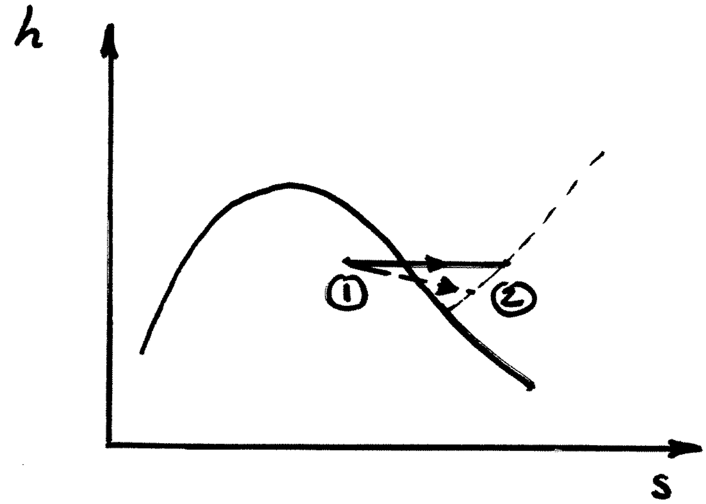
1B THERMO-FLUIDS 2001 – PAPER 4

Q.1. (a)



(6)

Reduction of pressure takes the state into the superheated vapour region. If KE changes can be ignored then process is isenthalpic. Measuring P & T at (2) then gives x_1. Otherwise, both SFEE and mass continuity are needed (dashed line).

(b) (i) $\dot{m} = \rho_2 A u_2$

$\therefore \; u_2 = 0.3/(0.0662 \times \pi \times 0.075^2)$

$\therefore \; \underline{u_2 = 256.5 \; ms^{-1}}$

INTERPOLATE FOR SPECIFIC VOLUME $\Rightarrow \rho_2 = 0.06618$

(3)

(ii) Essentially, due to the throttling (reduction in pressure) the density is much higher at (1), hence velocity and kinetic energy are much less.

(2)

(b) (iii) SFEE: $h_1 + \tfrac{1}{2}u_1^2 = h_2 + \tfrac{1}{2}u_2^2$

Neglect $\tfrac{1}{2}u_1^2$ term

$\therefore \quad h_{f_1} + x_1 h_{fg_1} = h_2 + \tfrac{1}{2}u_2^2$

$\therefore \quad x_1 = \left(2602.4 + \tfrac{1}{2} \times 256.5^2 \times 10^{-3} - 417.5\right)/2257.9$

$\underline{x_1 = 0.982}$ 2

(NOTE: Including $\tfrac{1}{2}u_1^2$ term and solving quadratic gives $x_1 = 0.9826$)

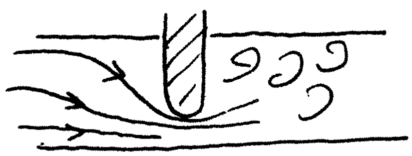
(iv) $s_1 = (1 - x_1)\overset{1.303}{s_{f_1}} + x_1 \overset{7.36}{s_{g_1}}$

$\quad = \underline{7.252 \text{ kJ/kgK}}$.

$s_2 = s\,(0.1\text{ bar}, 55°C)$

$\quad = \underline{8.204 \text{ kJ/kgK}}$.

Increase is due to highly irreversible dissipative processes (viscous dissipation + internal heat transfer) downstream of valve:



3

(c) By closing valve further, mass flow rate is reduced for given pressure drop. Hence KE terms become negligible and process is isenthalpic.

2

Q.2. (a) Exhaust gas may be assumed to have (at least approximately) constant heat capacity. Hence exhaust gas temperature falls linearly with % of heat transferred.
Water / steam temperature remains constant between P and Q due to boiling at constant pressure.

(b) (i) $T_P = T_s(25\text{ bar}) + 20$

$= 223.9 + 20$

$\underline{T_P = 243.9\ ^\circ C}$. [2]

(ii) $\dot{m}_g c_{pg}(650 - 243.9) = \dot{m}_s\left(h(25,450) - h_f(25)\right)$

$\therefore \dot{m}_s = 50 \times 1.01 \times (650 - 243.9) / (3351.25 - 962.0)$

$\underline{\dot{m}_s = 8.58\ \text{kg/s}}$ [3]

(iii) $\dot{m}_g c_{pg}(T_Q - T_P) = \dot{m}_s h_{fg}$.

$\therefore T_Q = T_P + \dot{m}_s h_{fg} / \dot{m} c_{pg}$

$= 243.9 + 8.58 \times 1839 / (50 \times 1.01)$ [2]

$\underline{T_Q = 556.5\ ^\circ C}$

(iv) $\Delta\dot{S} = \overset{-23.9}{\dot{m}_g c_p \ln\left(\frac{T_P}{T_Q}\right)} + \overset{31.75}{\dot{m}_s(s_g - s_f)}$

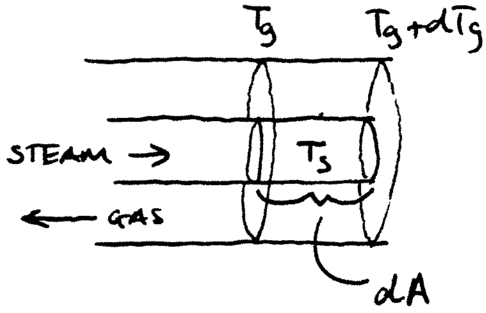
$= 50 \times 1.01 \times \ln\left(\frac{243.9 + 273.15}{556.5 + 273.15}\right)$

$+ 8.58 \times (6.254 - 2.554)$ [5]

$\underline{\Delta\dot{S} = 7.87\ \text{kJ/K}}$.

Increase due to heat transfer across large temperature differences.

(c)



$$\dot{m}_g c_{pg}\, dT_g = dQ = U (T_g - T_s)\, dA$$

$$\therefore \quad \frac{dT_g}{T_g - T_s} = \frac{U\, dA}{\dot{m}_g c_{pg}}$$

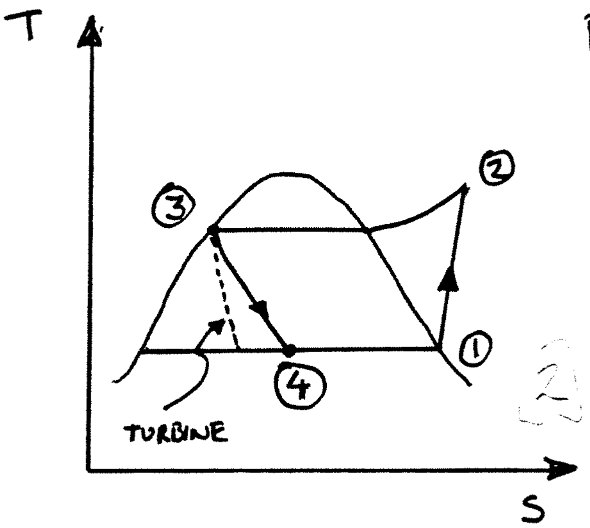
$$\therefore \quad A = \frac{\dot{m}_g c_{pg}}{U} \ln\left(\frac{T_a - T_s}{T_p - T_s}\right)$$

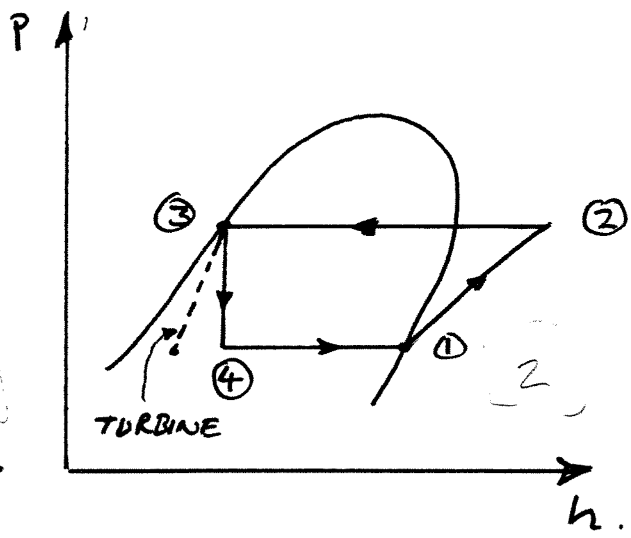
$$= \frac{50 \times 1.01}{0.2} \times \ln\left(\frac{556.5 - 223.9}{20}\right)$$

$$\underline{A = 710 \ m^2}$$

Q3.

(a)





[2] [2] [2] [2]

(b) $s_1 = 0.6991 \quad = s_g \ (\cdot 261 \ MN/m^2)$

$h_1 = 185.4 \quad h_g$

$$h_{2s} = 199.6 + \frac{(0.6991 - 0.6854)}{(0.7321 - 0.6854)} \times (214.3 - 199.6)$$

$= 203.91$ [i.e., superheated by 5.87 K)

$$\therefore \Delta h_c = \frac{(203.91 - 185.4)}{0.7} = 26.44 \text{ kJ/kg.}$$

$\therefore \ h_2 = 211.8$ kJ/kg.

$h_3 = 64.6$ kJ/kg. $\quad (h_3 = h_f)$

$\therefore \ \Delta h_{COND} = 147.24$ kJ/kg [5]

Mass flow rate, $\dot{m} = \dfrac{\dot{Q}}{\Delta h_{COND}} = \dfrac{5}{147.24}$

$= \underline{\underline{0.034 \text{ kg/s.}}}$

Power input, $\dot{W}_c = \dfrac{\Delta h_c}{\Delta h_{COND}} \times \dot{Q} = \dfrac{26.44}{147.24} \cdot 5000$

$= \underline{\underline{897 \text{ W}}} \quad (= \dot{m}\Delta h_{comp})$

[2]

(c) $PER = \frac{\Delta h_{COND}}{\Delta h_c} = \underline{\underline{5.58}}$.

$PER_{IDEAL} = \frac{T_c}{T_c - T_E} = \frac{303}{303-268} = \underline{\underline{8.65}}$

Differences: (i) Irreversibility in compressor and throttle

(ii) Temperature at inlet to condenser goes above 303 K. [3]

$s_3 = 0.2399$

(d) $h_{4s} = 31.4 + \frac{(0.2399 - 0.1251)}{(0.6991 - 0.1251)} \cdot (185.4 - 31.4)$

$= 62.2$ [20% ~~liquid~~] vapour / 80% liq.

$\therefore \Delta h_s = 64.6 - 62.2$

$= 2.4$ kJ/kg.

$\therefore \Delta h_{TURB} = 0.85 \times 2.4$

$= 2.04$ kJ/kg.

Net work input now $= w_c - w_T$

$= 26.44 - 2.04$

$= 24.4$ kJ/kg.

Heat output unchanged.

$\therefore$ New $PER = \frac{147.24}{24.4} = \underline{\underline{6.034}}$ (up by 8%

[4]

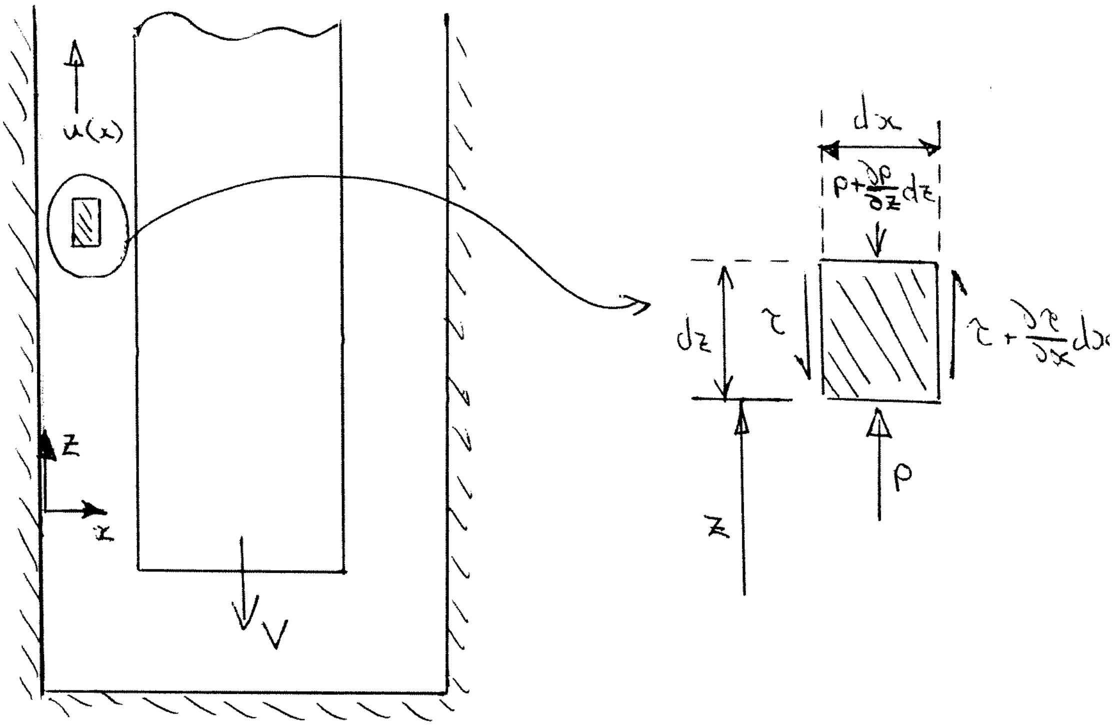
(e) Probably not: (i) Much greater complexity and cost for relatively small benefit.

(ii) High liquid content at turbine exit gives practical difficulties.

[2]

4.

(a)



Force balance on <u>non-accelerating</u> element of fluid. Sum forces in z direction;

$$0 = p\,dx - \left(p + \frac{dp}{dz}dz\right)dx - \tau\,dz + \left(\tau + \frac{\partial \tau}{\partial x}dx\right)dz$$

$$\Rightarrow \quad 0 = -\frac{dp}{dz}\,dz\,dx + \frac{\partial \tau}{\partial x}\,dx\,dz$$

$$\Rightarrow \quad \frac{\partial \tau}{\partial x} = \frac{dp}{dz}$$

Newtons law of viscosity $\tau = \mu \frac{\partial u_z}{\partial x}$

$$\Rightarrow \quad \underline{\underline{\mu \frac{\partial^2 u_z}{\partial x^2} = \frac{dp}{dz}}}$$

(b) Integrate, noting p a function of z only.

$$u_z = \frac{1}{\mu}\frac{dp}{dz}\left(\frac{x^2}{2}\right) + Ax + B$$

Boundary conditions : $\begin{cases} u_z = 0 \quad \text{at} \quad x = 0 \quad \Rightarrow \quad B = 0 \\ u_z = -V \quad \text{at} \quad x = W \quad \Rightarrow -V = \frac{W^2}{2\mu}\frac{dp}{dz} + AW \end{cases}$

Substitute for A :

$$\underline{\underline{u_z = \frac{-1}{2\mu}\frac{dp}{dz}(Wx - x^2) - V\frac{x}{W}}}$$

(c) Integrate to find mass flux across gap.

$$\dot{m}/\rho = \int_0^W u_z\,dx = -\frac{1}{2\mu}\frac{dp}{dz}\left(\frac{W^3}{2} - \frac{W^3}{3}\right) - V\frac{W}{2}$$

$$= -\frac{1}{12\mu}\frac{dp}{dz}W^3 - \frac{VW}{2}$$

But continuity of mass gives : $2\frac{\dot{m}}{\rho} = 2WV \Rightarrow \frac{\dot{m}}{\rho} = WV$

Thus,

$$-\frac{1}{12\mu}\frac{dp}{dz}W^3 - \frac{VW}{2} = VW$$

$$\Rightarrow \quad \frac{1}{\mu}\frac{dp}{dz} = -18\frac{V}{W^2} \quad \text{(negative, as required)}$$

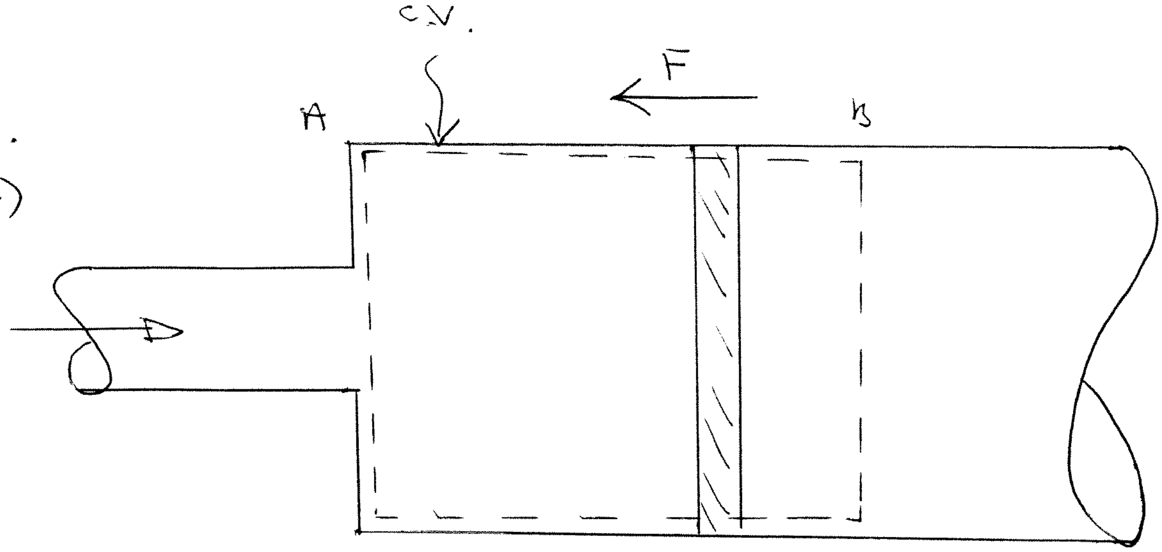
Substitute for dp/dz :

$$u_z = 9\frac{V}{W^2}(Wx - x^2) - V\frac{x}{W}$$

$$\Rightarrow \quad \underline{\underline{u_z = \frac{V}{W^2}(8Wx - 9x^2)}}$$

5.

(a)



Apply F.M.E. to C.V. in streamwise direction

$$p_A A_B - p_B A_B - F = \dot{m} V_B - \dot{m} V_A$$

clamping force needed to keep filter in place

$$F = c_D\left(\tfrac{1}{2}\rho V_B^2\right) A_B \quad , \quad \dot{m} = \rho A_B V_B$$

$$\Rightarrow \quad p_A \cancel{A_B} - p_B \cancel{A_B} - c_D\left(\tfrac{1}{2}\rho V_B^2\right)\cancel{A_B} = \rho \cancel{A_B} V_B (V_B - V_A)$$

$$\Rightarrow \quad \underline{\underline{p_A - p_B = c_D\left(\tfrac{1}{2}\rho V_B^2\right) + \rho V_B (V_B - V_A)}}$$

(b) Apply extended Bernoulli equation:

$$\left[p_A + \tfrac{1}{2}\rho V_A^2\right] - \left[p_B + \tfrac{1}{2}\rho V_B^2\right] = \text{energy lost } (A \to B)$$

$$\Rightarrow \quad \text{energy lost} = c_D\left(\tfrac{1}{2}\rho V_B^2\right) + \rho V_B (V_B - V_A) + \tfrac{1}{2}\rho\left(V_A^2 - V_B^2\right)$$

$$= \tfrac{1}{2} c_D \rho V_B^2 + \tfrac{1}{2}\rho\left(V_A^2 - V_B^2 - 2V_A V_B + 2V_B^2\right)$$

$$= \tfrac{1}{2} c_D \rho V_B^2 + \tfrac{1}{2}\rho\left(V_A^2 - 2V_A V_B + V_B^2\right)$$

$$= \underline{\underline{\tfrac{1}{2} c_D \rho V_B^2 + \tfrac{1}{2}\rho\left(V_A - V_B\right)^2}}$$

(e)

energy lost $= \frac{1}{2}\rho V_B^2 + \frac{1}{2}\rho(V_A - V_B)^2 = \frac{1}{2}\rho\left(\frac{1}{2}V_A\right)^2 + \frac{1}{2}\rho\left(\frac{1}{2}V_A\right)$

$(c_0 = 1)$ $\quad (V_B = \frac{1}{2}V_A)$

$\Rightarrow$ energy lost $= \frac{1}{4}\rho V_A^2$

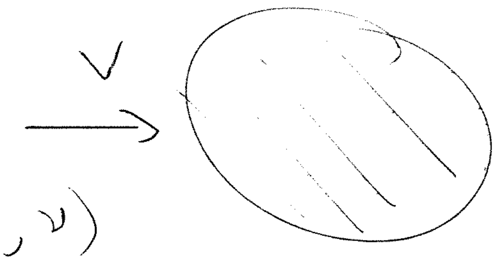
This loss of mechanical energy (per unit volume) leads to a rise of internal energy so,

$$\cancel{\rho} c_v \Delta T = \frac{1}{4}\cancel{\rho} V_A^2$$

$$\Rightarrow \quad \Delta T = \frac{V_A^2}{4 c_v} = \underline{\underline{0.1\,^\circ C}}$$

6.

(a)



$$\dot{Q} = f(\Delta T, V, d, \lambda, \alpha, \nu)$$

Π theorem: Parameters = 7

Dimensions = 4 (kg, s, m, °C)

Number of groups = 7 − 4 = 3.

Spot that $\left\{ \begin{array}{l} \nu/\alpha \text{ is dimensionless (Prandtl number)} \\ Vd/\nu \text{ is dimensionless (Reynolds number)} \end{array} \right.$

Need one more group. It must involve $\dot{Q}$, ΔT and λ.

$\dot{Q} \equiv$ Watts/m

$\Delta T \equiv$ °C

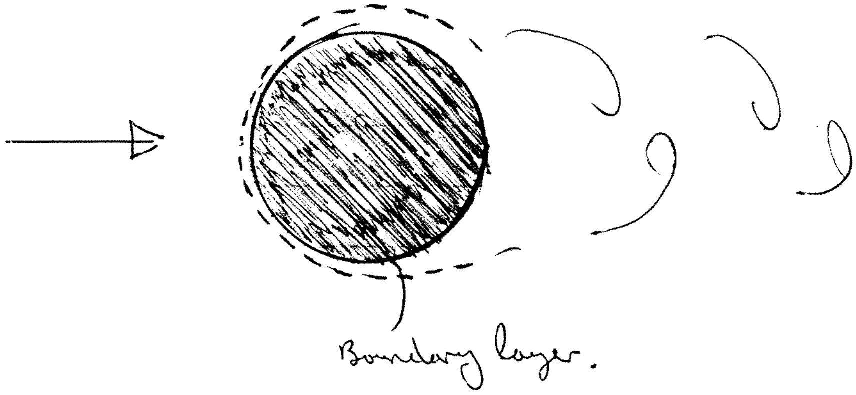
$\lambda \equiv$ Watts/m/°C

Thus third group is $\dfrac{\dot{Q}}{\lambda \Delta T}$ or $\dfrac{\lambda \Delta T}{\dot{Q}}$.

In summary,
$$\underline{\underline{\frac{\dot{Q}}{\lambda \Delta T} = f\left(\frac{Vd}{\nu}, \frac{\nu}{\alpha}\right)}}$$

Thus ΔT is proportional to $\dot{Q}$.

(b)



The boundary layer represents a significant part of the thermal resistance. The larger ν, the thicker the boundary layer, and hence the higher the thermal resistance.

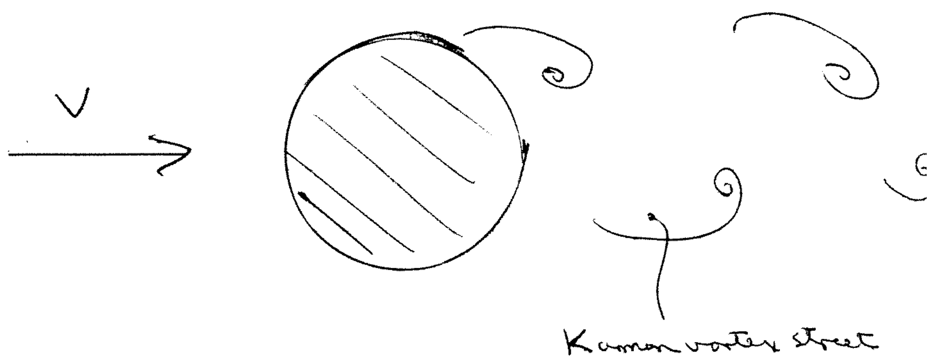
(c) If ν is not a relevant parameter,

$$\frac{\dot{Q}}{\lambda \, \Delta T} = f\left(\frac{Vd}{\alpha}\right)$$

Peclet number.

If $\dot{Q}/\lambda\Delta T$ is proportional to $V^{0.4}$ then it is also proportional to $d^{0.4}$ and $\alpha^{-0.4}$.

(d)



The oscillations are due to the periodic shedding of vortices from the rear of the cylinder which carry heat downstream.