IB - PAPER 6 - LINEAR SYSTEMS + CONTROL AND COMMUNICATIONS. - CRIB 2001

1. (a) $\bar{y}(s) = \bar{d}(s) + K(s)\, G(s) \left(\bar{r}(s) - \bar{y}(s)\right)$

(i) With $\bar{d}(s) = 0$:

$$H_r(s) = \frac{\bar{y}(s)}{\bar{r}(s)} = \frac{K(s)\, G(s)}{1 + K(s) G(s)}$$

(ii) With $\bar{r}(s) = 0$:

$$H_d(s) = \frac{\bar{y}(s)}{\bar{d}(s)} = \frac{1}{1 + K(s)\, G(s)}$$

$H_r(s)$ determines response to desired input signal.

Candidates could mention such things as:

- ensure the frequency response ($H_r(j\omega)$) is flat over the desired range of input frequencies
- in step response, design to give a good trade-off between overshoot and oscillation at output
- design so that steady state gain $H_r(0)$ is close to unity.

$H_d(s)$ is the sensitivity function, measuring sensitivity of system to noise disturbances $\bar{d}(s)$. Hence design to have low gain over all frequencies of interest in the system.

[can also answer in terms of sensitivity ~~to~~ of closed loop system to uncertainty about $G(s)$.

1. (b) $\quad H_r(s) = \dfrac{1}{s+4}$; $\quad H_d(s) = \dfrac{s+3}{s+4}$

(i) Use final value theorem:

$$\lim_{t \to \infty} y(t) = H_r(0) = 1/4$$

(ii) Note: many candidates used final value theorem here, which is wrong since the input is a sin-wave.

Should use frequency <u>response</u> with $\omega = 1$:

$$\lim_{t \to \infty} y(t) = |H_d(j)| \sin(t + \angle H_d(j))$$

where:

$$H_d(j\omega) = \frac{j\omega + 3}{j\omega + 4}$$

$$|H_d(j\omega)| = \frac{\sqrt{10}}{\sqrt{17}} = 0.767$$

$$\angle H_d(j\omega) = \tan^{-1} 1/3 - \tan^{-1} 1/4 = 0.0768$$

(iii) By linear superposition:

$$\lim_{t \to \infty} \{y(t)\} = 5/4 + \sqrt{10/17} \sin(t + \pi/4 + 0.0768)$$

Note: many candidates wasted a great deal of effort solving the differential equations in full using inverse Laplace transforms in part (b)

1. (c) (i) $k_I = 0$

No response, $y(t) = 0$.

(ii) & (iii)

$$H_d(s) = \frac{1}{s^2/k_I + 3s/k_I + 1} = \frac{k_I}{s^2 + 3s + k_I}$$

Compare with 2^{nd} order mechanical system (D-book p.6):

$$k_I = \omega_n^2$$

$$2\zeta/\omega_n = 3/k_I$$

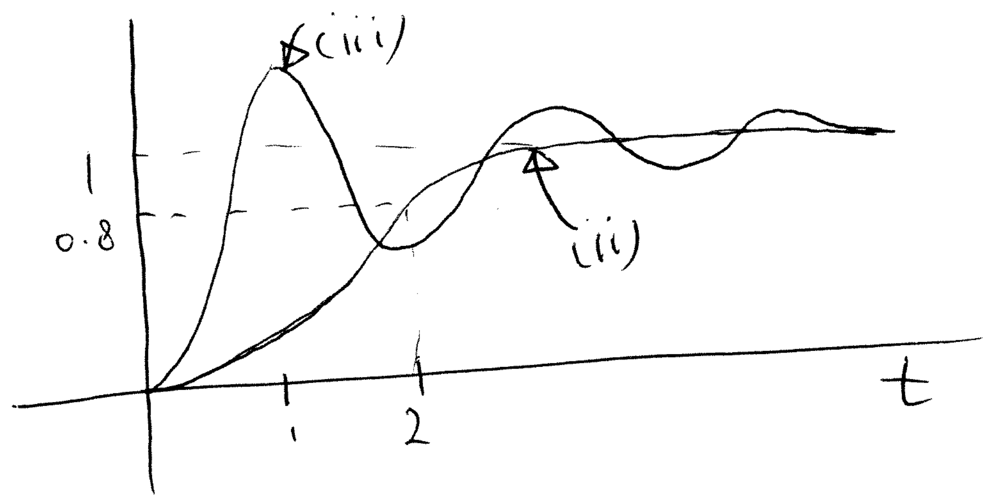
$$\Rightarrow \zeta = \frac{3}{2\sqrt{k_I}}$$

1. (c) (ii)&(iii) contd.

for $k_I = 2.25$ we have $\zeta = 1$, $\omega_n = 1.5$ rad/s

for $k_I = 10$ " " $\zeta = 0.237$, $\omega_n = \sqrt{10}$ rad/s

from Data book, p. 6, step response is:



Comment:

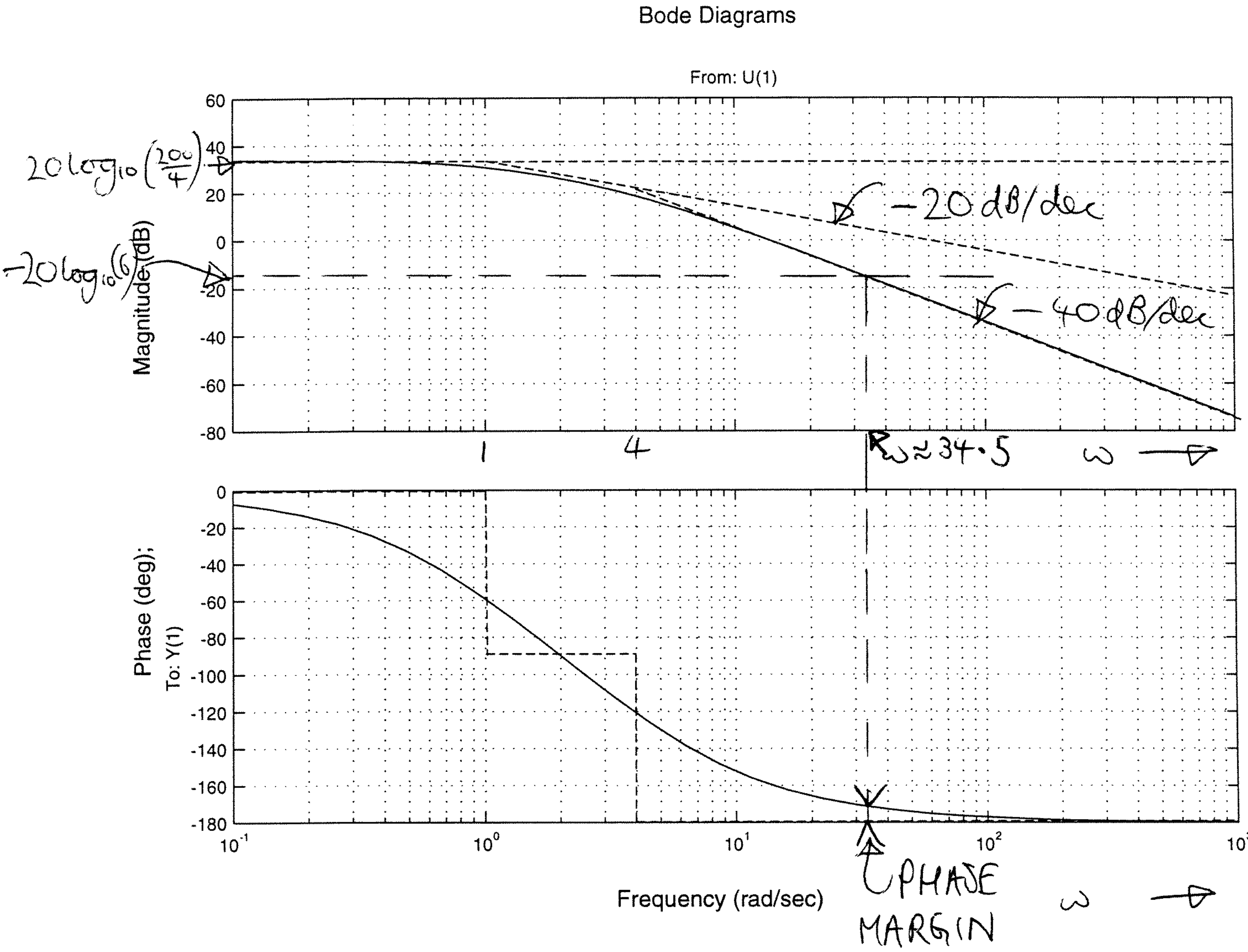
Integral feedback reduces steady state error to zero, which is desirable. However, as k_I increases the system becomes oscillatory and overshoot is a problem.

If a small amount of oscillation is acceptable then an intermediate k_I, say $k_I = 4$, may give a suitable trade-off ($k_I = 2.25$ was also acceptable).

NOTE: Many candidates again wasted time solving differential equations when all the necessary info. was available in the data book

Q2. (a) Asymptotes shown dotted:

Bode diagram is the plot of $G(j\omega)$, both magnitude and phase:



Note that $G(s)$ is a second order system, so valid answers could also be obtained by calculating mechanical 2^{nd} order parameters and using D-book.

Q2. (a) contd.

Phase margin for $k_p = 6$

$k_p = 6$ multiplies gain curve by 6, i.e. raises Bode plot by $20 \log_{10}(6)$ dB $\doteq +15.6$ dB

Hence phase margin will correspond to a gain of -15.6 dB on Bode plot for $G(s)$.

This is shown dotted on previous page.

Reading off phase margin gives $\approx 8°$.

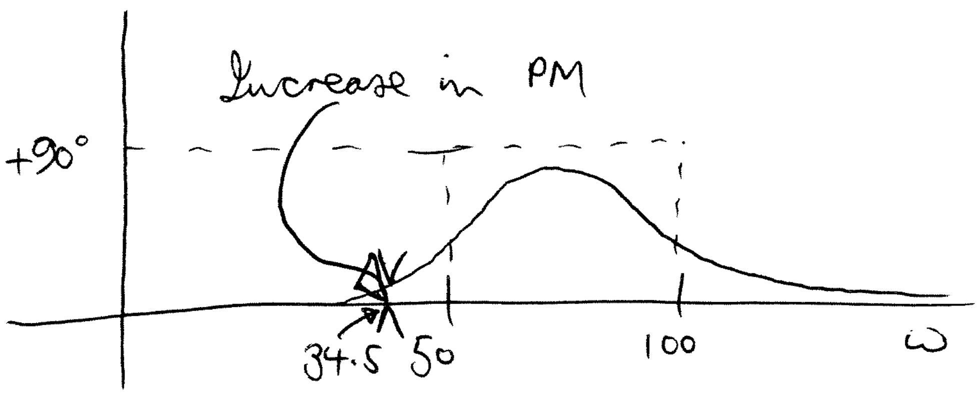
A more accurate calculation from $G(s)$ gives $\underline{PM = 8.3°}$, although correctly reasoned answers between $6-10°$ were given full marks.

This is a very small phase margin, so we can expect some overshoot and oscillation in response to rapid (e.g step) changes on the input.

Since phase never goes below $-180°$, increasing k_p has no effect on absolute stability (however, response becomes more oscillatory).

Q2(b). $K(s) = \dfrac{12(s+50)}{s+100}$

Note that $K(j0) = \dfrac{12 \times 50}{100} = 6$, hence dc gain is equal to that of $K_p G(s)$ in part (a). Note also that the corner frequencies are at $\omega = 50$, $\omega = 100$, hence gain will be almost flat up to at least $\omega = 40$ rad/s. Hence, frequency at which $|K(j\omega) G(j\omega)| = 1$ will be almost identica[l] to part (a) with $K_p = 6$. The phase response of $K(j\omega) G(j\omega)$ looks like:



Hence system remains stable and phase margin is increased by $\angle K(j34.5) \approx 13.7^\circ$.

Thus overall $\underline{\underline{PM \approx 21.9^\circ}}$

This is satisfactory and we expect no oscillation or overshoot.

Q3. (a)

To construct Nyquist diagram:

- Take open loop system $G(s)$.
- For a wide range of frequencies, input a sine-wave $\sin(\omega t)$ to the system $G(s)$.
- Once transients have died away, measure the amplitude gain and the relative phase from input to output of $G(s)$
- Plot amplitude gain and relative phase as complex numbers - the 'Nyquist' diagram.
- The Nyquist diagram should not encircle the point $-1/k_p$ for the system to be stable (or equivalently, leaves the point $-1/k_p$ ~~on~~ to the left of the curve).

[Note underlined quantities were key facts that many candidates omitted]

Q3.(b) Most of the required curve is given in Fig. 3b. We simply need to construct the section from $\omega = 0.3 \to \omega = 0$.

Consider:

$$\lim_{\omega \to 0} \frac{j\omega + 2}{2j\omega(j\omega + 3)(j\omega + 1)(j\omega + 0.5)}$$

Elementary errors in taking limits lead us to the incorrect answer of $-j\infty$.

Correct approaches take e.g. a Taylor expansion:

$$G(j\omega) = \frac{j\omega + 2}{2j\omega} \frac{(1 - j\omega/3 \cdots)(1 - j\omega \ldots)(1 - 2j\omega \ldots)}{1.5}$$

$$= \frac{2 + j\omega.(-5^{2}/_{3})}{2j\omega.1.5}.$$

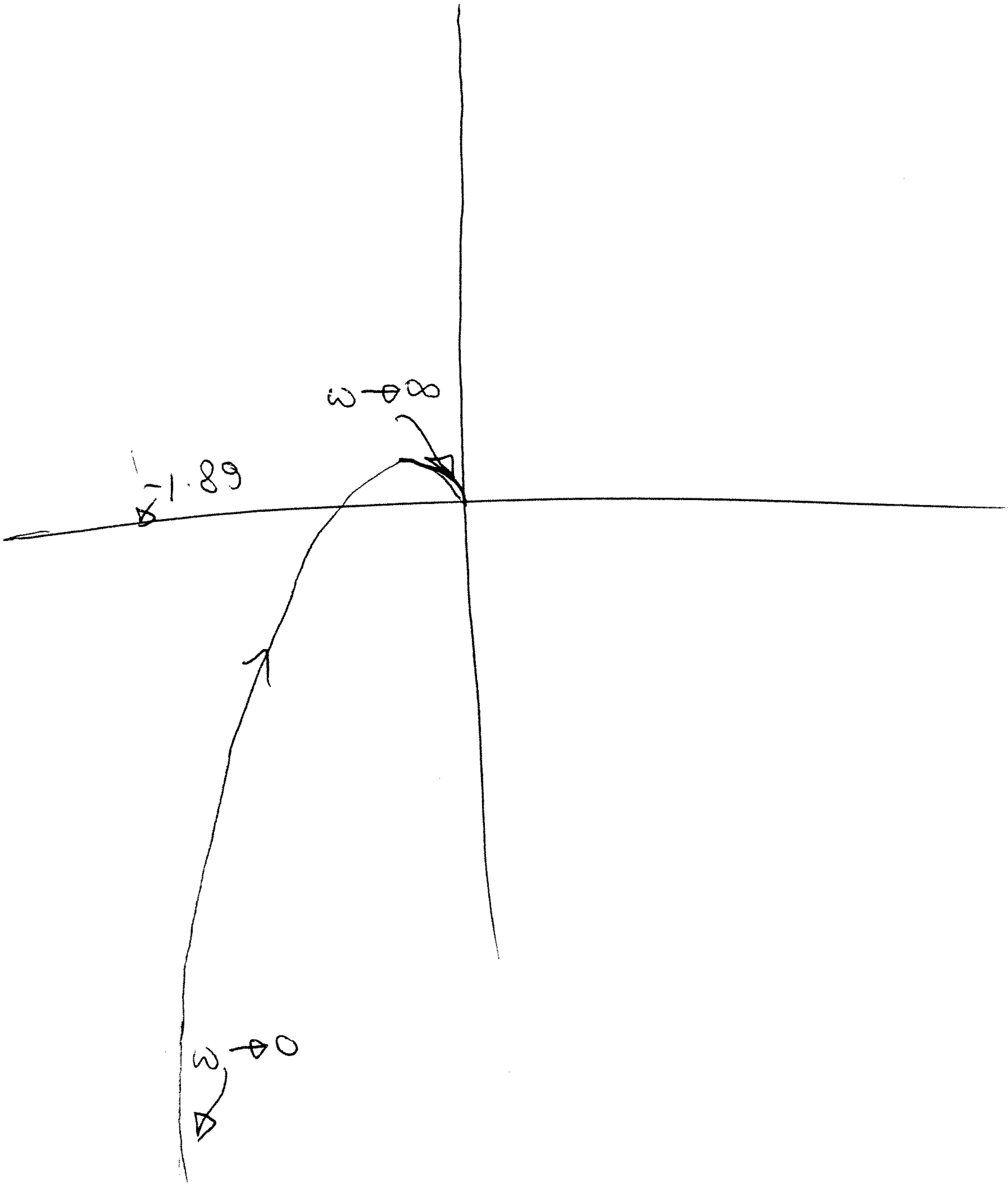
Hence

$$\lim_{\omega \to 0} G(j\omega) = \underline{\underline{-j\infty + -1.89}}$$

[Note: some credit was also given to candidate who calculated the limit directly for very small ω on a calculator]

Q3. (b) contd.

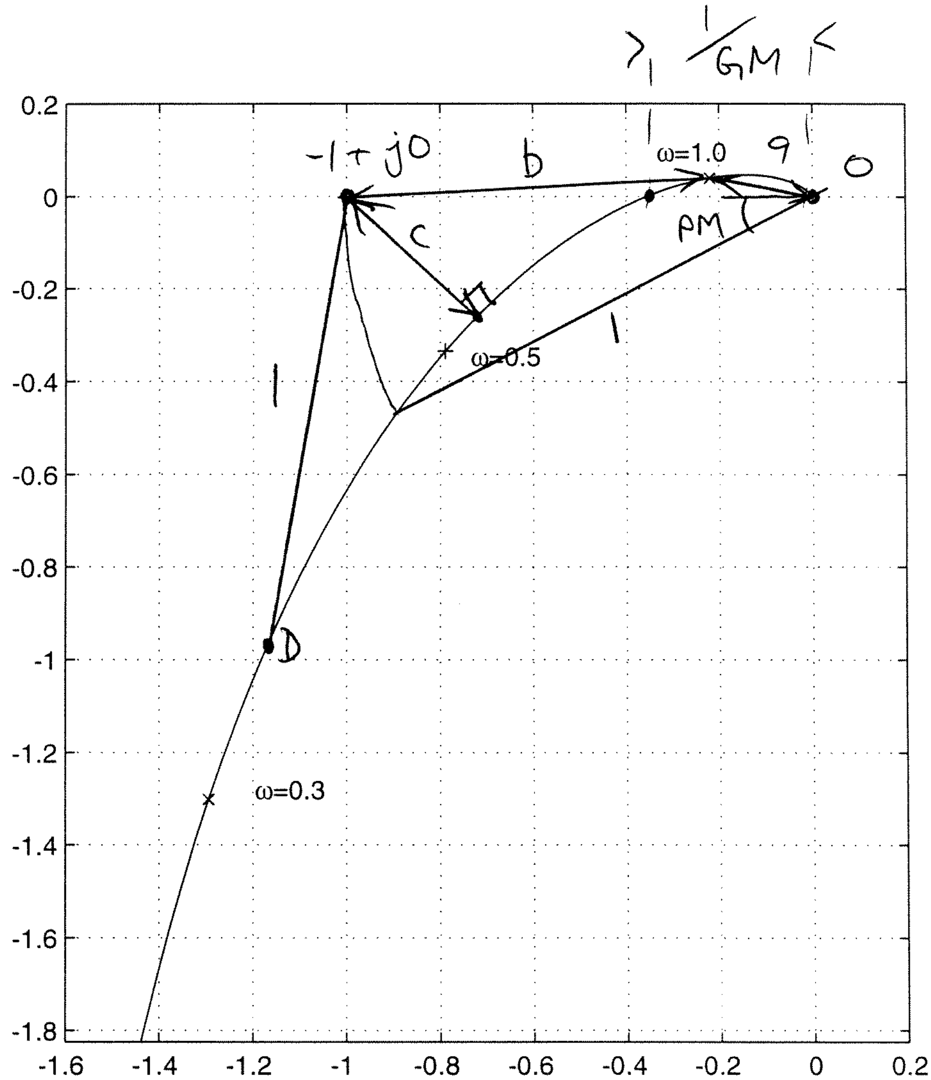
So, full Nyquist diagram looks like:



[A very complete answer would also check the angle of approach as $\omega \to \infty$, which is $\pi/2$]

3. (c)

All by measurement from the diagram:



(i) $GM = \frac{1}{0.36} \simeq 2.8$

$PM \simeq 27°$

(ii) $|H(j1)| = a/b \simeq 0.22$

(iii) $1/c \approx 2.6$

(iv) Point D corresponds to $|S(j\omega)| = 1$

Point D correspond to $\omega \simeq 0.34$ (after trying a few values in the equation)

$\Rightarrow$ $\underline{\omega > 0.34 \text{ rad/s}}$ gives $|S(j\omega)| > 1$

4. (a) Top mass: $F = M\ddot{y}$ $\quad \therefore \bar{F} = Ms^2\bar{y}$

Lower mass: $k(w-x) - F = m\ddot{x}$

$$\therefore k\bar{w} - k\bar{x} - \bar{F} = ms^2\bar{x}$$

$$\therefore \bar{x}(ms^2+k) = k\bar{w} - \bar{F}$$

$$\therefore \bar{x} = \frac{k\bar{w}-\bar{F}}{ms^2+k} \quad \& \quad \bar{y} = \frac{\bar{F}}{Ms^2}$$

$$\therefore \bar{z} = \bar{y} - \bar{x} = \bar{F}\left(\frac{1}{Ms^2} + \frac{1}{ms^2+k}\right) - \bar{w}\cdot\frac{k}{ms^2+k}$$

Hence $H_{FZ}(s) = \left.\frac{\bar{z}}{\bar{F}}\right|_{\bar{w}=0} = \frac{1}{Ms^2} + \frac{1}{ms^2+k}$

$$= \frac{ms^2+k+Ms^2}{Ms^2(ms^2+k)}$$

& $H_{WZ}(s) = \left.\frac{\bar{z}}{\bar{w}}\right|_{\bar{F}=0} = \frac{-k}{ms^2+k}$

(b) $\bar{z} = H_{FZ}\cdot\bar{F} + H_{WZ}\cdot\bar{w}$ $\quad \& \quad \bar{F} = -K(s).\bar{z}$

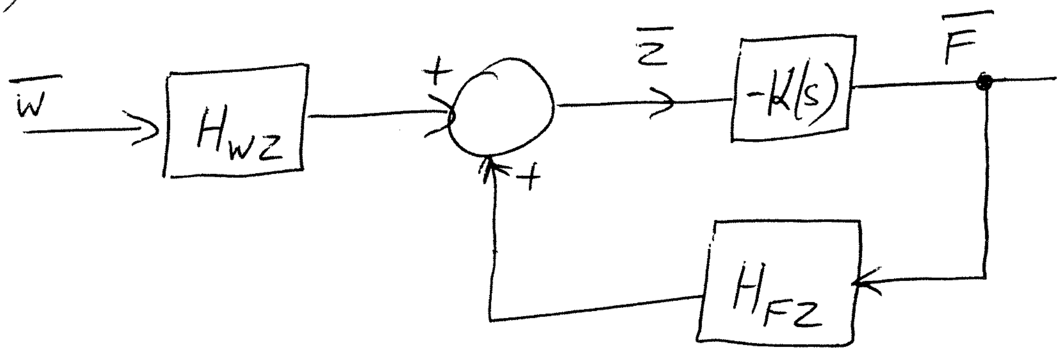
$$\therefore \bar{z} = -H_{FZ}\cdot K(s).\bar{z} + H_{WZ}\cdot\bar{w}$$

$$\therefore \frac{\bar{z}}{\bar{w}} = \frac{H_{WZ}}{1 + H_{FZ}\cdot K(s)} = \frac{-k}{ms^2+k+\left(\frac{ms^2+k}{Ms^2}+1\right).K(s)}$$

$$\frac{\bar{y}}{\bar{w}} = \frac{\bar{y}}{\bar{z}}\cdot\frac{\bar{z}}{\bar{w}} = \frac{\bar{y}}{\bar{F}}\cdot\frac{\bar{F}}{\bar{z}}\cdot\frac{\bar{z}}{\bar{w}} = \frac{1}{Ms^2}.(-K(s)).\frac{\bar{z}}{\bar{w}}$$

$$= \frac{-k.K(s)}{Ms^2(ms^2+k) + (ms^2+k+Ms^2).K(s)}$$

4 (b) (cont) From above expression for $\frac{\bar{z}}{\bar{w}}$:



Many variations on this are possible & correct.

(c) For steady state response to $\bar{w} = A\cos(10t)$, set $s = j\omega = j10$, and calculate the gain of each transfer function at this frequency.

$$\left|\frac{\bar{y}(j10)}{\bar{w}(j10)}\right| = \left|\frac{-k.K(j10)}{M.(-100).(-100m+k) + (-100m+k-100M).K}\right|$$

$$= \left|\frac{-5.10^5(1+10j)}{-5.10^4(5.10^5-10^3) + (5.10^5 - 5 \cdot 1.10^4)(1+10j)}\right|$$

The second denominator term can be neglected, giving:

$$\left|\frac{\bar{y}}{\bar{w}}\right| = \frac{10 \cdot 10.05}{4.99.10^5} = 2.01.10^{-4}$$

Similarly $\left|\frac{\bar{z}}{\bar{w}}\right| \simeq \frac{k}{k-100m} = 1.002$

$\therefore$ Since amplitude of w is A, amplitude of y is $\underline{\underline{2.01.10^{-4}A}}$ and amplitude of z is $\underline{\underline{1.002A}}$.

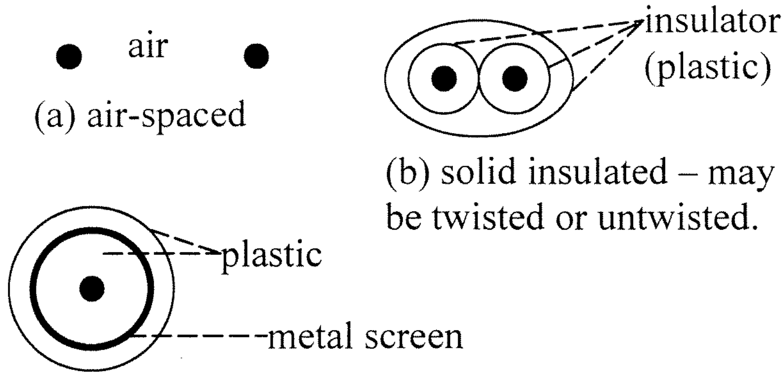
Since $x = y - z$, and the ampl. of y is so small, the amplitude of x will also be $\underline{\underline{1.002A}}$ (to 4 s.f.).

4. (c) (cont)

We note that at this frequency, y is strongly attenuated, giving good comfort of ride, and that x closely follows w ($x = 1.002\, w$, phase $\simeq 0$), so roadholding will be good.

5(a) From lecture notes:

This figure shows cross-sections of three common types of metal cable.



(a) air-spaced

(b) solid insulated – may be twisted or untwisted.

(c) co-axial

The maximum data rate over a cable circuit is proportional to its bandwidth, which in turn is inversely proportional to the product of the cable's shunt capacitance and its loop (series) resistance. The **air-spaced pair cable** has much lower capacitance per unit length, and hence higher bandwidth, than the solid-insulated pair cable. Such cables (strung between "telegraph poles") were the original choice for long-distance telegraphs (and then telephones).

Solid insulation (nowadays polythene or PVC), with an overall sheath is used in most applications (e.g. underground telephone wiring into buildings, wiring around and within buildings).

Both of the above types of cable suffer from interference from any external changing magnetic (EM) field; they also emit EM interference; finally, they have poor high frequency performance – due to the loss of energy by EM radiation, increasing with frequency.

To reduce EM interference, a solid pair cable is therefore often twisted (a "**twisted pair**" cable) to minimise the net voltage difference induced between the two wires.

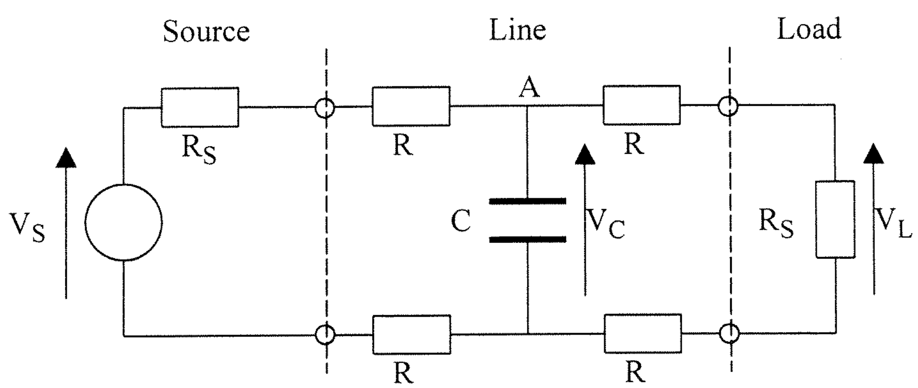
However, the loss of energy by EM radiation is still a serious limitation of pair cables for long distance, high bandwidth, communications. The solution is the coaxial cable. This configuration traps (or 'guides') E-M fields inside the tubular screen, so it emits little E-M energy. Coaxial cables have a wide and flat frequency response. Coaxial cables are used for some "telegraphy" applications (e.g. Ethernet networks). Because of their better high frequency performance, coaxial cables are almost always used at radio frequencies (*RF*), e.g. for domestic TV aerial connections. However, a coaxial cable is more expensive than a pair cable.

Because it prevents high frequency E-M fields entering the cable, a coaxial cable has low sensitivity to interference from external E-M fields.

For the most E-M sensitive applications at lower frequencies (e.g. long microphone leads), a twisted pair may be enclosed in a flexible screen like that of a coaxial cable, to prevent E-M fields from getting into cable. Interference is further minimised by using 'balanced' (i.e. differential) transmission.

Cables may also incorporate steel tension-bearing strands, to take the stress in suspended cables, or steel armouring round the outside, for protection of buried cables.

5(b) From lectures, this is the simplified equivalent circuit for a long pair cable:



The *Loop resistance* of the cable $R_{loop} = 4R$.
The following analysis is a single frequency analysis. Alternatively, do a Laplace analysis and substitute $s = j\omega$.

Sum currents at A: $$j\omega C V_C + \frac{V_C}{2R + R_S} = \frac{V_S - V_C}{2R + R_S}.$$

Hence $$V_C\left(j\omega C + \frac{2}{2R + R_S}\right) = \frac{V_S}{2R + R_S}.$$

Define $R_P = (2R + R_S)\|(2R + R_S) = (R + R_S/2)$ and multiply through by R_P, giving

$$V_C\left(j\omega C R_P + 1\right) = \frac{V_S}{2}$$

Thus the transfer function from the source to the capacitor is

$$\frac{V_C}{V_S} = \frac{1}{2\left(j\omega C R_P + 1\right)}.$$

Since $\frac{V_L}{V_C} = \frac{R_S}{2R + R_S}$, the transfer function from the source to the load (output) is

$$\frac{V_L}{V_S} = \frac{R_S}{4R + 2R_S} \times \frac{1}{\left(j\omega C R_P + 1\right)}$$ as required.

5(c) The term $1/(j\omega CR_P+1)$ is a lowpass filter with a -3 dB cutoff frequency ω_C given by $\omega_C CR_P=1$, or $f_C=1/(2\pi CR_P)$. We must set f_C equal to the required bandwidth.

If the length of the cable is x km, then $C=30x\times10^{-9}$ F and
$R_P=\frac{15}{4}x+\frac{600}{2}=3.75x+300\ \Omega$.

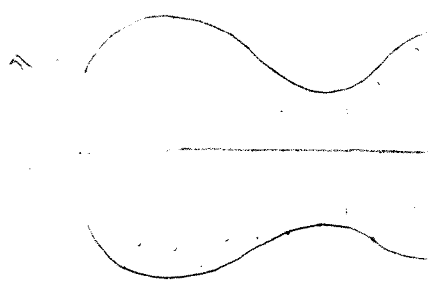
Hence for $f_C=3400$, $(3.75x+300)(30x\times10^{-9})=\frac{1}{2\pi 3400}$.

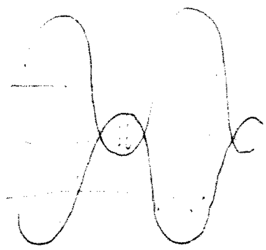
The positive root of this quadratic equation is $x=4.90$ km.

For $f_C=51200$ Hz, the positive root of the quadratic equation is $x=0.344$ km.

6. (a) DSB-AM: simple to generate and demodulate. The earliest modulation system. Disadvantages: not bandwidth efficient (SSB is twice as efficient); not power efficient (SSB is 3 times as power efficient); poor noise performance (FM is much better, though requires much greater bandwidth).

The left-hand picture below illustrates AM with $m_A < 1$. The RH picture with $m_A > 1$.





We see that provided $m_A < 1$, the information bearing signal can be extracted by an envelope detector. Otherwise, much more complicated demodulation is required.

6. (b) we have $x(t) = b\cos(\omega_M t)$ where $b = a_0 m_A$. Hence

$$
\begin{aligned}
s(t) &= \left[a_0 + b\cos(\omega_M t)\right]\cos(\omega_C t) \\
&= a_0\cos(\omega_C t) + b\cos(\omega_M t)\cos(\omega_C t) \\
&= a_0\cos(\omega_C t) + \frac{b}{2}\cos((\omega_C + \omega_M)t) + \frac{b}{2}\cos((\omega_C - \omega_M)t)
\end{aligned}
$$

The first term is a (co)sinusoid at the carrier frequency. The second and third terms are the *upper and lower sidebands*.

6. (c) Put $x(t) = \cos(\omega_M t)$. The square-law circuit output is

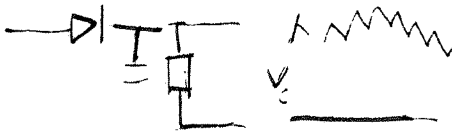
$$
\begin{aligned}
y(t) &= \left(b + a\cos(\omega_C t) + \cos(\omega_M t)\right)^2 \\
&= b^2 + a^2\cos^2(\omega_C t) + \cos^2(\omega_M t) + 2ba\cos(\omega_C t) + 2b\cos(\omega_M t) + 2a\cos(\omega_C t)\cos(\omega_M t) \\
&= b^2 + \frac{a^2}{2} + \frac{a^2}{2}\cos(2\omega_C t) + \frac{1}{2} + \frac{1}{2}\cos(2\omega_M t) + 2ba\cos(\omega_C t) + 2b\cos(\omega_M t) + 2a\cos(\omega_C t)\cos(\omega_M t)
\end{aligned}
$$

This contains constant terms (0 frequency), terms at frequencies $\omega_M, 2\omega_M, 2\omega_C$, plus

$$z(t) = 2ba\cos(\omega_C t) + 2a\cos(\omega_C t)\cos(\omega_M t)$$

Putting $a_0 = 2ba,\ a_0 m_A = 2a$ - hence $m_A = 1/b$ - makes $z(t)$ equal the required DSB-AM signal. Hence a bandpass filter is required with a passband centred on the carrier frequency, and a bandwidth equal to twice the bandwidth of $x(t)$. Provided $\omega_C \gg \omega_M$, this will filter out the unwanted terms.

6. (d) Another common type of DSB-AM demodulator is the "envelope detector", as shown below:



In the positive half-cycles of the AM signal, the capacitor is charged to the peak value of the modulated carrier wave. Between the peaks, the resistor discharges the capacitor. The result is that the capacitor voltage roughly follows the true envelope. The error, which is modulation of the capacitor voltage at the carrier frequency, is easily removed by a simple lowpass filter.

SJG, NGK,
MDM,
July 2001

Paper 6, June 2001, Answers:

1. (b) (i) 0.25 (ii) 0.767sin(t+0.0768) (iii) 1.25+0.767sin(t+π/4+0.0768)
2. (a) Phase margin is approximately 8 degrees, hence system is oscillatory. (b) New PM is approximately 22 degrees.
3. (c)(i) GM=2.8, PM=27 degrees. (ii)0.22 (iii) 2.6 (iv) w>0.34 rad/s.
4. (c) Amplitude of y = $2.01.10^{-4}A$, amplitude of z is 1.002A, amplitude of x=1.002A.
5. (c) (i) 4.9km (ii) 0.344 km.
6. (c) passband frequency range = twice the bandwidth of x. $a_0 = 2ba$. $a_0 m_A = 2a$, hence $m_A = 1/b$