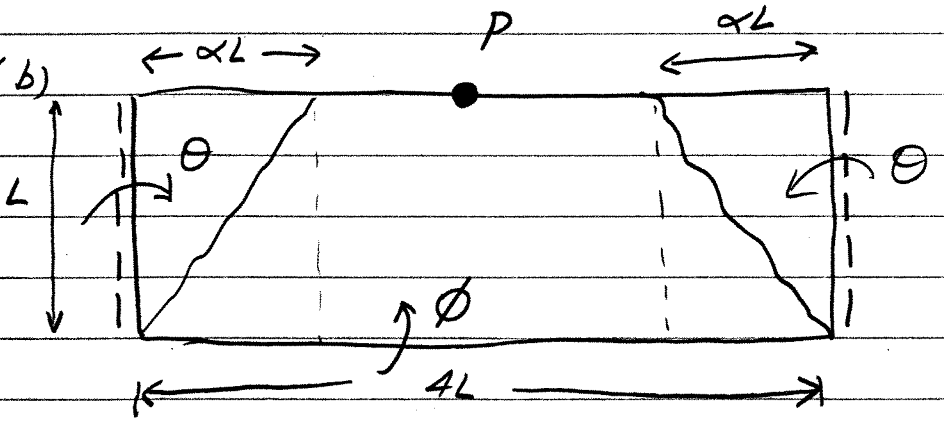
WD $= P\delta$

ED $= \underbrace{m.4L.\phi}_{\text{sagging y.l.}} + \underbrace{m.L.\theta.2 + m.L.\theta.2}_{\text{hogging y.l.}} = mL(4\phi + 4\theta)$

$= 4m\cancel{L}\left(\frac{\delta}{\cancel{L}} + \frac{\delta}{2\cancel{L}}\right) = 4m\delta\left(1 + \frac{1}{2}\right) = 6m\delta$

WD = ED $\quad P\delta = 6m\delta \Rightarrow P = 6m$ [4]

2(b)



$\theta = \frac{\delta}{\alpha L}$

$\phi = \frac{\delta}{L}$

WD $= P.\delta$

ED $= m.L.\theta.2 + m.\alpha L.\phi.2 + m.L.\theta.2 = 2mL(2\theta + \alpha\phi)$

$= 2m\cancel{L}\left(\frac{2\delta}{\alpha\cancel{L}} + \frac{\alpha\delta}{\cancel{L}}\right) = 2m\delta\left(\frac{2}{\alpha} + \alpha\right)$

WD = ED $\quad P.\cancel{\delta} = 2m\cancel{\delta}\left(\frac{2}{\alpha} + \alpha\right) \quad \therefore P = 2m\left(\frac{2}{\alpha} + \alpha\right)$

$\therefore \frac{dP}{d\alpha} = 2m\left(\frac{-2}{\alpha^2} + 1\right) = 0$ when $\alpha^2 = 2 \quad \alpha = \sqrt{2}$

$\therefore P = 2m\left(\frac{2}{\sqrt{2}} + \sqrt{2}\right) = 5{\cdot}66m \quad < P = 6m$ from (a)

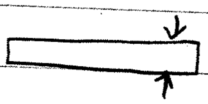
$\Rightarrow$ (b) is critical at $\alpha = \sqrt{2}$.

[6]

Q2 (cont.)

2(c) Best estimate of the collapse load is the least upper bound of the two results i.e. $P = 5.66m$ for mechanism in Fig 2(b) with $\alpha = \sqrt{2}$. [2]

2(d) Increasing BC would have no effect since there is no energy dissipation term associated with the rotation of the slab about support BC & hence the work equation would remain unchanged for the mechanism shown in Fig. 2(b). Decreasing the length BC down until $BC = 2\sqrt{2}L = 2.83L$ would also have no effect on the critical collapse load in fig. 2(b) for the same reasons. At lower values of BC the critical mechanism geometry changes & a new calculation would be needed to confirm the critical failure load. [5]

2(e)  $t = 10mm$ $\sigma_y = 350 N/mm^2$

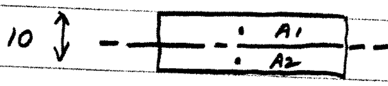
$$m = \frac{\sigma_y t^2}{4} = \frac{350 \times 10^3 \times 0.01^2}{4}$$

$$= 8.75 \, kNm/m$$

$$P = 5.66m = 5.66 \times 8.75 = 49.5 \, kN$$ [3]

Long hand derivation

$$Z_p = \Sigma A_i |y_i|$$



$$= 2(1 \times 0.005 \times 0.0025)$$

$$= 25 \times 10^{-6} \, m^3$$

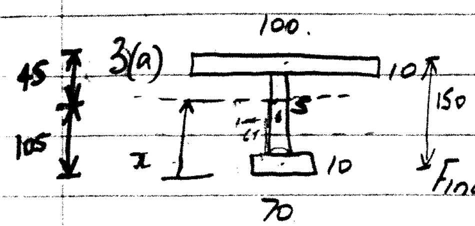
$$M_p = \sigma_y . Z_p = 350 \times 10^6 \times 25 \times 10^{-6}$$

$$= 8.75 \times 10^3 \, Nm$$

$$= 8.75 \, kNm.$$

$$P = 4\sqrt{2}.m = 5.66m = 5.66 \times 8.75 = 49.5 \, kN.$$

3(a)



$\sigma_y = 245\ N/mm^2$

Find centroid. $A = 100\times10 + 130\times5 + 70\times10 = 2350\,mm^2$

NA divides into 2 halves. $\therefore A_{\frac{1}{2}} = \frac{2350}{2} = 1175\,mm^2$

Assume x in web. $70\times10 + (x-10)5 = 1175$

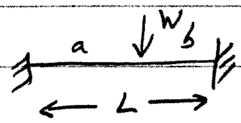
$x = \frac{1175-700}{5} + 10 = 105\,mm$ OK in web.

$Z_p = \Sigma A_i y_i$ (Data book.) $Z_p = 100\times10\times40 + 70\times10\times100 + 95\times5\times\frac{95}{2} + 35\times5\times\frac{35}{2}$

$= 40{,}000 + 70{,}000 + 22562.5 + 3062.5 = 135{,}625\,mm^3$

$M_p = Z_p.\sigma_y = 135{,}625\times245 = 33.23\times10^6\,Nmm \equiv 33.2\,kNm.$ [4]

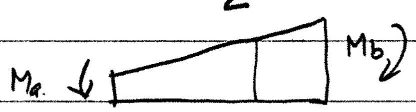
3(b)



$M_c = -\frac{Wb.a}{L}$ Particular Sol^n

$R_A.L = W.b \quad \therefore R_A = \frac{Wb}{L}$

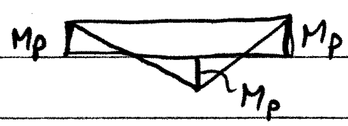
$R_B = \frac{Wa}{L}$



Self-stress

$M_A = \frac{Wb^2a}{L^2}$ $M_B = \frac{Wa^2b}{L^2}$

For optimum want $M_A = M_B = M_C = M_p$. (3 hinges needed for failure)



$M_c = -\frac{Wba}{L} = 2M_p$.

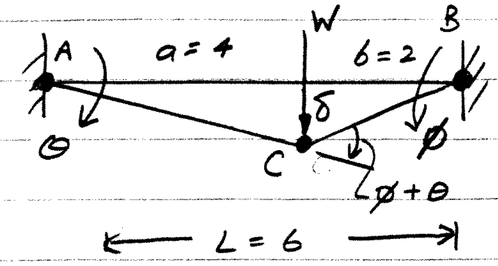
$\boxed{W_c = \frac{2M_pL}{ba}}$ $\left(= \frac{2\times6}{4\times2}M_p = \frac{12}{8}M_p = \frac{3}{2}M_p \right)$ [6]

3(c) $a = 4m$, $b = 2m$ $L = 6m$. $W_c = \frac{2M_pL}{ba} = \frac{\cancel{2}M_p\,6}{\cancel{2}.4} = \frac{3M_pL}{2} = \frac{3\times33.2}{2}$

$= 49.8\,kN$

$\therefore$ Safe Load for FoS = 2 is $W_{safe} = \frac{49.8}{2} = 24.9\,kN$. [2]

3 (d) Postulate a failure mechanism with hinges at A, C & B & load W displacing δ downwards.



$\theta = \frac{\delta}{a}$ $\phi = \frac{\delta}{b}$

WD = ED $W.\delta = M_p.\left[\theta + (\phi+\theta) + \phi\right]$

$$W.\cancel{\delta} = M_p\left[\frac{2\cancel{\delta}}{a} + \frac{2\cancel{\delta}}{b}\right]$$

$$W = M_p\frac{(2b+2a)}{ab} = 2M_p\frac{(a+b)}{ab} = \frac{2M_pL}{ab}$$ Same as lower bound.

$\therefore$ For a = 4, b = 2, L = 6 $W = \frac{2M_p 6}{4.2} = \frac{3M_p}{2} = 49.8\,kN$.

Thus the lower & upper bound solutions are identical showing that this solution is the exact solution for the collapse load of this beam.

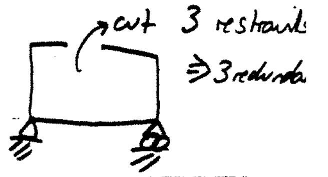
[5]

(e) The collapse load would remain unchanged at $W_c = 49.8\,kN = \frac{3M_p}{2}$
Displacement of A relative to B may cause plastic hinges to form at A or B or both, however until a third hinge at C forms, collapse cannot occur. The key assumption here is that there is sufficient ductility in the beam to allow rotations at constant M_p. This shows that initial or residual stress has no influence on the final collapse load of ductile beam structures.

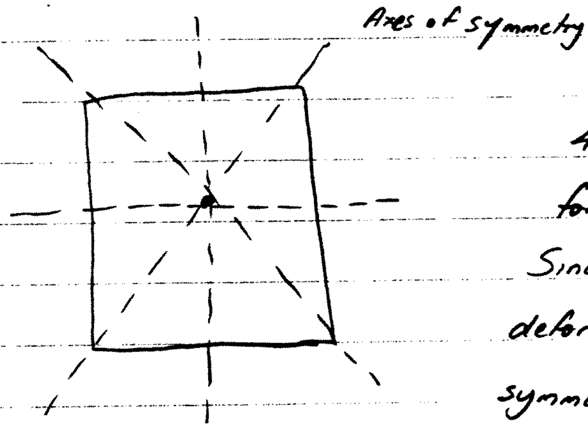
[3]

Q4

(a) 3 redundancies (closed loop) Check



4 (b) (i)



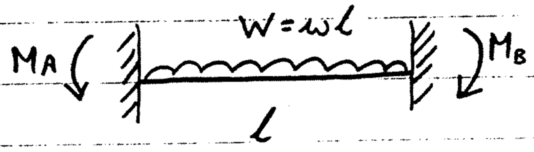
4 planes of mirror symmetry for structure & loading.

Since loading is symmetric then deformation response must also be symmetric.

Only possible symmetric response is for the corners to move either in or out along diagonals. However if the bars do not change length this movement is zero hence corners do not rotate.

4 (b)(ii) Consider databook case applied to one side of frame. From part (b)(i) there is not rotation at the corners hence ends of each bar in the frame will have no rotation at the corners. Can thus use databook case for fixed end beam with UDL loading.

i.e.

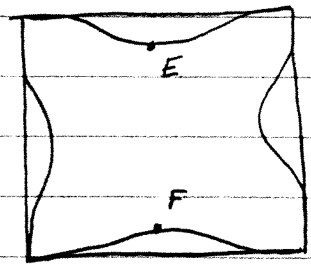


$M_A = M_B = \dfrac{Wl}{12}$

Here $l = 2L$ $\quad W = w.2L$ $\quad M = \dfrac{w.2L.2L}{12} = \dfrac{wL^2}{3}$

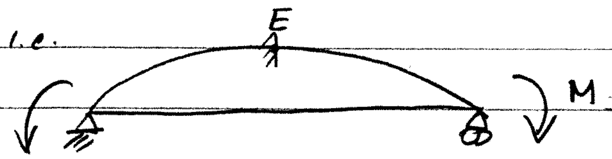
Q4 (cont.)

4(b) (iii)



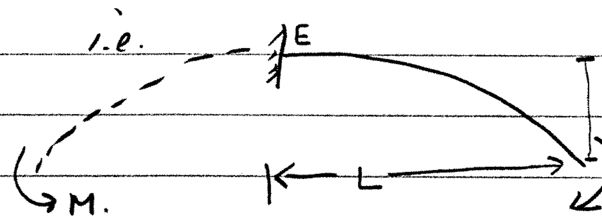
Central deflection is due to inward load w + outward moment M at corners.

i.e.



To get displacement consider slope at E is zero hence displacement can be modelled by fixed cantilever with M at tip.

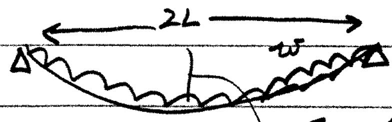
i.e.



$\delta_1 = \dfrac{ML^2}{2EI}$ from databook.
Upwards (−ve)

Also have deflection from UDL.

$$24 \times 16 = 144,\quad 144 \times \ldots \; 24 \quad 384$$



$$\delta_2 = \frac{5.(w.2L)(2L)^3}{384EI} = \frac{5wL^4}{24EI}$$

$\therefore$ Displacement at E is $\delta_1 + \delta_2 = \dfrac{-ML^2}{2EI} + \dfrac{5wL^4}{24EI}$

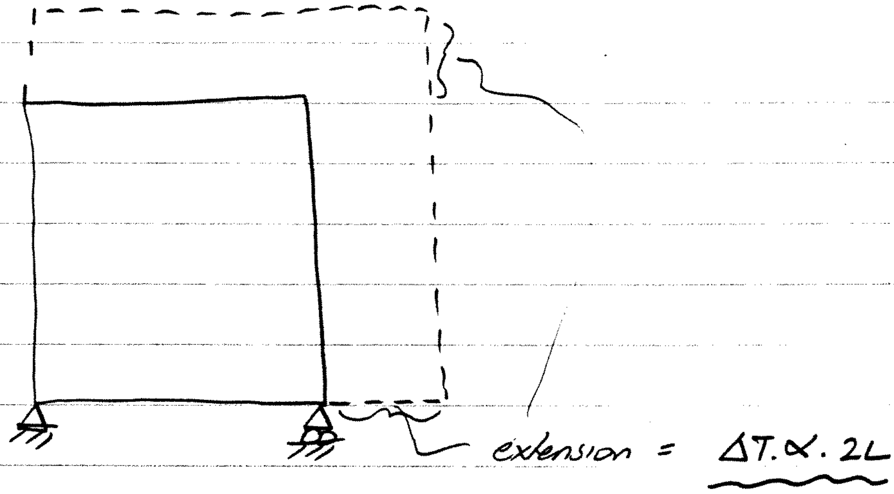
But from (ii) $M = \dfrac{wL^2}{3}$

$$\therefore \delta_E = \frac{-wL^4}{6EI} + \frac{5wL^4}{24EI} = \underline{\frac{wL^4}{24EI}}$$

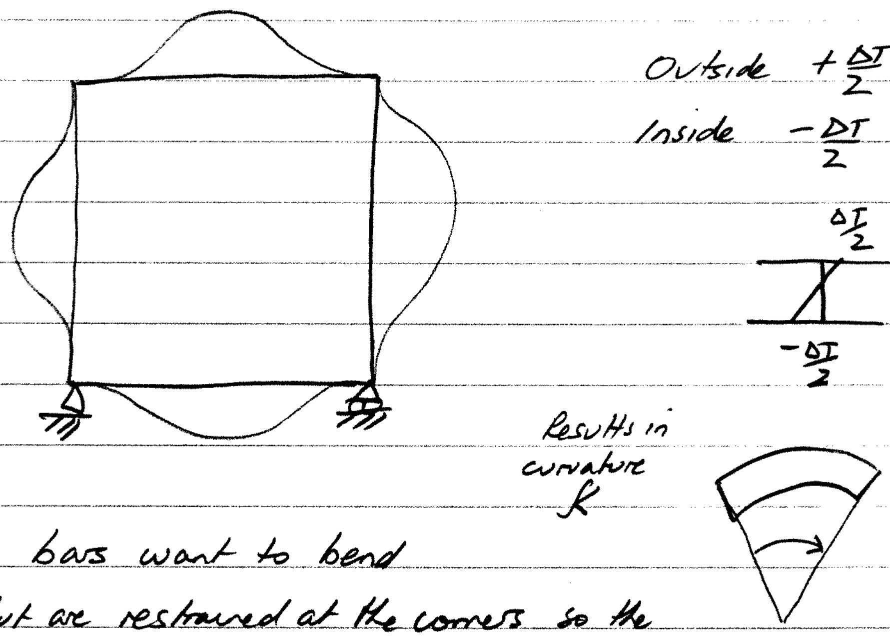
$\therefore$ Change in displacement EF $= 2 \times \delta_E = \underline{\dfrac{wL^4}{12EI}}$

Q4 (cont.)

4(c)(i)

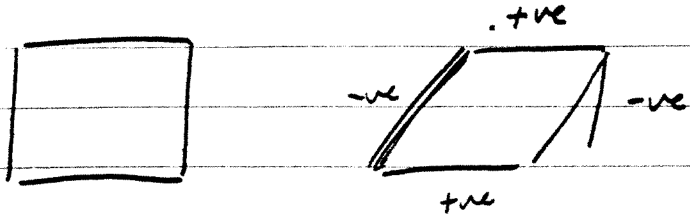


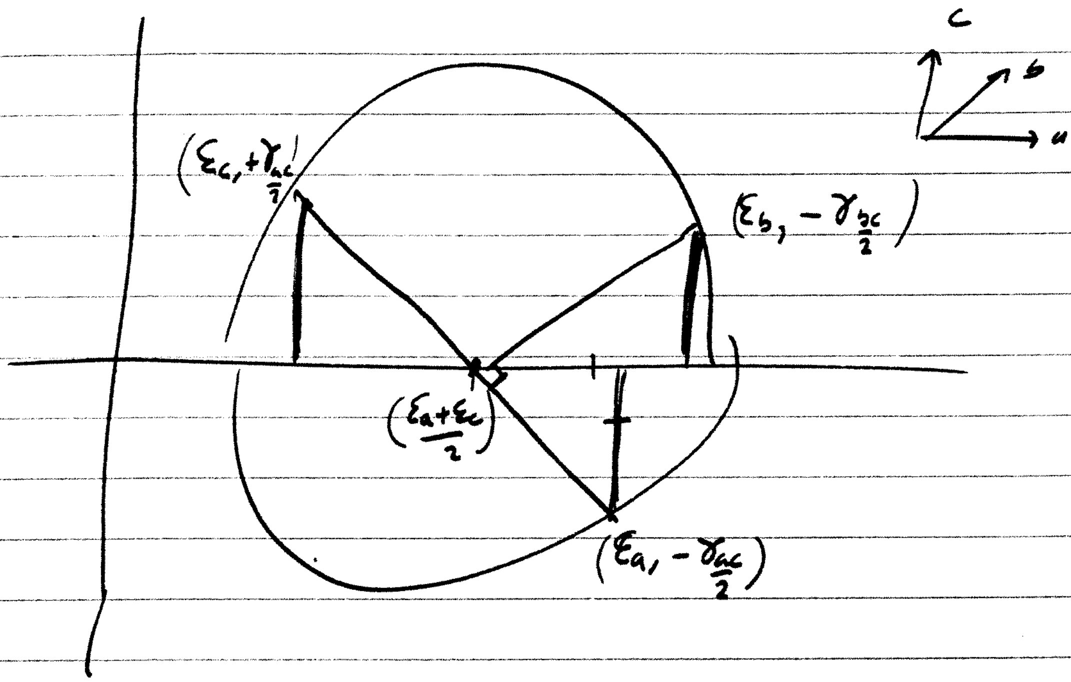
4c(ii)



As in (i) bars want to bend outwards but are restrained at the corners so the corners must not rotate. Hence the deformed shape will look as shown.

Q5 (a) Bookwork





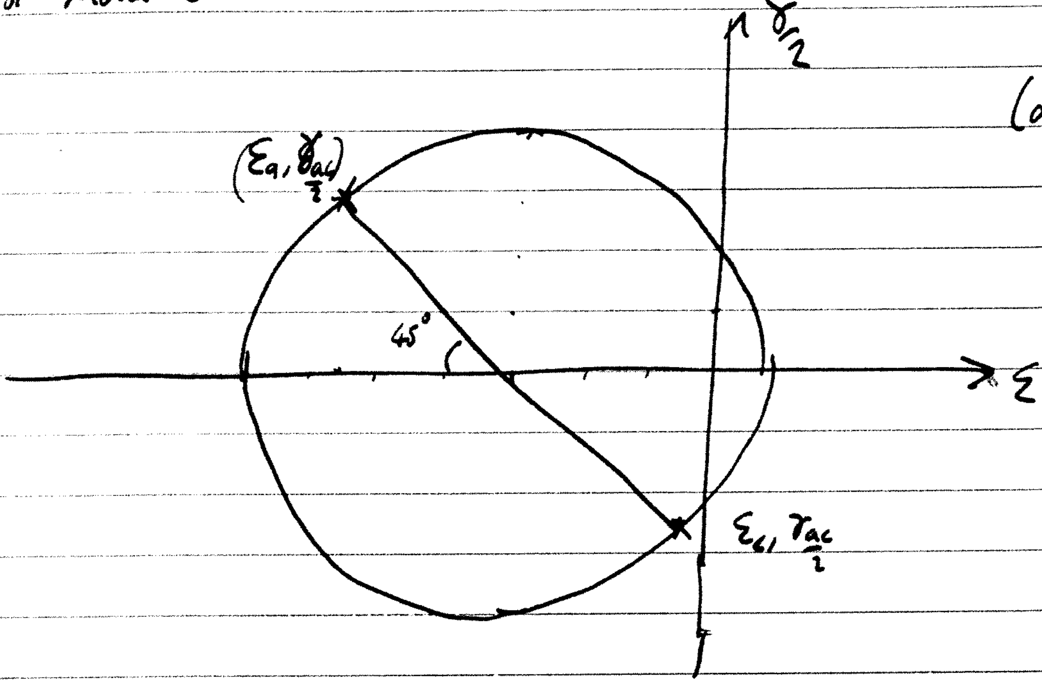
congruent Δ's
$$\frac{\gamma_{ac}}{2} = \varepsilon_b - \frac{\varepsilon_a - \varepsilon_c}{2}$$

$$\gamma_{ac} : 2\varepsilon_b - \varepsilon_a - \varepsilon_c \quad ✓$$

05

b $\quad 10^6 \times \gamma_{ac} = -640 + 320 + 12$
$\quad\quad\quad\quad\quad = -308$

Plot Mohr ○



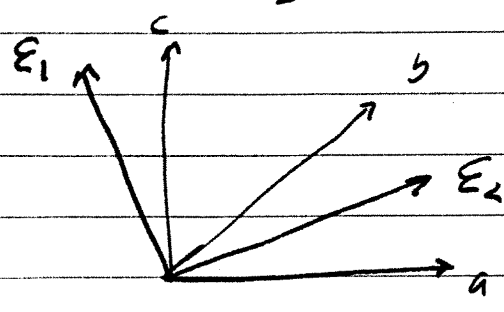
Centre at $\frac{\varepsilon_a + \varepsilon_c}{2} = -166 \times 10^{6}$

$$R^2 = (166-12)^2 + \frac{308}{2}^2 = 154^2 + 154^2$$

$$R = 218 \times 10^{-6}$$

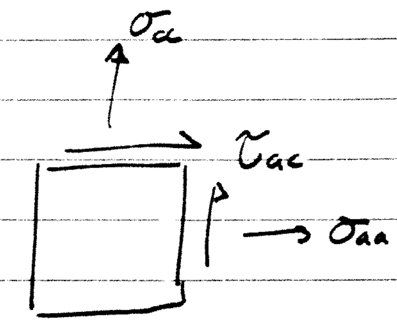
$\underline{\underline{\varepsilon_1 = 52 \times 10^{-6}}}$, $\underline{\underline{\varepsilon_2 = -384 \times 10^{-6}}}$ principal strains.

Principal strains at $\frac{45°}{2} = 22.5°$ to a, c axes



Q5

(c) Find stresses



$$\sigma_{aa} = \frac{E}{(1-\nu^2)}(\varepsilon_a + \nu\varepsilon_c)$$

$$= \frac{210\times10^9}{0.91}\left(-320\times10^{-6} + 0.3\times-12\times10^{-6}\right)$$

$$= \underline{-74.7\times10^6\ N/m^2}$$

$$\sigma_{cc} = \frac{210\times10^9}{0.91}\left(-12\times10^{-6} - 0.3\times320\times10^{-6}\right)$$

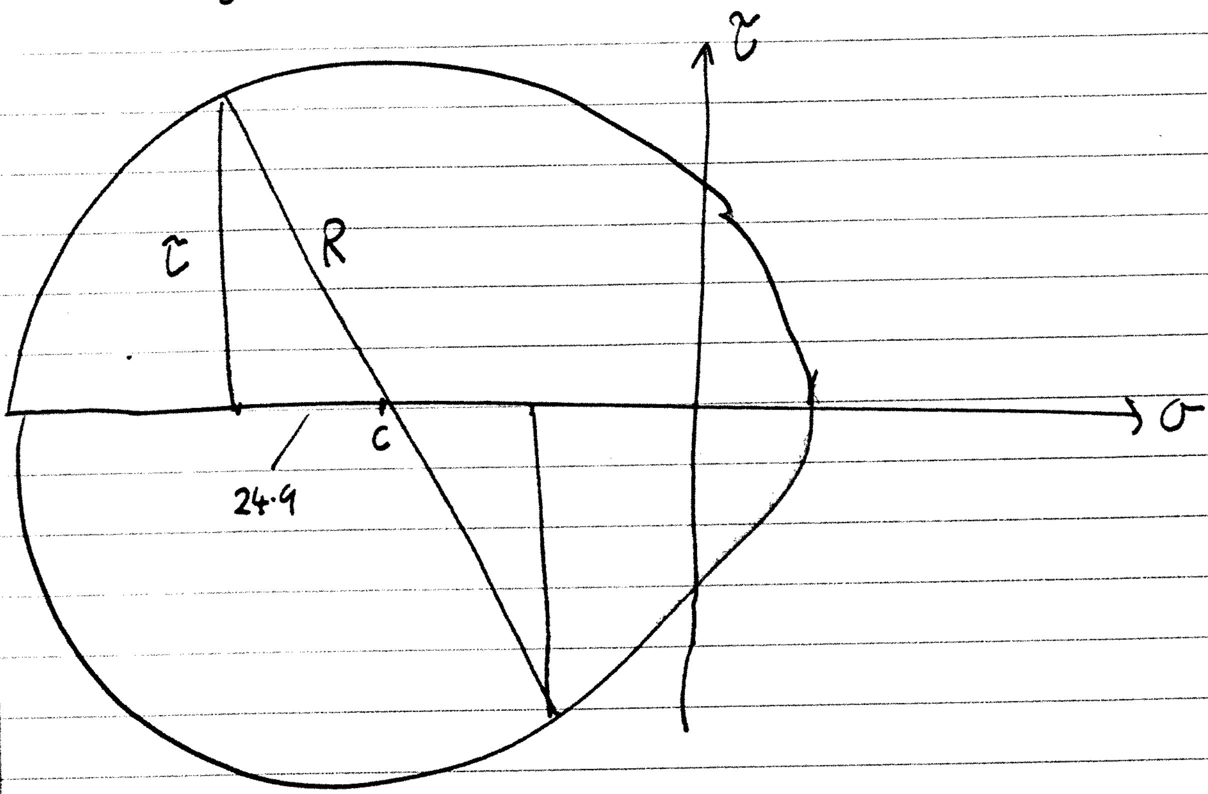
$$= \underline{-24.9\times10^6\ N/m^2}$$

unchanged by change in T

$$\tau_{ac} = G\gamma_{ac} = 81\times10^9 \times -308\times10^{-6} = -24.9\times10^6\ N/m^2.$$

Q5

Assume yield occurs in-plane



Centre of Mohrs O = $\frac{\sigma_{aa} + \sigma_{cc}}{2}$ = ~~-~~ $-49.8\ N/mm^2$

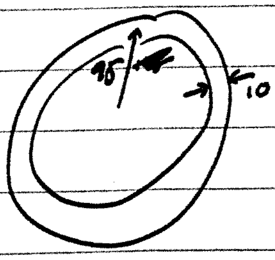
At yield, radius of Mohrs O = $\frac{Y}{2}$ = $125\ N/mm^2$

(assumption ok.)

~~τ =~~

$\tau^2 + 24.9^2 = 125^2$

$\tau = 122.5\ N/mm^2$.



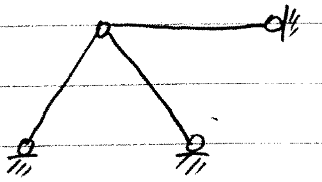
$T = \tau \times 2\pi \times 95 \times 10 \times 95$

$=$ ~~293 ×10⁶ Nmm~~ 69.5×10^6

$=$ ~~2932 kNm~~ 69.5 kNm

Question 6 Use Maxwell's equation $s - m = b + r - D_j$:

(a)(i)



$b = 3$

$D_j = 2 \times 4 = 8$

$r = 6$

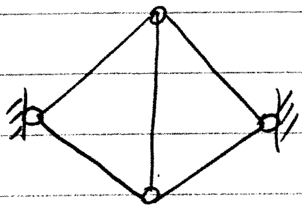
$m = 0.$

$\therefore s - m = b + r - D_j$

$s = 3 + 6 - 8 = 1$

$s = 1$

(ii)



$b = 5$

$D_j = 2 \times 4$

$r = 4$

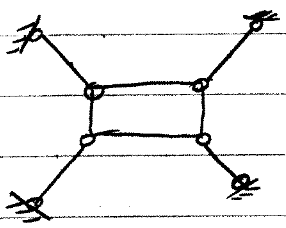
$m = 0$

$s - m = b + r - D_j$

$s - 0 = 5 + 4 - 8$

$s = 1$

(iii)



$b = 8$

$D_j = 2 \times 8 = 16$

$r = 8$

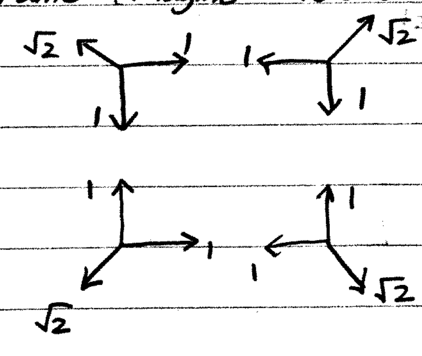
$s = m = ?$ Unknown

$s - m = b + r - D_j$

$s - m = 8 + 8 - 16 = 0$

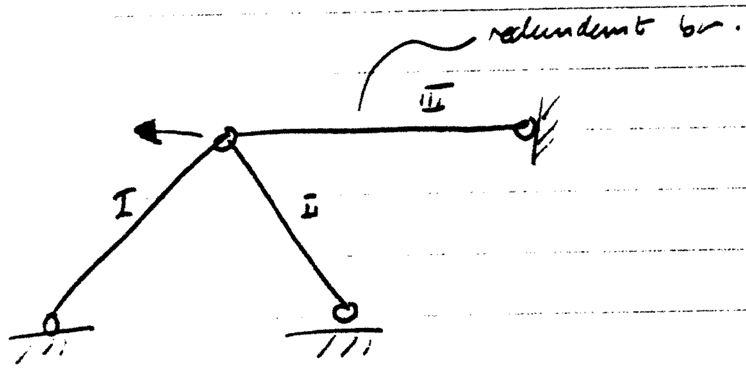
A state of self-stress is possible in this frame. (imagine turnbuckles in inner frame)

e.g.



$\therefore s = 1$ hence $m = 1$

6(b)



$$\underset{\sim}{t_0} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} F$$

State of self-stress

$$\underset{\sim}{s} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 1 \end{bmatrix}$$

find from compatibility.

General equilibrium soln. $\underset{\sim}{t} = \underset{\sim}{t_0} + x\underset{\sim}{s}$

flexibility $= \dfrac{L}{AE}$

$$\underset{\sim}{F} = \begin{bmatrix} \sqrt{2}L & & \\ & \sqrt{2}L & \\ & & 2L \end{bmatrix} \frac{1}{AE}$$

$$\underset{\sim}{e} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} F + x \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Use v.w. to find e, $\underset{\sim}{s} \cdot \underset{\sim}{e} = 0$

$$\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 0 \right) F + x \cdot \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 2 \right) = 0$$

$$-\sqrt{2} F + x (2 + \sqrt{2}) = 0$$

$$x = \frac{\sqrt{2} F}{(2 + \sqrt{2})}$$

$$\therefore \underset{\sim}{t} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} F + \begin{bmatrix} \left(\frac{1}{2+\sqrt{2}} \right) \\ -\frac{1}{(2+\sqrt{2})} \\ \frac{\sqrt{2}}{2+\sqrt{2}} \end{bmatrix} F$$

$t_I = -0.41 F$

$t_{II} = +0.41 F$

$t_{III} : 0.41 F$

10.27 Q6 (cont)

(c) $$\underset{\sim}{t} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} F + x \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ -F/2 \\ ? \end{bmatrix}$$

$$\therefore \quad \frac{F}{\sqrt{2}} - \frac{x}{\sqrt{2}} = -\frac{F}{2}$$

$$x = +\frac{F}{\sqrt{2}} + F$$

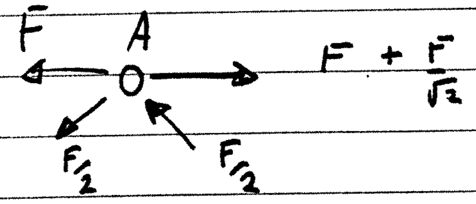
$$= \frac{(\sqrt{2}+1)}{\sqrt{2}} F$$

$$\therefore \quad t_I = -\frac{F\sqrt{2}}{2} + F\frac{(\sqrt{2}+1)}{2}$$

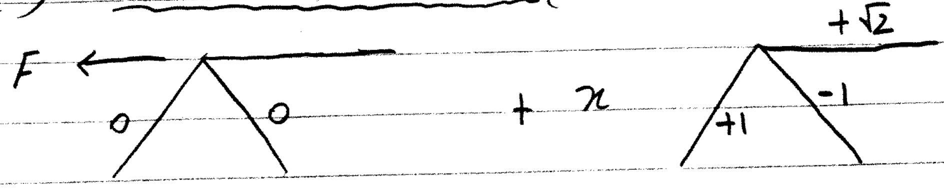
$$= \frac{F}{2}$$

$$t_{II} = \frac{(\sqrt{2}+1)}{\sqrt{2}} F$$

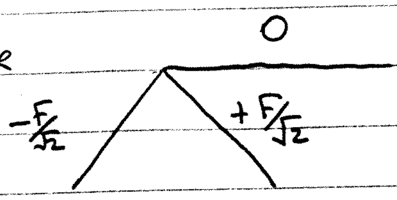
Check:



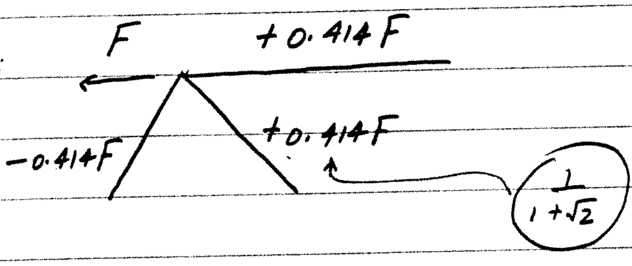
6 (b) Short version of answer. (alternative approach)



OR



Answer:



6 (c) Short version of answer (alternative approach)