IB - PAPER 5 - ELECTRICAL ENGINEERING - CRIB. 2001

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Ib Paper 5 Q1 (a) For the case h_{FE} essentially infinite then $I_C = I_E$

Base voltage $V_B = \frac{R_2}{R_1+R_2} V_{cc} = \frac{1.5}{20+1.5} \times 15$

$= 1.05$ volts

Emitter voltage $V_E = V_B - 0.85$

$= 0.2$ volts

$I_E = \frac{V_E}{R_E} = \frac{0.2}{150}$

$I_C = I_E = 1.33 \times 10^{-3}$ amp

(b) case where $h_{FE} = 150$ ie $I_C = 150\, I_B$

$I_E = I_C + I_B = 151\, I_B$

At the base $\frac{15 - V_B}{R_1} = I_B + \frac{V_B}{R_2}$ ①

But $V_B = 0.85 + I_E R_E$

$= 0.85 + 151\, I_B \times 150$

$= 0.85 + 22650\, I_B$

Substitute in ① $\frac{15 - (0.85 + 22650\, I_B)}{20000} = I_B + \frac{0.85 + 22650\, I_B}{1500}$

$\therefore I_B = 8.17 \times 10^{-6}$ Amp

$I_C = h_{FE} I_B = 150 \times 8.17 \times 10^{-6}$

$= 1.226 \times 10^{-3}$ Amp

$I_E = I_C + I_B = \underline{\underline{1.234 \times 10^{-3} \text{ Amp}}}$

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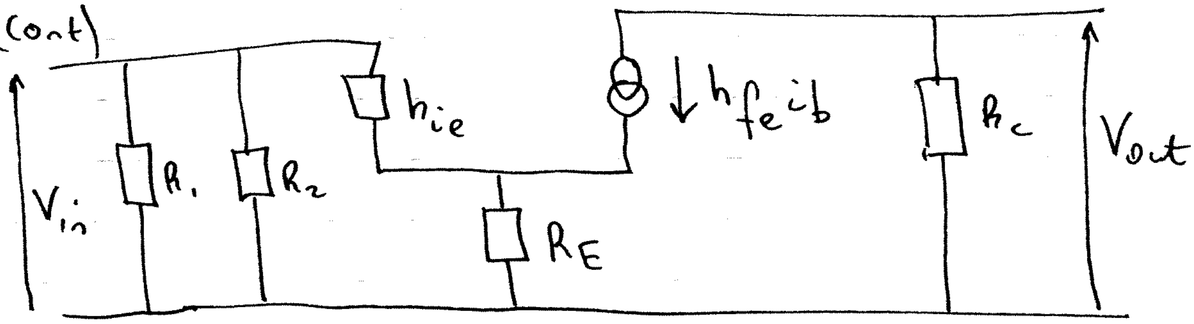
Q1 (cont)



$$V_{in} = i_b h_{ie} + (h_{fe} i_b + i_b) R_E$$

$$V_{out} = - h_{fe} i_b R_c$$

$$\text{Voltage gain} = \frac{V_{out}}{V_{in}} = - \frac{h_{fe} i_b R_c}{i_b(h_{ie} + h_{fe} R_E + i_b R_E)}$$

But in this case $h_{fe} R_E \gg h_{ie}$

$$\therefore \text{ gain} = - \frac{h_{fe} R_c}{(h_{fe}+1) R_E}$$

the maximum value corresponds to large h_{fe} ie. $h_{fe} \gg 1$

$$\text{max. gain} = - \frac{h_{fe} R_c}{h_{fe} R_E} = - \frac{R_c}{R_E} = - \frac{3000}{150}$$

$$= \underline{\underline{-20}}$$

(d) the voltage gain (for ac signals) is increased when R_E is bypassed with a capacitor in which case

$$V_{in} = i_b h_{ie}$$

$$V_{out} = - h_{fe} i_b R_c \quad \text{(as before)}$$

and the gain $= \dfrac{V_{out}}{V_{in}} = - \dfrac{h_{fe} R_c}{h_{ie}}$

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Ib Paper 5 Q 2.

(a) For an op-amp circuit in differential mode both inputs are out of phase i.e. $V_1 = -V_2$

The circuit output is $A_{diff}(V_1 - V_2)$

in common mode the inputs are in phase i.e. $V_1 = V_2$

The circuit output is $A_{com}\left(\frac{V_1 + V_2}{2}\right)$

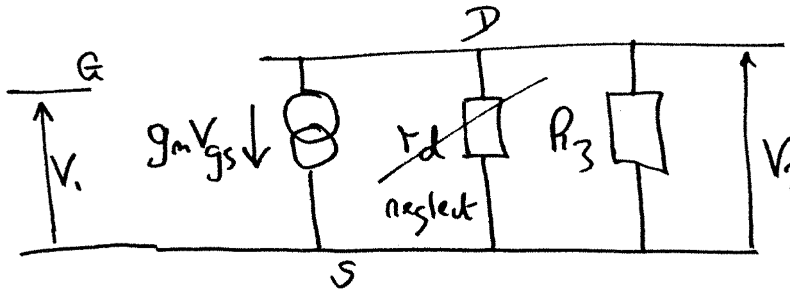
(b) $\text{CMRR} = \dfrac{\text{differential mode gain}}{\text{common mode gain}}$

which is the measure of performance of an op-amp

(c) circuit is symmetric

use half circuit

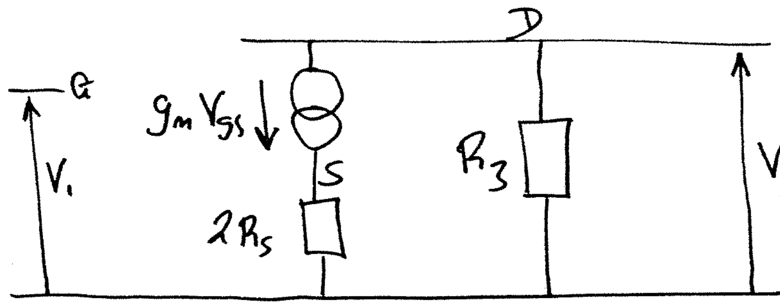
for differential case the voltage across R_S is dc



$$V_3 = -g_m V_{gs} R_3$$

$$\text{gain} = \frac{V_o}{V_i} = \frac{V_3}{V_1} = \frac{-g_m V_{gs} R_3}{V_1} = -g_m \cancel{V_{gs}} R_3$$

for common mode case the value of R_S is doubled for the half circuit



$$V_3 = -g_m V_{gs} R_3$$

$$V_1 = g_m V_{gs} 2R_S + V_{gs} = V_{gs}(g_m 2R_S + 1)$$

$$\text{gain} = \frac{V_o}{V_i} = \frac{V_3}{V_1} = -\frac{g_m V_{gs} R_3}{V_{gs}(g_m 2R_S + 1)} = -\frac{g_m R_3}{g_m 2R_S + 1}$$

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Q2(cont.) $$CMRR = \frac{-g_m R_3}{-g_m R_3 / (g_m 2R_s + 1)} = \underline{\underline{g_m 2R_s + 1}}$$

(d) For the case $g_m = 7\,mS$ $\quad R_s = 15\,k\Omega$ $\quad R_3 = R_4 = 2\,k\Omega$

$$\begin{aligned}\text{differential gain} &= -g_m R_3 = -7\times10^{-3} \times 2\times10^{3} \\ &= \underline{\underline{-14}}\end{aligned}$$

$$\begin{aligned}CMRR &= g_m 2R_s + 1 \\ &= 7\times10^{-3} \times 2 \times 15\times10^{3} + 1 \\ &= 210 + 1 \\ &= 211\end{aligned}$$

(e) To obtain an output relative to ground take <u>either</u> V_3 <u>or</u> V_4 as the output.

then the differential gain is -7

and CMRR is 106.

Ib Paper 5

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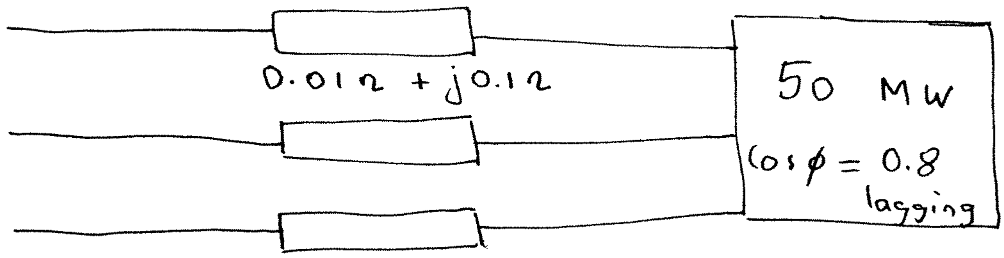
Q3 (a) The cost of wiring a three phase system to a town is less than an equivalent dc system because the fourth return conductor can be relatively thin. ac electric power can also be conveniently transformed up to a high voltage to minimise the galvanic loss for long distance transmission.

$$V_{line} = \sqrt{3}\, V_{phase}$$

A balanced load is one where a similar load is placed on each of the three phases of the system.

(b)



Line current per phase $I_{line} = \frac{1}{\sqrt{3}} \frac{50 \times 10^6}{0.8 \times 33 \times 10^3}$

$= \underline{\underline{1093 \text{ amp.}}}$

(c) Let the feeder line length be n kilometers.

Total power dissipated in three lines each n km long

$$3n \times 0.01 \times (1093)^2 = 35870\,n \text{ watt}$$

To dissipate 10% of the power in the line at most

$$\frac{1}{10}\, 50 \times 10^6 = 35870\,n$$

$$\underline{\underline{n = 139 \text{ km}}}$$

Ib Paper 5 Q3 (continued)

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(c) For 139 km long feeder the impedance is $1.39 + i\,13.9$

$$Q_{in} = 50 \times 10^6 \tan(\cos^{-1} 0.8) + 3 \times (1093)^2 \times 13.9$$
$$= 37.5 \times 10^6 + 49.8 \times 10^6$$
$$= 87.3 \times 10^6$$

$$P_{in} = 50 \times 10^6 + 5 \times 10^6$$
$$= 55 \times 10^6$$

$$S_s = \left((87.3 \times 10^6)^2 + (55 \times 10^6)^2 \right)^{1/2}$$
$$= 103.2 \times 10^6$$

But $S_s = \sqrt{3}\, I_\ell V_s$

Hence $$V_s = \frac{103.2 \times 10^6}{1.732 \times 1093}$$

$$= 54500$$

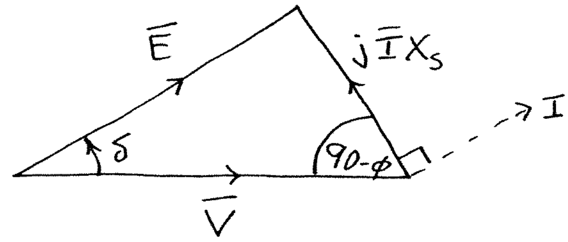
The line voltage at the sending end is 54.5 kV

d) If the phase angle of the load is reduced to require less reactive power, then the current in the feeder line would be reduced and there would be less galvanic loss.

By using a higher transmission line voltage the current in the feeder line for a given power is reduced and it is feasible to site a power station further from the town.

Ib Paper 5 — DFm 6/2001

Q 4 (a) Synchronous ac generator (three phase)



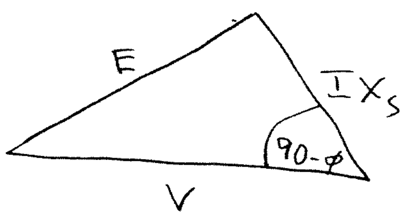
The machine carries a field winding supply with dc on its rotor and a three-phase armature winding on its stator.

When underexcited the generator has a leading power factor.
When overexcited by adjusting the supply to the field winding it has a lagging power factor. Hence the reactive power to the load can be adjusted.

(b) Power delivered $= \sqrt{3}\ V_L\ I_L\ \cos\phi$

$$500 \times 10^6 = 1.732 \times 33 \times 10^3\ I_L \times 0.7$$

$$\therefore I_L = 12500 \text{ amp}$$



Apply cosine rule $\quad V = V_{line}/\sqrt{3} = 19050$ volt

$$E^2 = V^2 + (IX_s)^2 - 2\,V\,IX_s \cos(90-\phi)$$

$$= \frac{(33 \times 10^3)^2}{3} + (12500 \times 0.5)^2 - \frac{2 \times 33 \times 10^3}{\sqrt{3}}(12500 \times 0.5)\sin\phi$$

$$= 363 \times 10^6 + 39 \times 10^6 - 238 \times 10^6 \times 0.714$$

$$\therefore E^2 = 232 \times 10^6$$

generator excitation phase voltage $E = 15{,}230$ volt

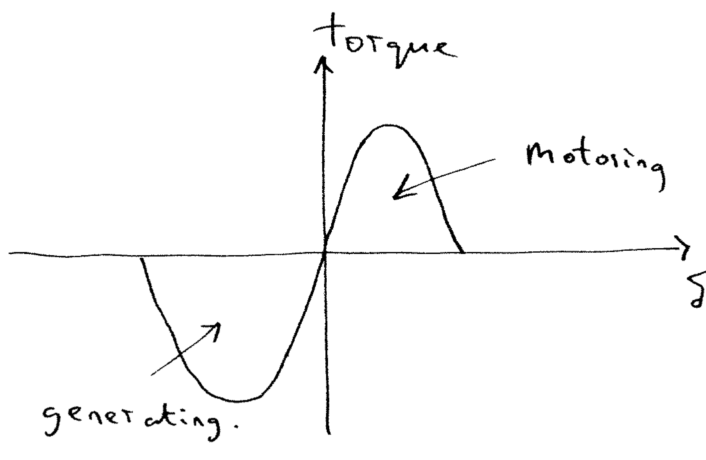
(ii) to find the load angle apply the sine rule

$$\frac{\sin\delta}{IX_s} = \frac{\sin(90-\phi)}{E}$$

$$\therefore \sin\delta = 0.7 \times \frac{12500 \times 0.5}{15230} = 0.287$$

load angle $\delta = 16.7^\circ$

Torque characteristic of a Synchronous machine



The maximum torque corresponds to $\delta = \pm 90^\circ$

From data book $T_{max} = \frac{3\,VE}{\omega_s X_s}$

$$= \frac{3 \times 19.05 \times 10^3 \times 15.2 \times 10^3}{2\pi \times 50 \times 0.5}$$

$$= 5.5 \times 10^6 \text{ Nm}$$

Ib Paper 5 Q5

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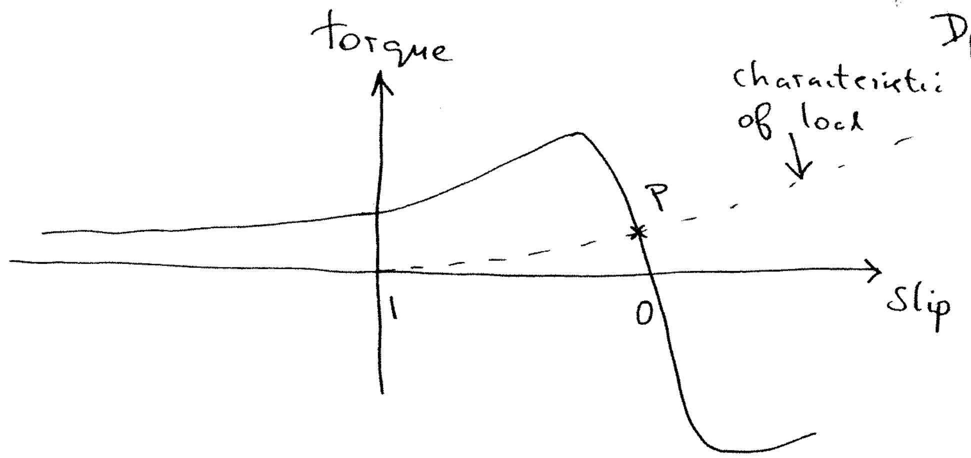
(a)

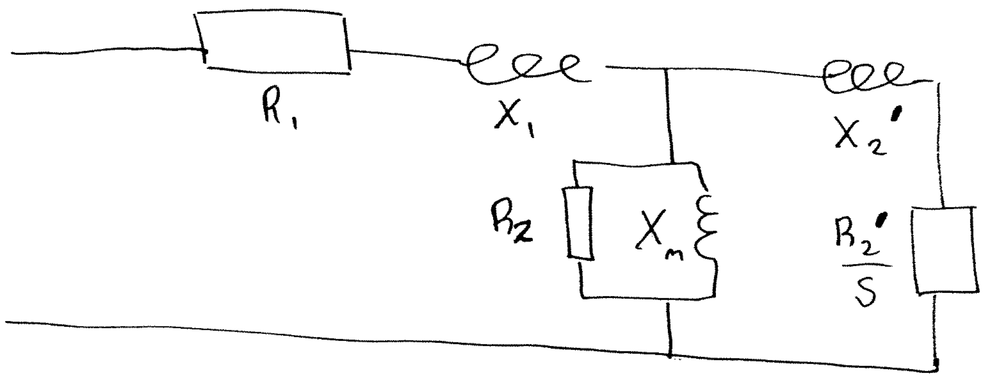


P is the operating point of the induction motor where the load characteristic intersects the motor characteristic.

The value of the slip S is positive.

In the figure at point B the slip S = 0 and the motor produces no torque.

(b)



R_1 is the stator winding resistance of the motor

X_1 is the stator leakage reactance

X_2' is the referred rotor leakage reactance

R_2 is the iron loss resistance

X_m is the three phase magnetizing reactance

R_2'/S is the referred rotor resistance divided by the slip.

Ib Paper 5 Q5 (continued)

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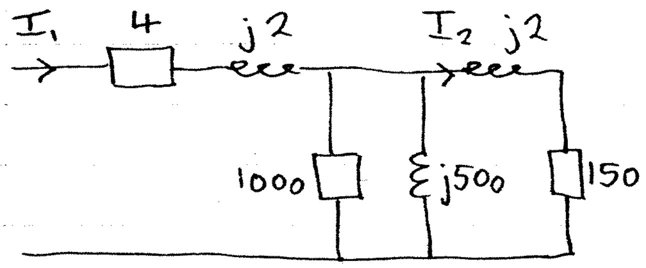
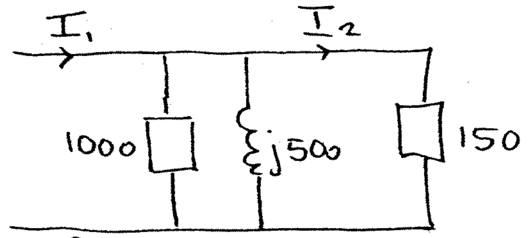
c) From data book slip $S = \dfrac{\omega_s - \omega_r}{\omega_s}$

$\omega_s = \dfrac{\omega}{p}$; $p = 4$ $\quad \therefore S = \dfrac{2\pi\, 50 \times 60/4 - 2\pi 735}{2\pi\, 50 \times 60/4} = 0.02$





APPROXIMATE CIRCUIT

Find input impedance Z_T neglecting the small impedance (various approx. are possible)

$$\frac{1}{Z_T} = \frac{1}{1000} + \frac{1}{j500} + \frac{1}{150}$$

$$= 7.667 \times 10^{-3} - j\,2 \times 10^{-3}$$

$$\underline{\underline{Z_T = 122 + j32}} \qquad |Z_T| = 126$$

(iii) stator line current $= \dfrac{\sqrt{3} \times 415}{|Z_T|} = \underline{\underline{5.7 \text{ amp}}}$

phase current $|I_1| = \dfrac{415}{|Z_T|} = 3.29$ amp

current $|I_2| = \dfrac{415}{150} = 2.77$ amp.

torque developed by the motor (from data book)

$$T = \frac{3 I_2^2}{\omega_s} \frac{R_2'}{S}$$

$$= \frac{3 \times (2.77)^2 \times 150}{2\pi \times 50/4}$$

$$= \underline{\underline{44 \text{ Nm}}}$$

At 765 rpm the machine is behaving like a generator since $\omega > \omega_s$

Part IB Paper 5 Q6

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a) A transmission line is used to transmit an electrical signal between two points with the minimum of signal loss or distortion.

The characteristic impedance of a transmission line is the ratio between the voltage and current of a unidirectional wave on the line and it is defined at a point.

For uniform line with inductance L per unit length and capacitance C per unit length the impedance $Z = \sqrt{\frac{L}{C}}$

A load is matched if its impedance is the same as that of the line.

For a line which is shorter than $\frac{\lambda}{16}$ time delays and propagation effects are negligible at frequencies corresponding to λ; hence the total capacitance C_{total} and inductance L_{total} as discrete circuit elements may be used to describe the behaviour of the line.

b) Wave velocity $u = \frac{1}{\sqrt{LC}}$

Wavelength $\lambda = \frac{u}{f} = \frac{1}{f\sqrt{LC}} = \frac{1}{10^9\sqrt{200\times10^{-9}\times 80\times10^{-12}}}$

$= \frac{1}{10^9\sqrt{16\times10^{-18}}} = 0.25$ metres.

Hence maximum length of line for which transmission line effects can be ignored with 1GHz signals is $\frac{250}{16}$ = 16 millimetres

Part Ib Paper 5 Q6 (continued)

(c)

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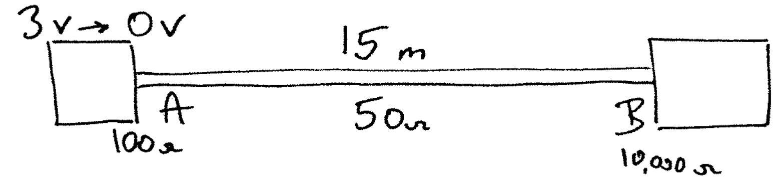


Impedance of the transmission line

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{200\times10^{-9}}{80\times10^{-12}}} = \sqrt{2.5\times10^{3}} = 50\ \Omega$$

Consider point A where the 100 Ω output impedance mismatches the line.

Change of voltage on the device $\Delta V_{out} = -3$ volt

Subsequent change of voltage on the line is ΔV_{line}

Match currents at the boundary.

$$\frac{\Delta V_{out} - \Delta V_{line}}{Z_{out}} = \frac{\Delta V_{line}}{Z_{line}}$$

$$\frac{-3 - \Delta V_{line}}{100} = \frac{\Delta V_{line}}{50}$$

$$\therefore -3 - \Delta V_{line} = 2\Delta V_{line}$$

Hence $\Delta V_{line} = -1$ Volt

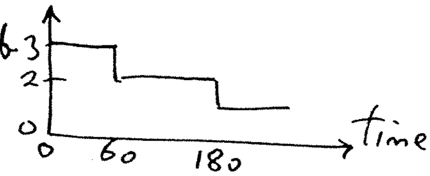
ie. the first wave that travels from the source to the load reduces the potential from 3 volt to 2 volt.

The delay in reaching the load is $\frac{\text{length}}{u} = \frac{15}{\frac{1}{\sqrt{200\times10^{-9}\times80\times10^{-12}}}}$

$$= \frac{15}{2.5\times10^{8}} = 60\times10^{-9}\ \text{sec.}$$

After 60 nsec there is a reflection at the load where the input impedance is high, then 60 nsec later a reflection back from the source ie after 180 nsec a second pulse reaches the load.

Eventually the signal settles to near zero.



Ib Paper 5 Q7

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(a) Wave equation $\frac{\partial^2 E}{\partial z^2} - \varepsilon_o \mu_o \frac{\partial^2 E}{\partial t^2} = 0$ (1)

ε_o is the permittivity of free space $8.8\ 10^{-12}$ Fm^{-1}

μ_o is the permeability of free space $4\pi \times 10^{-7}$ Hm^{-1}

for a plane transverse electromagnetic wave travelling with velocity C in free space in the Z direction then

E is the transverse electric field (eg. E_x)

and $\frac{\partial^2 E}{\partial z^2}$ is the second derivative with respect to the direction of propagation. $\frac{\partial^2 E}{\partial t^2}$ is the 2nd time derivative

Trial function $E = E_x \exp j(\omega t - \beta z)$

$$\frac{\partial^2 E}{\partial z^2} = (-j\beta)^2 E_x \exp j(\omega t - \beta z)$$

$$= -\beta^2 E_x \exp j(\omega t - \beta z)$$

$$\frac{\partial^2 E}{\partial t^2} = (j\omega)^2 E_x \exp j(\omega t - \beta z)$$

$$= -\omega^2 E_x \exp j(\omega t - \beta z)$$

Substitute into equation (1) LHS

$$-\beta^2 E_x \exp j(\omega t - \beta z) + \varepsilon_o \mu_o \omega^2 \exp j(\omega t - \beta z)$$

$$(-\beta^2 + \varepsilon_o \mu_o \omega^2) \exp j(\omega t - \beta z)$$

Satisfies (1) if $\beta^2 = \varepsilon_o \mu_o \omega^2$

$$\frac{\beta}{\omega} = \sqrt{\varepsilon_o \mu_o}$$

From the trial function the speed of propagation is

$$\frac{\omega}{\beta} = \underline{\underline{\frac{1}{\sqrt{\varepsilon_o \mu_o}}}}$$

Ib Paper 5 Q7 (continued)

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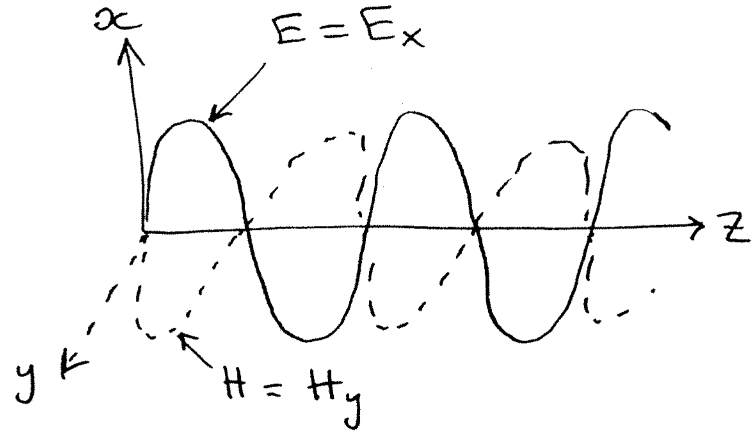
(b) electric and magnetic fields are orthogonal.



The impedance $\eta = \frac{|\underline{E}|}{|\underline{H}|} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$; $H_y = \frac{E_0}{\eta} \exp j(\omega t - \beta z)$

Power per unit area $= \left| \text{Real}\left(\frac{1}{2} \underline{E} \wedge \underline{H}^*\right) \right|$

$= \left| \frac{1}{2} E_0 \exp j(\omega t - \beta z) \frac{E_0}{\eta} \exp -j(\omega t - \beta z) \right|$

$= \underline{\underline{\frac{1}{2} \frac{E_0^2}{\eta}}} = \underline{\underline{\frac{1}{2} E_0^2 \sqrt{\frac{\varepsilon_0}{\mu_0}}}}$

(c) Intensity in the optimum direction at distance r from an antennae with gain G is $\frac{G \text{ Power}}{4\pi r^2}$

In this case intensity is $\frac{1000 \times 20}{4\pi (20000 \times 10^3)^2}$ W m^{-2}

$= \frac{2 \times 10^4}{4\pi \times 4 \times 10^{14}} = 4 \times 10^{-12}$

For an optimally orientated antenna with effective area 0.5 m^2 the power received is

$0.5 \times 4 \times 10^{-12} = \underline{\underline{2 \times 10^{-12}}}$ watt