1B Paper 3 – Materials – 2001

1. (a)
Steel is in carburising atmosphere, so carbon will dissolve in the steel surface until the steel is saturated. C_s is therefore found from the C-Fe phase diagram (databook fig 4.7) as the limit of carbon solubility in austenite at 1000°C, about 1.4 wt%.
D is related to D_0 by $D = D_0 \exp(-Q/RT)$.
Q is given; R from the databook is 8.31 J mol^{-1}.

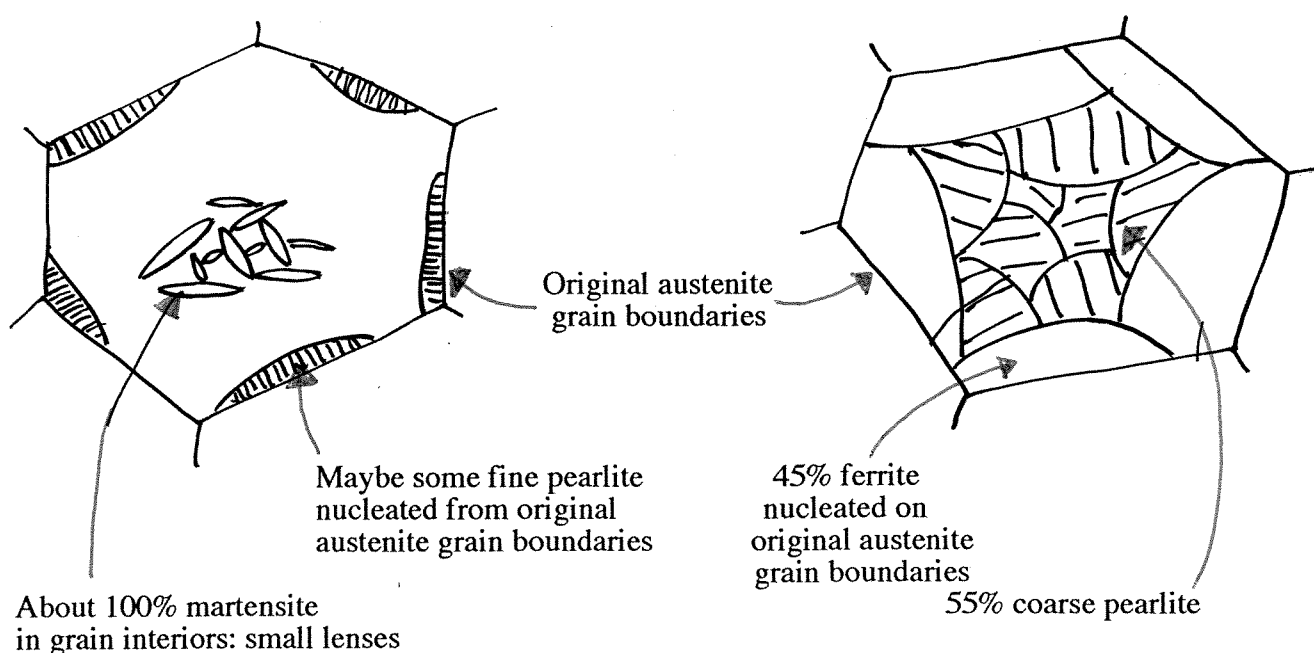$$D = 9.1 \times 10^{-6} \exp\left[-\frac{125 \times 10^3}{8.31 \times 1273}\right] = 6.72 \times 10^{-11} m^2 s^{-1}$$

So

$$erf\left[\frac{x}{2\sqrt{Dt}}\right] = 1 - \frac{0.45 - 0.2}{1.4 - 0.2} = 0.79$$

From chart, and remembering x=1.5mm
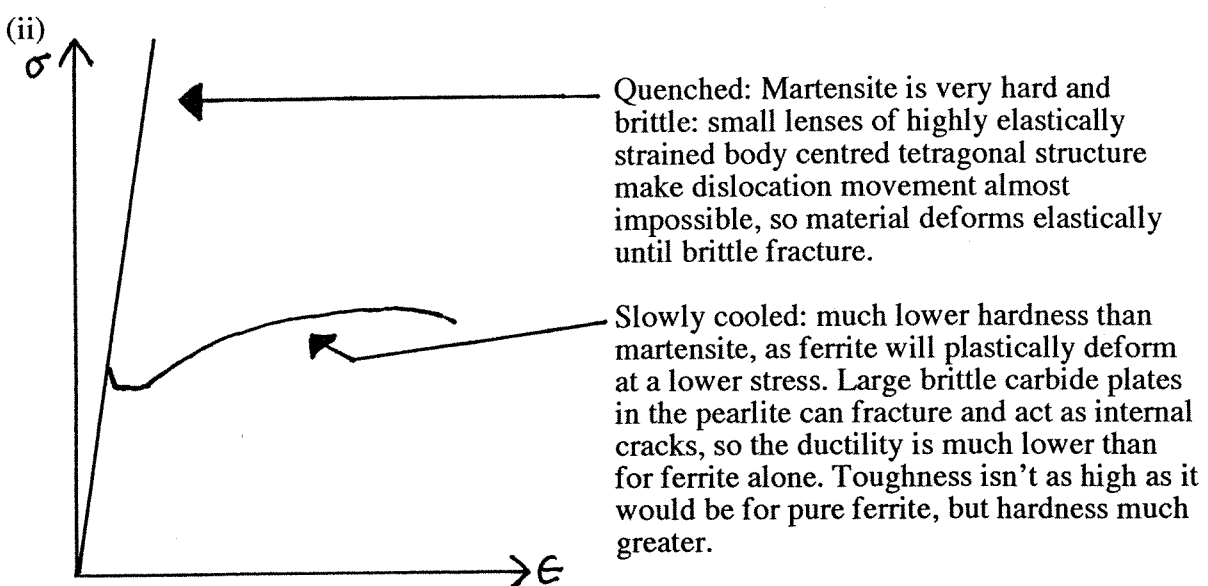
$$\frac{x}{2\sqrt{Dt}} = 0.895 \quad \Rightarrow \quad t = 1.04 \times 10^4 s = 2.9h$$

(b) Exact cooling rates are not given, so a range of answers are acceptable. Quenching (typically by dropping component into water) may give a completely martensitic structure, or there may be a small amount of pearlite which nucleates on the austenite boundaries as the cooling curve passes through the 'nose' on the TTT diagram. Air cooling may take minutes or up to an hour or so to cool to room temperature, depending on specimen size. Structure will contain equilibrium amount of ferrite (about 45%) and coarse pearlite (alternating plates of ferrite and cementite).

(i)

Original austenite grain boundaries

Maybe some fine pearlite nucleated from original austenite grain boundaries

45% ferrite nucleated on original austenite grain boundaries

About 100% martensite in grain interiors: small lenses

55% coarse pearlite

(ii)

Quenched: Martensite is very hard and brittle: small lenses of highly elastically strained body centred tetragonal structure make dislocation movement almost impossible, so material deforms elastically until brittle fracture.

Slowly cooled: much lower hardness than martensite, as ferrite will plastically deform at a lower stress. Large brittle carbide plates in the pearlite can fracture and act as internal cracks, so the ductility is much lower than for ferrite alone. Toughness isn't as high as it would be for pure ferrite, but hardness much greater.

(iii) Furnace cooling means turning off the furnace and letting the sample cool in the furnace to room temperature. Very slow cooling: may take 24h. The initial austenite grain size is of course the same as with the other specimens. The transformation microstructures will be in the same volume fractions as the air-cooled sample, but much coarser: ferrite grains fewer and bigger; pearlite plates will be thicker. Material will be softer (lower yield stress: less hardening from coarse carbide plates) and rather less ductile (cracks in carbide plates will be bigger), so less tough.

Examiner's comments:
Only a small proportion got the correct boundary condition for carbon concentration. Many mistakes of a factor 1000 in both Q and R were made in calculating Q/RT. The term 'air cooling' was a bit vague as to the precise cooling rate so a number of answers were acceptable, provided they contained some ferrite. A depressingly large majority said that the Young's modulus of the quenched material was much greater than in the air cooled. Only a minority could give the effect of furnace cooling which seemed a new term to them, although the meaning is pretty obvious.

2 (a) (i)
Two-phase mixture 87% α, 13% β

Matrix is a solid solution, α, with composition 97%Pb, 3% Sn.

β precipitates are almost pure Sn.

Precipitates may be on grain boundaries or within grains, or both.
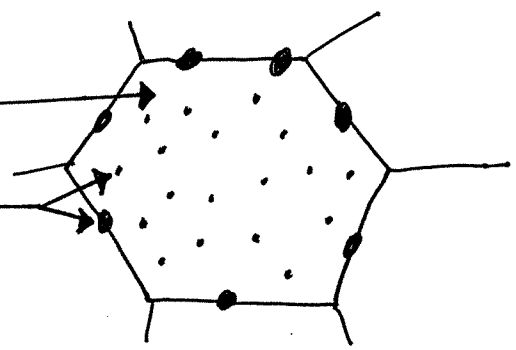
(ii) (All compositions given in weight%)
Two-phase mixture, but more important than the overall α and β proportions are the proportions of eutectic mixture and α .

Just above the eutectic temperature (in the α plus liquid phase field), there would have been 28% α and 72% liquid; α has the composition 19%Sn, 81%Pb: these are the 'primary' α precipitates. The liquid solidifies to the eutectic mixture with 46% α and 54% β , in alternating plates. α has the same composition as the primary precipitates; β plates have the composition 97.5%Sn, 2.5%Pb. Both α and β are solid solutions.
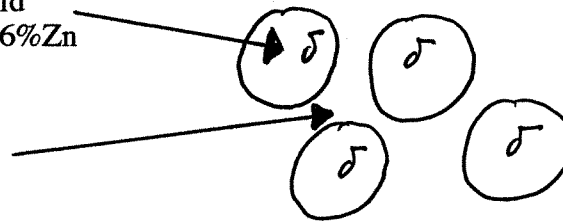
28% Primary precipitates: α containing 19%Sn
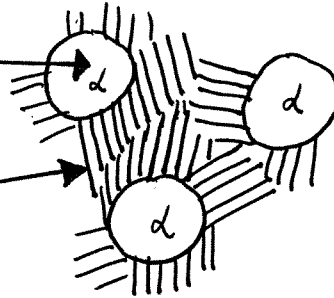
72% Eutectic mixture: alternating plates of α and β , compositions given above.

(iii)
At 610°C:
64% δ solid: single-phase solid
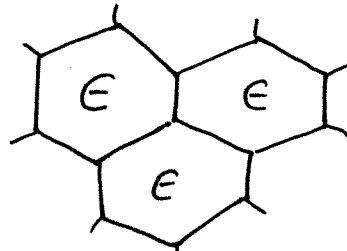solution containing 24%Cu, 76%Zn

36% Liquid solution,
composition 13% Cu, 87%Zn

At 500°C:
A peritectic reaction has taken place, δ+ liquid combine to form ϵ . We therefore have 100% single-phase ϵ , which is a solid solution composition 80%Cu, 20%Zn.

Single-phase grains of ϵ solid solution

(b) More rapid cooling:
Pb-15%Sn at 100°C: More nuclei form so finer grains of α; similarly, finer (and more) β precipitates. Volume fraction of β unchanged.
Cu-80%Zn at 500°C: Peritectic reaction requires diffusion of Zn into and Cu out of the δ precipitates, and diffusion needs to happen all the way through. Under anything other than very slow cooling, this won't take place completely. So two major changes will be seen under non-equilibrium cooling:
   – the δ precipitates will be smaller and more numerous (more nucleation, as above);
    – the peritectic reaction will have happened only on the outsides of the δ precipitates.
The remaining liquid will solidify to form ϵ solid solution of variable concentration (between 79% and 85%Zn).
Examiners comments:
The better candidates lost marks by not putting in everything asked for. Many candidates wasted time describing the microstructures at all temperatures down to the one asked for. More worrying was the large number of candidates who seem never to have heard of a eutectic and just applied the same rules to the 50% composition as to the 15% composition. Most answers on the rapid cooling just talked about supersaturated solutions.

3 (a) 2000 series Al alloys are heat-treatable, so they produce precipitates (of $CuAl_2$, θ) dispersed through the grains. Heat alloy into single-phase field to form α solid solution (about 550-600°C from phase diagram in databook fig. 4.3). Quench to form nuclei. Age at 120-250°C to grow precipitates.
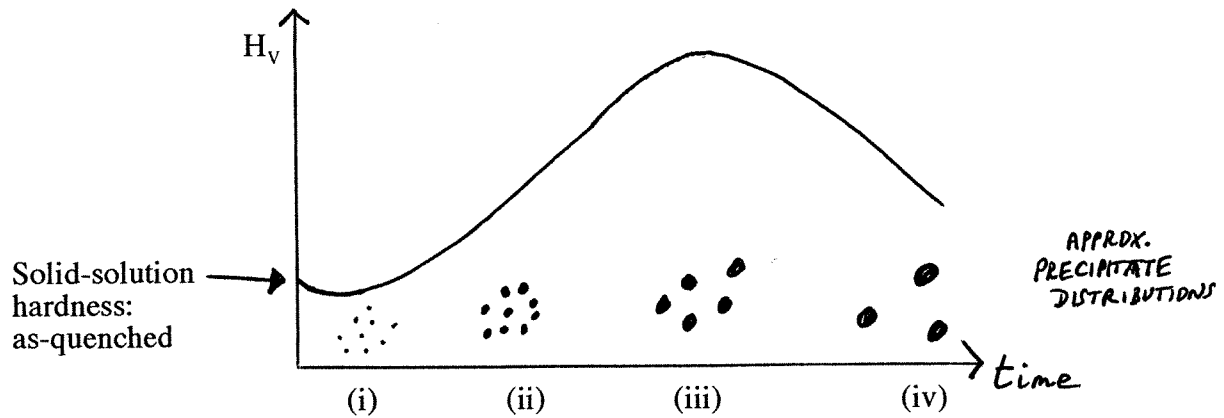
At (i) many nuclei are present; they all grow up to (ii). Precipitate radius increases; spacing decreases; volumes fraction increases.

From (ii) onwards, the volume fraction of precipitates in constant, so precipitates now grow competitively. Radius increases; spacing increases; number decreases, as smaller precipitates dissolve. Dislocations cut through precipitates.

(iii) Maximum hardness, 'peak aged'. Cutting and bowing equally difficult.

(iv) Alloy now 'over-aged'; dislocations bow round precipitates.

To obtain maximum hardness, age to point (iii). In fact, for safety, it is usual to age to just before this point (slightly 'under-aged').

Maximum ductility will be obtained in the soft (low hardness) solid solution, the as-quenched condition.

Once precipitates are present, particularly the larger ones, dislocation debris builds up as the material deforms (all those dislocation loops left behind when dislocations bow round precipitates) and the material work-hardens rapidly. The over-aged material initially has low hardness, but it is actually rather brittle because of this rapidly increasing dislocation density.

(b) The alloy is wrought rather than heat-treatable, so no precipitates form. Need to cold-deform and then recrystallise. The grain size is determined by the amount of cold-deformation and by the recrystallisation temperature.

More deformation means increased driving-force for recrystallisation, so number of nuclei increases
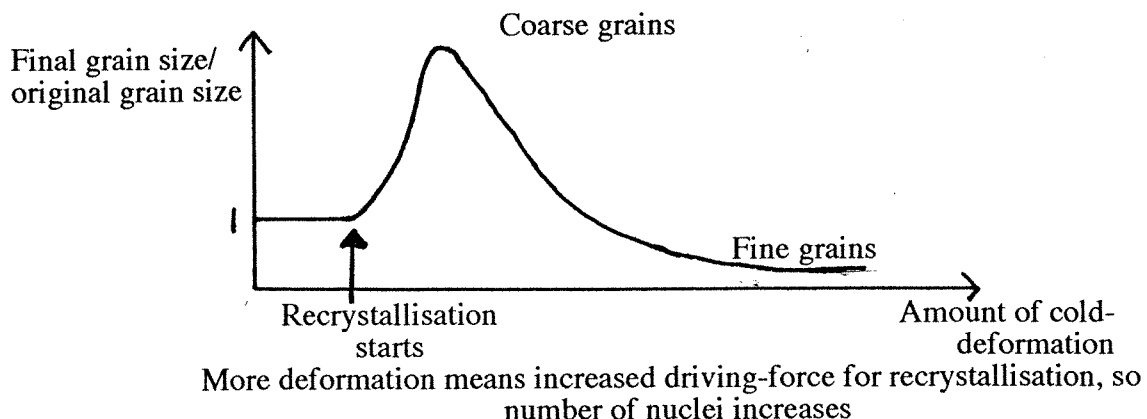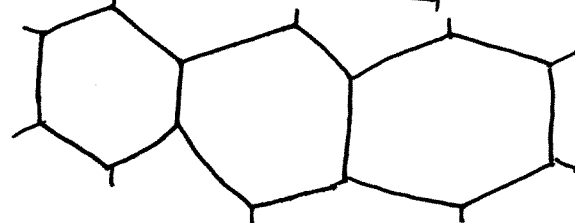
Rolled structure with
high dislocation density.
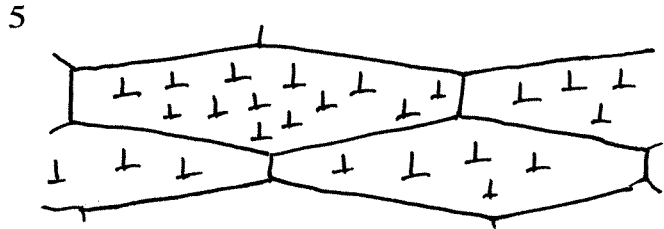
Heat to allow first recovery
(rearrangement of dislocations),
then to nucleate new grains
which are dislocation-free
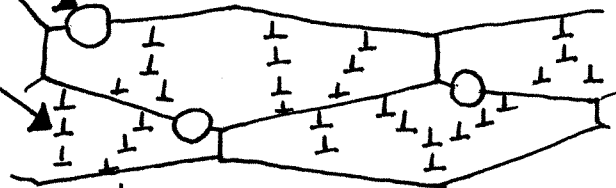
New grains grow
(recrystallisation) until
they run into each other.
More nuclei means smaller
grain size

Experiment: For a chosen recrystallisation temperature and time, vary the amount of cold-deformation. Metallurgical analysis to look for start of recrystallisation and so to plot the grain-size/strain curve shown in (a). If necessary, repeat both for different recrystallisation times and different temperatures.

Grain size important for: Mechanical properties (finer means harder but tougher); also surface finish (coarse-grained structures can give a rough surface, as all the grains deform in different ways).

Examiner's comments:
There were a lot of good answers to the first part. A number of candidates solution treated above the melting point. There was also considerable confusion between the behaviour of Al and Steel. Most candidates thought that quenching produced a hard brittle alloy. They also talked about 'tempering' the alloy.
*Part (b) was based closely on the practical experiment on recrystallisation. Most candidates had a reasonable shot at this one. Knowledge of the effect of grain size on mechanical properties was generally vague.*

4 (a) Ceramics have strong directional covalent bonds, or strong ionic bonds: they do not normally contain dislocations and the stress to move dislocations is very high. So they fail in a brittle way, and strength is determined by the propagation of pre-existing cracks and flaws. In tension the fracture stress is determined by fast fracture starting from the largest favourably-oriented flaw, and $\sigma_f = K_{IC}/\sqrt{\pi a}$ where interior flaws have length $2a$, surface flaws length a. In compression, failure is caused by the slow growth and linking of many cracks, so is determined by the mean crack length. Failure stress in compression is 10-15 times higher than tensile fracture stress.

ONE CRACK LEADS TO FAST FRACTURE

Tension

MANY CRACKS EXTEND PARALLEL TO STRESS AXIS

Compression
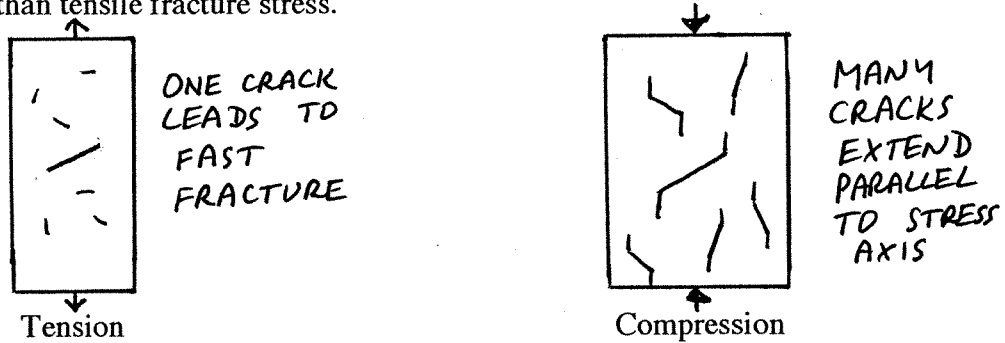
Flaw sizes are related to grain size. Cracks may be between grains (intergranular) or across grains (transgranular). Porosity (which is a residue of the manufacturing process, see below) is also influenced by grain size. So small grain size leads to higher strength by limiting the size of pre-existing cracks. In addition, grain boundaries themselves can act as a strengthening mechanism. Cracks propagating through the material may be stopped or deflected by grain boundaries, so more grain boundaries (smaller grain size) leads to higher strength.

Ceramics have very high melting-temperatures and so cannot be made by processes involving melting. They are normally made from powder particles, which are then bonded together. The powder particles are mixed with a binder (something to hold them together temporarily) e.g. water (for clays) or polymer (for many engineering ceramics) and formed into the required shape, typically by processes such as pressing or extruding. The 'green' compact can easily be machined. It is then heated to burn off the binder and to bond the ceramic particles together. Heating to around 0.7 of the melting temperature (sintering) allows the particles to bond together by diffusion (particle surfaces become grain boundaries). The sintering process can be accelerated by using pressure as well as heat, as in hot isostatic pressing. Alternatively, a low-melting point phase can be used to flow between the particles, 'gluing' them together.

PARTICLES

POROSITY
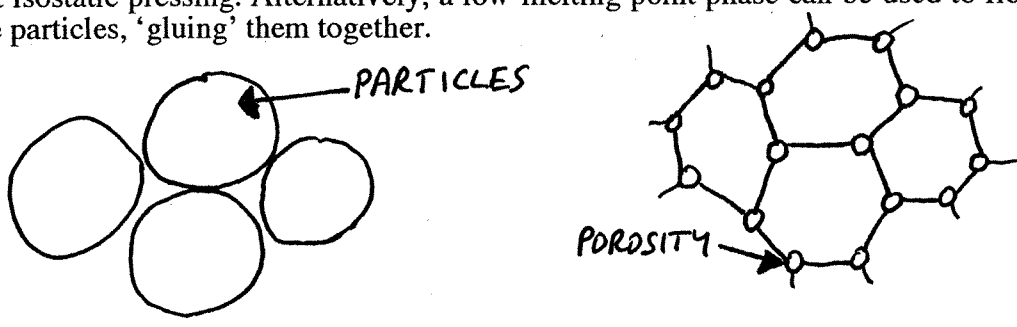
The grain size in ceramics is mainly determined by the initial particle size: grains cannot be bigger than the particles, so fine powder leads to fine-grained ceramic.

Once the particles have bonded together to form grains, there may be grain growth if it is kept at high temperature for a long time.

Porosity results from the gaps between particles. The number and size of pores reduce during sintering.

As ceramics cool down, they contract different amounts in different crystallographic directions. This means that stresses build up, and most ceramics crack as they cool to relieve the stress.

(b) (i)

$$\ln P_S = -\frac{V}{V_0}\left[\frac{\sigma}{\sigma_0}\right]^m$$

Stress σ is proportional to ρV

Increase dimensions uniformly by factor β, so volume now $\beta^3 V$

Note that equation is in terms of probability of survival; question gives failure probability

Eliminating the pair of constants σ_0 and V_0 between the initial condition (survival probability 0.9) and final condition (survival probability 0.1); m=10: Final/Initial condition gives:

$$\frac{\ln 0.1}{\ln 0.9} = \frac{\beta^3 V}{V}\left[\frac{\rho\beta^3 V}{\rho V}\right]^{10}$$

β^{33}=21.85

$\underline{\beta = 1.098}$ Increase all cylinder dimensions by this factor.

(ii) Mass of water-containing bottle is

$$\rho V + \alpha\frac{\rho}{3}\pi r^2 h = \rho\pi r^2 h\left[\frac{2+5\alpha}{15}\right]$$

(Note that V is volume of hollow cylinder so V=2πrth, or V=(2/15) πr^2 h

Again, eliminating the pair of constants σ_0 and V_0 between the initial condition (survival probability 0.9) and final condition (survival probability 0.1):

$$\frac{\ln 0.1}{\ln 0.9} = \frac{V}{V}\left[\frac{\rho\pi r^2 h(2+5\alpha)}{2\rho\pi r^2 h}\right]^m$$

2+5α =2.723

$\underline{\alpha = 0.145}$ Fill bottle to height 0.145h with water

Examiner's comments:
Part (a)Very badly done. Porosity was generally agreed to lead to low strength; opinions were mixed about grain size (where it was mentioned); cracks (inter- or trans-granular) were often not mentioned. Many candidates were entirely clueless about ceramics processing; it was not uncommon to talk about melting the ceramic and casting to shape. Many mentioned sintering, but clearly didn't know what it meant.
(b) Very few used the simplest method, used here, which is always to eliminate σ_0 and V_0 between the pair of equations. It was not uncommon for people to use three pages of algebra to arrive at (or fail to arrive at, more likely) a solution which should have taken them a few lines.
Common identifiable mistakes were:
Part (i): Failure to notice that stress was proportional to volume, and assuming that stress was the same for the two cases.
Muddling probability of survival and probability of failure.
(ii) A number of people equated the final mass calculated in (i) to the required mass in this part. This is of course incorrect because the survival probability depends on volume as well as stress (which was proportional to mass, in this case), The volume of glass in the bottles in (ii) is the original volume from (i) rather than the increased volume; mass increase is achieved by other means.

5 (a)

Maximise energy per unit weight; free variable to be eliminated is rotation velocity ω

Weight $W = \pi R^2 t \rho\, g$

$$\frac{U}{W} = \frac{\pi}{4} \frac{\rho R^4 t \omega^2}{\pi R^2 t \rho g} = \frac{R^2 \omega^2}{4g}$$

Limiting stress condition

$$R^2 \omega^2 = \frac{\sigma_F}{k \rho}$$

So substituting in equation will eliminate ω and give the required condition

$$\frac{U}{W} = \frac{1}{4kg}\left[\frac{\sigma_F}{\rho}\right]$$

Merit index: Maximise $\sigma_F / \rho = M$

(b) Taking logs of merit index equation, $\log \sigma_F = \log \rho + \log M$, so a line of slope 1 has intercept $\log M$ on the y-axis. Maximising the intercept is achieved by taking the line closest to the top left-hand corner of the chart.

Materials will be ranked in decreasing order of suitability according to the merit index alone:

Al_2O_3 (alumina, an engineering ceramic), GFRP uniply, GFRP laminates (engineering composites), high-strength steel as the top of the steel balloon, tungsten alloys (W alloys), lead alloys (all of which are engineering alloys).

The box in the top left-hand corner of the chart announces that the compressive strength is given for ceramics. The material in a flywheel will be mainly in tension, so the appropriate strength for this application would be 10 - 15 times lower. This would bring alumina down to be about as good as tungsten on the merit index.

(c) Mass-production implies that material costs as well as manufacturing costs must be considered.

GFRP uniply is now the most suitable material. The properties on the chart (in tension, as it states) will correspond to those in the fibre direction, so the flywheel would have to be constructed with fibres lying along maximum stress directions. There are two alternatives: radial or circumferential. The latter could be achieved quite easily by rolling up composite tape into a reel; radial is rather more difficult but could also be managed by filament or tape winding. GFRP (fibreglass) is not a prohibitably expensive material, but a large fraction of costs come in the labour-intensive processing of the composite required to produce structures. Composites are therefore not generally suitable for high-volume applications. However, the market for town buses cannot be huge, so GFRP flywheels could be suitable (indeed, one candidate asserted they had seen them being produced!).

High-strength steel is perhaps an obvious choice: steel is easily (and so cheaply) processed by conventional shaping routes (casting, forging or machining); material costs are reasonable.

Alumina is a ceramic and so is liable to fast fracture under tension. This would be unacceptable in a town bus. Manufacture costs for a large component like this would also be high.

Tungsten has a very high melting-point (used for light-bulb filaments), so processing is difficult and expensive (made from powder, in the same way as ceramics). The material itself is also costly, and toughness is low.

Lead is an expensive raw material, but processing would be cheap. However, the merit index is poor so this is unlikely to be a viable option.

(d) GFRP dicussed above.

Steel: Various equally acceptable possibilities. Most obvious is forging (i.e. using heat and pressure to squeeze to shape) followed by machining to improve tolerances, then heat-treating for strength. Maximum strength (with appreciable toughness) in steels often achieved in tempered martensite. Something as large as a flywheel will be difficult to quench fast enough to form an entirely martensitic structure, so probably settle for forming martensite on the surface. Could alternatively machine from a billet then heat-treat, or cast to shape then machine and heat-treat.

Examiner's comments:

(a) Two common errors. One was to work out the merit index for U rather than U/mass. Another was to eliminate R rather than ω from the equation. This actually gave the required result because ω cancelled anyway, but marks were deducted.

(ii) Several candidates couldn't find tungsten on the map, and guessed wildly. Some assumed that Al₂O₃ was an aluminium alloy, which generated rather bizarre answers.

(iii) Once alumina has been eliminated by tensile strength (few people noted that a brittle material would anyway be unsuitable) the best material according to the charts was GFRP (uniply or laminates). Few commented on the anisotropic properties of composites, and noted the need to align fibres along principal stress directions; many dimly knew that composites are difficult to manufacture. A surprising number of answers assumed that the flywheel, by storing kinetic energy, would become hot.

6. (a) HDPE is above its T_G at room temperature. Under the bending stress, the initial response is elastic, with intermolecular bonds (van der Waals) stretching. With time, the molecules are able to move around (creep, plastic deformation) to relieve the elastic stress (spring-and-dashpot model). The molecules have now attained elastic-stress-free configurations, and the pipe retains its shape on removal from the ground. It is worth noting that this will happen even though HDPE is quite highly crystalline – 70-80% according to the databook.

(b) (i) Need a transparent polymer, so it must be amorphous (semi-crystalline polymers are opaque because crystals and amorphous polymer have different refractive indices and so light is scattered at the interfaces between the two regions). Toughness (rather then strength) is the key property - a high toughness material will allow the goggles to be made thinner and hence lighter. The polymer must also be rigid, so below its T_G. Most polymers below T_G are brittle, but polycarbonate has two T_G 's, one below and the other above room temperature. Polycarbonate is therefore the best choice. PMMA is acceptable, but its toughness is half that of polycarbonate.

(ii) To seal properly, we need a soft polymer rather than a rigid one, so it must be above its T_G. We also need dimensional stability in the presence or absence of water. HDPE is the best choice: it is above T_G over the whole temperature range and it is also unaffected by water. Nylon absorbs water and swells, so is less suitable. In addition, dry nylon has a higher T_G than wet nylon, so there is a possibility that the T_G of the dry nylon will be above 0°C.

(c)

σ

BRITTLE FRACTURE

III

SOME LOCAL DUCTILITY

II

I

STABLE NECK

SEVERAL 100% STRAIN

ϵ
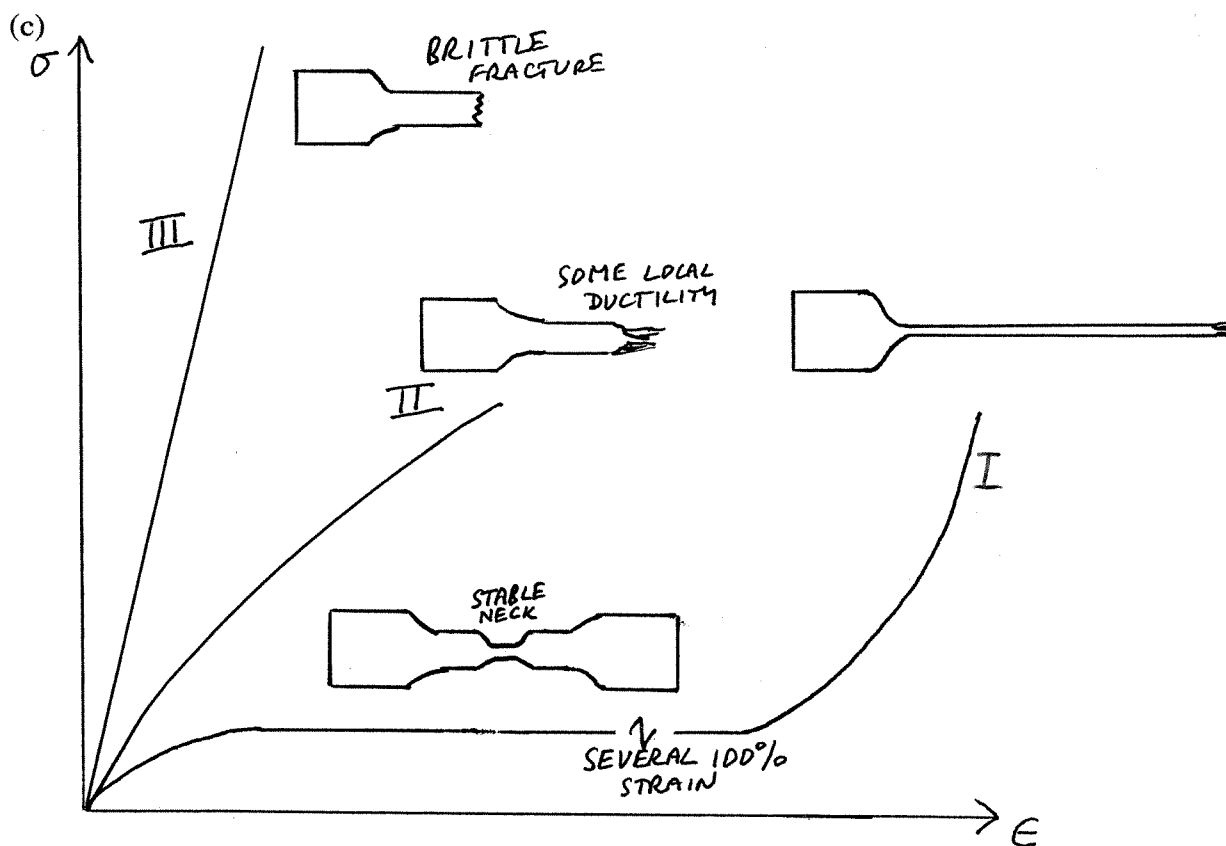
I. Room temperature, low strain-rate. Molecules can move over each other readily. Initial part of the curve 'elastic' but even here some molecular movement takes place so we get a curve rather than a straight line. There is then a long linear region, deformation at constant stress, and a stable neck propagates through the specimen. Molecules in the neck are aligned, so (even though the cross-section area is very much reduced) the necked region is very much stronger than the undeformed material. Once all the molecules are aligned (neck extends over whole specimen; maybe several hundred% strain) the curve goes up steeply, and finally fracture takes place at a high stress.

II. Room temperature, high strain-rate. Less time for molecules to move, so the elastic region is steeper (closer to the true elastic modulus). The 'stable neck' region may be absent altogether if the strain-rate is high enough, so specimen breaks with only local plastic strain. Alternatively, there may be some necking, but at a higher stress than in (I), and leading to less plastic strain.

III. Polymer is now well below T_G and polymer chains are held rigidly by the van der Waals bonds. There is no inter-molecular movement and so no plastic deformation. The initial part of the curve shows almost true linear elastic behaviour. Brittle fracture takes place.

Examiner's comments:
Drew heavily on material covered in the lab. Answers suffered from lack of detail, as well as generally muddled thought. Clearly a somewhat desperate fourth question for some.
(c) Answer required sketches of stress-strain curves as well as explanations. Very few drew completely correct curves. Explanations rarely matched the curves.