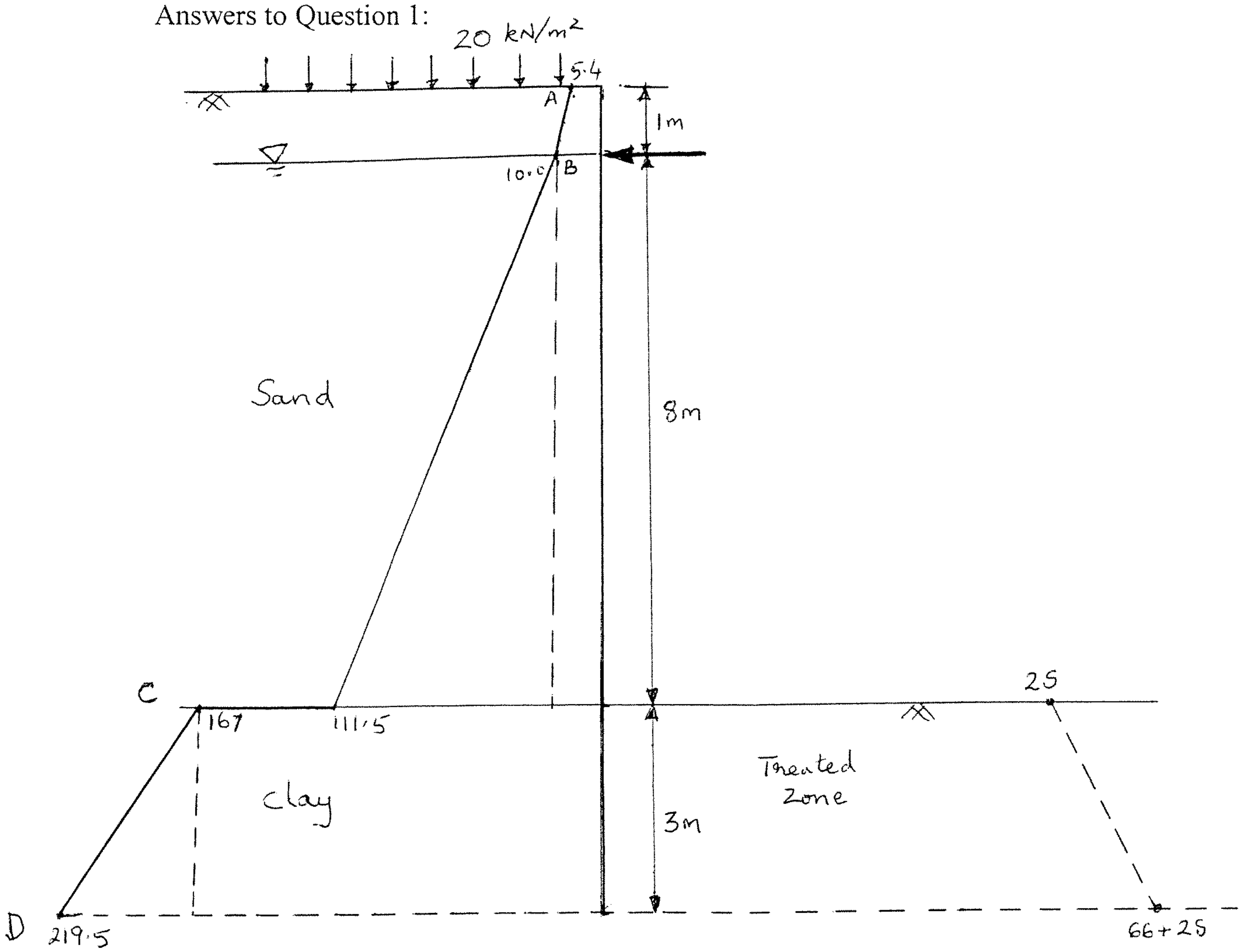
① IB PAPER 8: SELECTED TOPICS – CRIBS 2001.

Part IB Selected Topics - Civil Engineering - June 2001

Answers to Question 1:



a) For Sand $\quad K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'}$ and $\sigma_h' = K_a\,\sigma_v'$

$\phi' = 35°$ $\quad \therefore \quad K_a = \dfrac{1-0.57}{1+0.57} = 0.27$

Total horizontal pressure $\sigma_h = \sigma_h' + u$

where σ_h' = effective horizontal pressure and u = pore water pressure

For clay, assuming undrained behaviour

Total horizontal pressure (active) $\sigma_h = \sigma_v - 2c_u$

where σ_v = total vertical pressure and c_u is the undrained shear strength

At ground surface (Level A):

$\sigma_v = \sigma_v' = 20$ kPa $\Rightarrow \sigma_h = \sigma_h' = K_a\,\sigma_v' = 0.27 \times 20 = \underline{5.4}$ kPa

②

At the water table (level B):

$\sigma_v = \sigma_v' = 20 + 17 \times 1 = 37$ kPa

$\sigma_h = \sigma_h' = K_a \sigma_v' = 0.27 \times 37 = \underline{10.0}$ kPa

At the bottom of the sand layer (level C):

$\sigma_v = 37 + 8 \times 20 = 197$ kPa

Pore pressure $u = 8 \times 10 = 80$ kPa.

$\therefore \sigma_v' = \sigma_v - u = 197 - 80 = 117$ kPa

$\sigma_h' = K_a \sigma_v' = 0.27 \times 117 = \underline{31.6}$ kPa

$\sigma_h = \sigma_h' + u = 31.6 + 80 = \underline{111.6}$ kPa

At the top of clay layer (level C):

$\sigma_v = 197$ kpa (as above)

$\sigma_h = \sigma_v - 2c_u = 197 - 2 \times 15 = \underline{167}$ kPa

At the toe of the wall (level D):

$\sigma_v = 197 + 3 \times 17.5 = 249.5$ kPa

$\sigma_h = \sigma_v - 2c_u = 249.5 - 2 \times 15 = \underline{219.5}$ kPa [10]

1 b) Treated ground in front of the wall has unit weight $\gamma = 22$ kN/m³ and undrained shear strength S kN/m².

Passive total horizontal pressure $\sigma_h = \sigma_v + 2S$

At surface of excavation, $\sigma_v = 0 \quad \therefore \sigma_h = 2S$

At the toe of the wall $\sigma_v = 3 \times 22 = 66$ kPa

$\therefore \sigma_h = 66 + 2S$

Taking moments about prop:

Total active moment (pushing wall out), M_A

$$M_A = 10 \times 8 \times 4 + \tfrac{1}{2}(111.6 - 10)\, 8 \times \tfrac{2}{3} \times 8 + 167 \times 3 \times 9.5 + \tfrac{1}{2}(219.5 - 167) \times 3 \times 10$$

$$= 320 + 2167 + 4760 + 788 = \underline{8035} \text{ kN-m/m}$$

Total passive moment (preventing wall moving out), M_P:

$$M_P = (2S \times 3 \times 9.5) + \tfrac{1}{2} \times 66 \times 3 \times 10 = \underline{57S + 990} \text{ kN-m/m}$$

For equilibrium to be just maintained, $M_p = M_A$

$$57\,S + 990 = 8035$$

$$S = \underline{123.6} \text{ kPa} \qquad [10]$$

[In practice a Factor of Safety of at least 2 would be required, so that $M_p = 2M_a$]
ie $57\,S + 990 = 2 \times 8035 \Rightarrow S = \underline{\underline{264.6}}$ kPa

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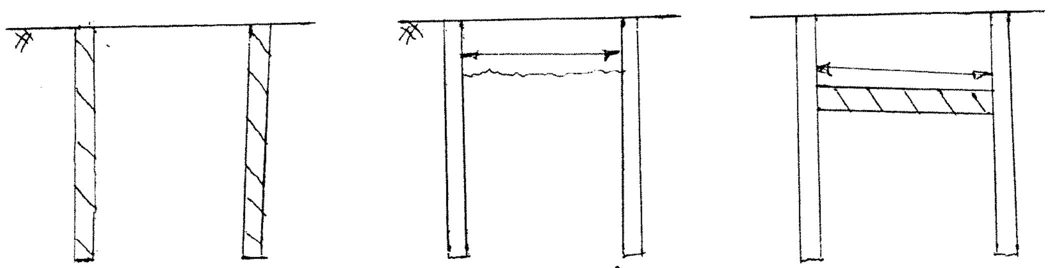
Question 2 Solution :

(a) Temporary <u>sheet piling</u> could be driven to support the ground, and the box constructed within the sheet piles. There could be problems with driving the sheet piles into the coarse dense gravel, and noise and vibration would be a problem (particularly the vibration effects on Building B).

A preferable option would be <u>diaphragm walls</u> or <u>secant bored pile</u> walls on the line of the box walls so that they can be incorporated into the permanent structure. Both types of wall have the advantage of being <u>water tight</u>. (necessary because of the water present in the course sand and gravel strata).

Either a <u>'bottom-up'</u> sequence or a <u>'top-down'</u> sequence could be adopted. 'Bottom-up' involves excavating to the base slab level, propping the walls with temporary props, and constructing the box starting with the base slab and then working upwards. 'Top-down' involves construction of the roof slab of the box first and then working downwards, as illustrated below:

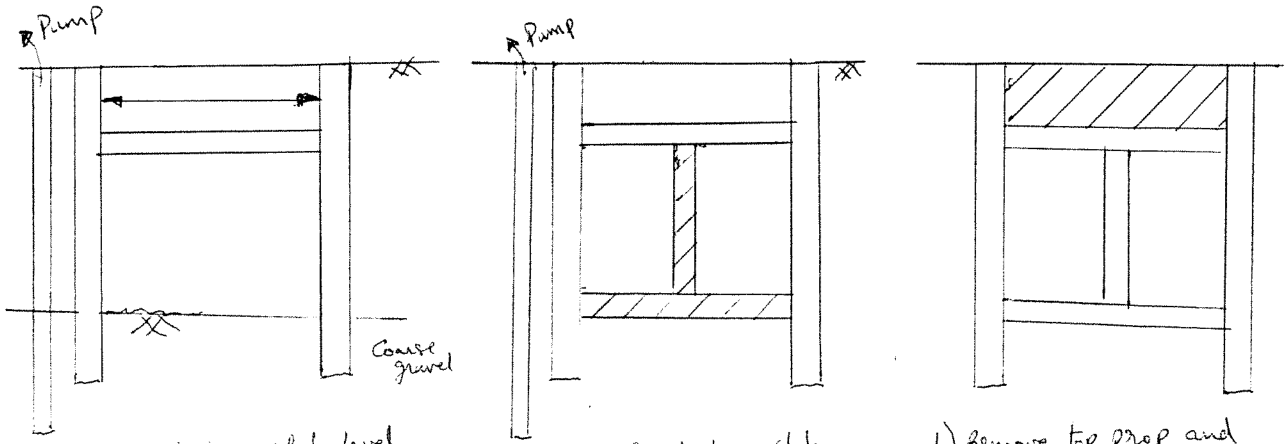


a) Install walls

b) Excavate several metres and install temporary prop

c) Excavate to the underside of the roof and cast roof slab (connecting it to the walls)

④



d) Excavate to base slab level, having first installed the borehole wells to pump and lower water table in gravel [see part (c) of the question]

e) Construct base slab and central wall

f) Remove top prop and backfill, then cease pumping. [8]

2 b) Axes of bored tunnels at a depth of 10 m

Unit weight of soil ≃ 20 kN/m³

Undrained strength of stiff clay ≃ 100 kPa.

∴ stability ratio (without face support) $= \frac{10 \times 20}{100} = 2$

This is very low, which means that the tunnels can be constructed without face support. The method could be open face tunnelling machine + lining segments (precast concrete or spheroidal graphite cast iron), or open face excavation + sprayed concrete linings (NATM - New Austrian Tunnelling Method). [3]

2c) There is a need for the walls to be watertight to prevent water leakage from the sand layer (see sketch (b)) and from the gravel layer. It is essential that the water level in the gravel layer is lowered by pumping before the excavation is taken too close to the top of the gravel (see sketch (d)). The water bearing sand and gravel layers would not effect the tunnelling in the stiff clay (provided tunnel construction is relatively rapid). [3]

2d) The ground movements are unlikely to affect Building A because it is on deep piled foundations (there may be minor horizontal movement of the nearest piles).

Building B could be significantly affected by both vertical and horizontal ground movements. Possible techniques to prevent damage are:

- Installation of tie rods

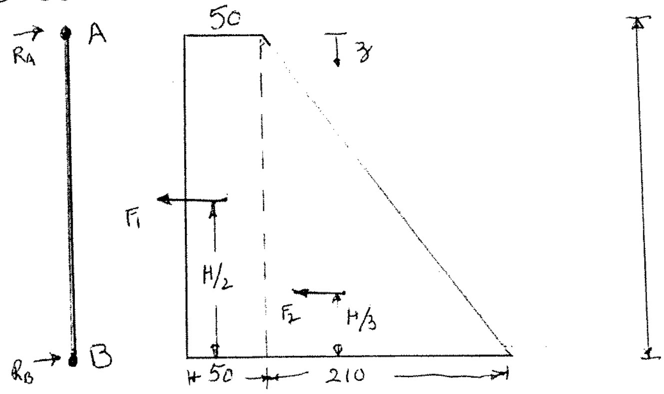
- under pinning the building (or Jacking)
- Grouting of the sand layer beneath the building to create a 'stiff raft'.
- Compensation grouting (grouting from grout tubes - TAM's and monitoring).

[3].

2e) Instrumentation should be installed to monitor:

- Settlement of both buildings
- Settlement of the ground surface
- Horizontal movements of the grounds (inclinometers)
- Horizontal movements of the walls in the case of cut-and-cover option. (inclinometers in the walls)
- Sub surface settlements of the ground (extensometers) in the case of tunnels.

Question 3 Solution:



3(a)

$F_1 = 50 \times 7 = 350$ kN @ H/2

$F_2 = \frac{1}{2} \times 210 \times 7 = 735$ kN @ H/3

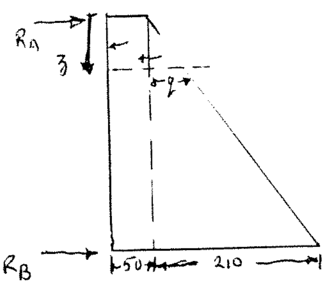
} $F_1 + F_2 = 1085$ kN

Take moments about B;

$R_A \times H = F_1 \times H/2 + F_2 \times H/3 \Rightarrow R_A = \frac{F_1}{2} + \frac{F_2}{3} = \underline{420 \text{ kN}}$

$R_B = F_1 + F_2 - R_A = 1085 - 420 = \underline{665 \text{ kN}}$ [3]

3(b) Method 1 Derive expression for M at z and find maximum from $\frac{dM}{dz} = 0$



At any depth z: $\frac{z}{7} = \frac{q}{210} \Rightarrow z = q \times \frac{7}{210}$ or $q = 30z$

$M_z = 420z - 50 \times z \times \frac{z}{2} - \frac{1}{2} \times z \times 30z \times \frac{z}{3}$

$= 420z - 25z^2 - 5z^3$

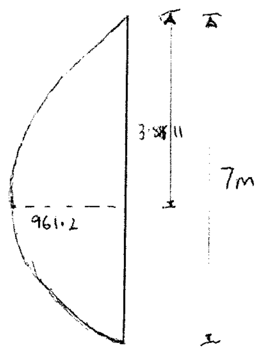
$\frac{dM_z}{dz} = 420 - 50z - 15z^2 = 0$

or $z^2 + 3.33z - 28 = 0$ $\therefore z = \frac{-3.33 \pm \sqrt{3.33^2 + 4 \times 28}}{2}$

$\boxed{z = 3.8811 \text{ m}}$

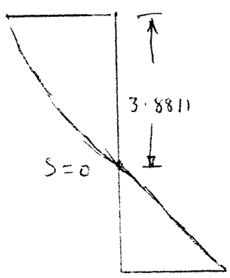
$\therefore M_{max} = 420 \times 3.8811 - 25 \times 3.8811^2 - 5 \times 3.8811^3$

$= \underline{961.2 \text{ kN·m}}$



[5]

Method 2 : Find depth at which shear force = 0 and then sub into equation for moments.



Shear $S_z = 420 - 50z - \frac{1}{2} z \times 30z$

$= 420 - 50z - 15z^2$

$S_z = 0$ when $420 - 50z - 15z^2 = 0$

i.e. $z^2 + 3.33z - 28 = 0$

$z = \dfrac{-3.33 \pm \sqrt{3.33^2 + 4 \times 28}}{2} = \boxed{3.8811}$ as before.

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3 c) From Data sheets $M = 0.15 f_{cu} bd^2$

$d = \sqrt{\dfrac{M}{0.15 f_{cu} b}} = \sqrt{\dfrac{961.2}{0.15 \times 40 \times 1000}} = 400 \text{ mm}$

Allow 50 mm cover hence $t_{wall} = 450$ mm (minimum)
(Probably better to use 500 mm as tw)

Reinforcement Assume N.A at $x = 0.5$

From Data sheets $M = 0.87 f_y A_s d(1 - x/2)$

$\therefore A_s = \dfrac{M}{0.87 f_y d(1 - x/2)} = \dfrac{961.2 \times 10^6}{0.87 \times 460 \times 400 \times (1 - \frac{0.5}{2})} = 8006 \text{ mm}^2$

Check N.A Data sheet gives $x = 2.175 \dfrac{f_y}{f_{cu}} \dfrac{A_s}{bd} \leq 0.5$

$= 2.175 \times \dfrac{460}{40} \times \dfrac{8006}{1000 \times 400} = 0.5$ OK

Bar options :-

	A bar	Spacing	A_s		
ø 32	804	100	8042	> A_{req}	OK
40	1256	150	8373	> A_{req}	OK

Adopt 32ø bars @ 100 mm spacing

Check x $\quad x = 2.175 \times \dfrac{460}{40} \times \dfrac{8042}{1000 \times 400} = 0.503 \simeq \underline{0.5}$ OK

(7)

3 d) Curtailment is useful for reducing the amount of steel used in areas where the BM is reduced. When determining whether or not to curtail steel one must consider

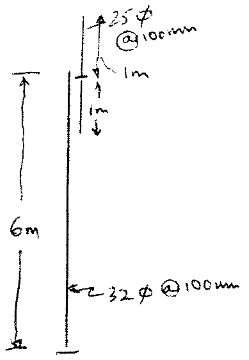
i) The standard length of r/c bars - usually 6m; It is expensive to have to cut bars + splice bars so try to curtail at standard lengths.

ii) Anchorage length - must allow for anchorage bond length when determining the location of the effective area of steel. Approximately 40 ϕ can be used here.

In this question $40\phi = 40 \times 32 = 1280$ mm so the bars must extend this far beyond the location at which they are required. For the wall of 7m height it will be necessary to splice the bars anyway, unless 7m lengths are available.

One common option is to alternate bars so that there is a halving of A_{st} at 1m & 6m in this case. However after allowing for development lengths this would mean steel was effective from 2280 mm - 4720 m. ie 2440 mm central zone. Would need to check moment capacity at this location is adequate.

Here it seems more practical to use smaller bars for top zone



Say 25ϕ @ 100 mm spacing = 4909 mm²

$L_B = 40 \times 25 = 1000$ mm

check $x = 2.175 \times \frac{460}{40} \times \frac{4909}{1000 \times 400} = 0.307$

$M_{cap} = 0.87 \times 460 \times 4909 \times 400 (1 - 0.307) \times 10^{-6}$

$= 545$ kN-m

$M_{3\,(3=1)} = 420 - 50 - 15 = 355$ kN-m $< M_{cap}$ OK

[3]

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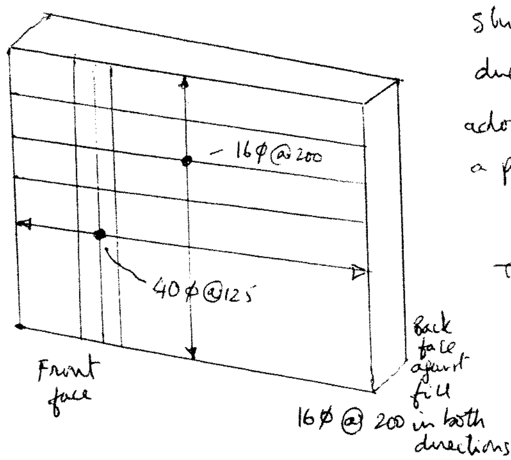
3 e) In theory there is no need for any horizontal bending r/f in the front face of this wall since the loading on the wall is assumed constant along the tunnel length, hence there is no transverse bending set up. However, in practice, some minimum steel should always be provided to inhibit cracking due to temperature changes and shrinkage. Typically adopt a minimum of 500 mm²/m although codes give a percentage (0.13% for $f_y = 460$ MPa)

ie 8ϕ @100 mm or 16ϕ @200 mm. Adopt 16ϕ @ 200 mm

This should be placed vertically and horizontally on back face and horizontally on the front face (inside the main flexural reinforcement)



[3]

3 b) Shear Capacity

$$v_c = 0.68 \left(\frac{100 A_s}{bd}\right)^{0.33} \left(\frac{400}{d}\right)^{0.25} \text{ MPa}$$

$$= 0.68 \times \left(\frac{100 \times 5042}{400 \times 1000}\right)^{0.33} \left(\frac{400}{400}\right)^{0.25}$$

$$= \underline{0.86 \text{ MPa}}$$

Applied shear stress: $v = \frac{V}{bd} = \frac{R_b}{bd} = \frac{665 \times 10^3}{1000 \times 400} = \underline{\underline{1.7 \text{ MPa}}}$

$v > v_c \Rightarrow$ Require shear links

Modify design Either 1) increase wall thickness
or 2) Put in shear links to take
$v - v_c = 1.7 - 0.9 = 0.8$ MPa

Preferably increase wall thickness as placing shear links is very expensive & labour intensive.

SECTION B: MECHANICAL ENGINEERING

4\. a) Outer structure: strong, stiff material to provide strength and stiffness in bending and torsion. Must also provide resistance to knocks, protect the core from moisture, and serve as a base for decorative transfers, metal edges, and binding points.

Inner core: acts as a spacer for the outer structure, and provides damping. Need not be as strong or stiff as the outer structure, but must provide resistance against crushing, especially under the binding area.

b) From data book, p17:

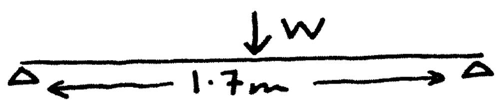
Material	ρ	**E**
CFRP	1.5 Mg/m^3	50 GPa
Spruce (parallel to grain)	0.35 Mg/m^3	6 GPa

N.B. 1) Don't use uni-ply CFRP, as structural properties are required in all directions.
2) Choose E_{Spruce} parallel to grain, as the grain runs *along* the ski.

Then mass of ski $= 1.95\{.08 \times .018 \times \rho_{CFRP} + .072 \times .014\,(\rho_{Spruce} - \rho_{CFRP})\}$
$= 1.95$ kg (to 3 s.f.)

$$EI = \frac{0.08 \times 0.018^3}{12} E_{CFRP} + \frac{0.072 \times 0.014^3}{12}\left(E_{Spruce} - E_{CFRP}\right) = 1.22 \text{kNm}^2 \text{ (to 3 s.f.)}$$

c)

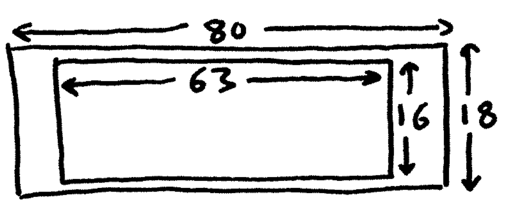


$$\delta = \frac{W\ell^3}{48EI} = \frac{45 \times 9.81 \times 1.7^3}{48 \times 1.22 \times 10^3} = 0.037\text{m}$$

i.e. a deflection of 37mm.

Net deflection from load is unaffected by either self-weight or camber, assuming the ski behaves as a linearly elastic structure.

d) If no change in materials is allowed, the stiffness can be reduced by reducing the amount of CFRP at the top and bottom of the ski. Weight can be maintained (if required) by increasing the amount of CFRP at the sides of the ski, to compensate, e.g:



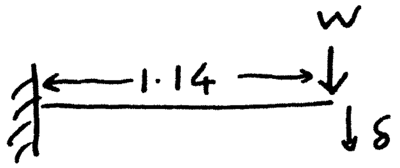
$M = 1.95$ kg (as before)
$E = 998$ Nm^2

If the core can be changed, choose a material with a higher density, to allow weight to be maintained without adding CFRP to the sidewalls, as above. (It does not matter if the value of E is slightly higher, as the core only has a small effect on the overall bending stiffness of the ski.)

The new material would need to be crush-resistant in the binding area, and to have good damping and fatigue properties. A coefficient of thermal expansion similar to that of CFRP would also prevent the ski from de-laminating when its temperature changes.

5. a) Using an equivalent mass-spring system to model the fundamental vibration mode of a ski allows the use of the response curves for single-degree-of-freedom damped systems, as shown in the Mechanics data Book – and also simplifies the modelling of tuned-mass dampers on the ski.

b) Forebody length $= \frac{1.5}{2.5} \times 1.9 = 1.14\text{m}$



$$\delta = \frac{W\ell^3}{3EI}$$

$$EI = \frac{W}{\delta} \times \frac{\ell^3}{3} = 1800 \times 0.494 = 889\text{Nm}^2$$

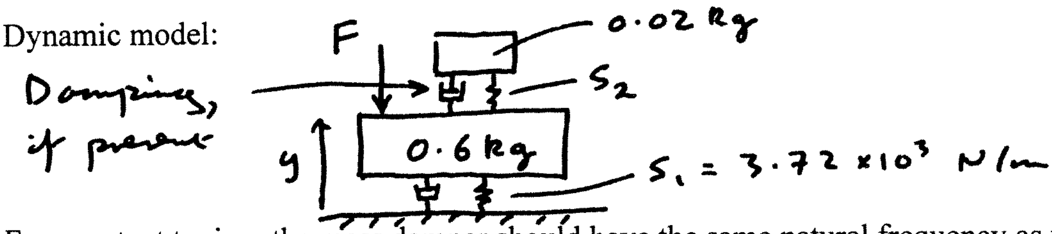
c) Actual mass of forebody $= \frac{1.5}{2.5} \times 2 = 1.2\text{kg}$

$$\omega_n = 3.52\sqrt{\frac{889}{1.2 \times 1.14^3}} = 78.7 \text{ rad/s} = \underline{12.5 \text{ Hz}}$$

For equivalent mass, $\omega_n = \sqrt{\frac{s}{m}} \Rightarrow s = m\omega_n^2$

Equiv. mass = 0.6 kg, so $\underline{s = 3.72 \text{ kN/m}}$

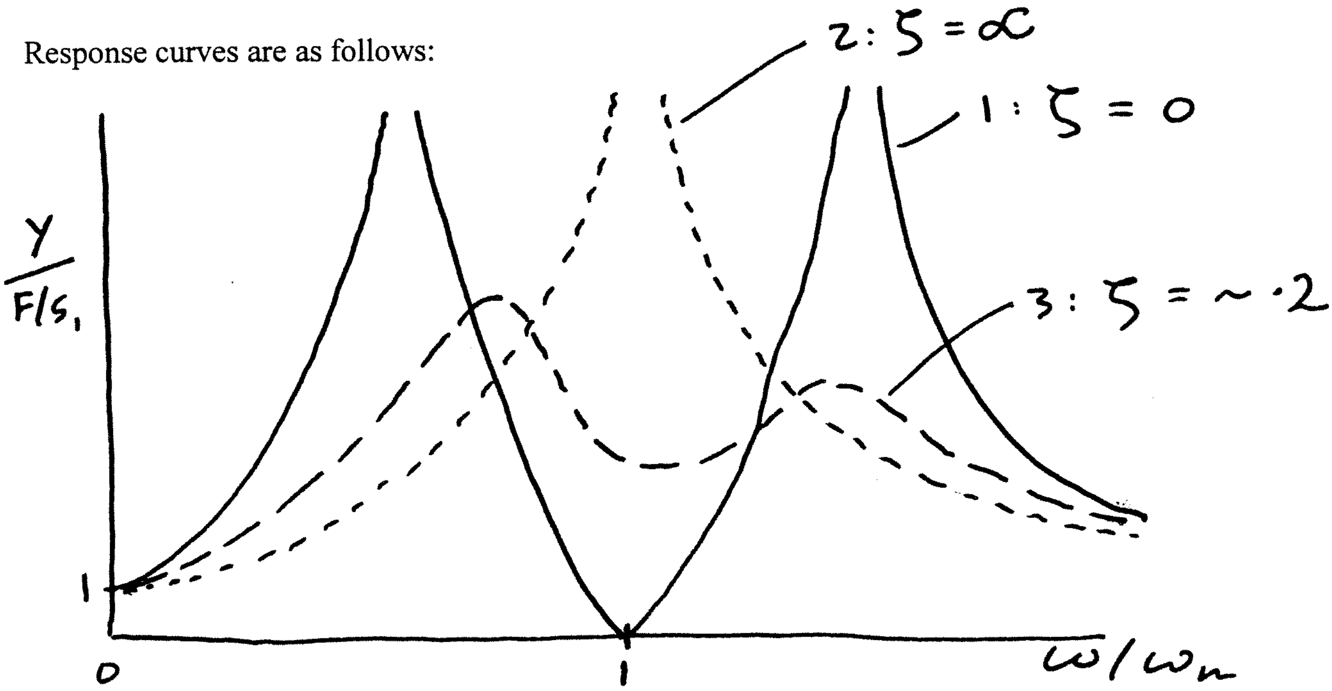
d) Dynamic model:



For constant tuning, the mass damper should have the same natural frequency as the original system, i.e.

$$s_2 = 0.02 \times 78.7^2 = \underline{124 \text{ N/m}} \text{ (to 3 s.f.)}$$

e) Response curves are as follows:



Optimal tuning gives the mass damper a slightly different natural frequency, to make the two 'humps' in curve (3) more equal in height.

6. A competent answer to this question would cover the following points:

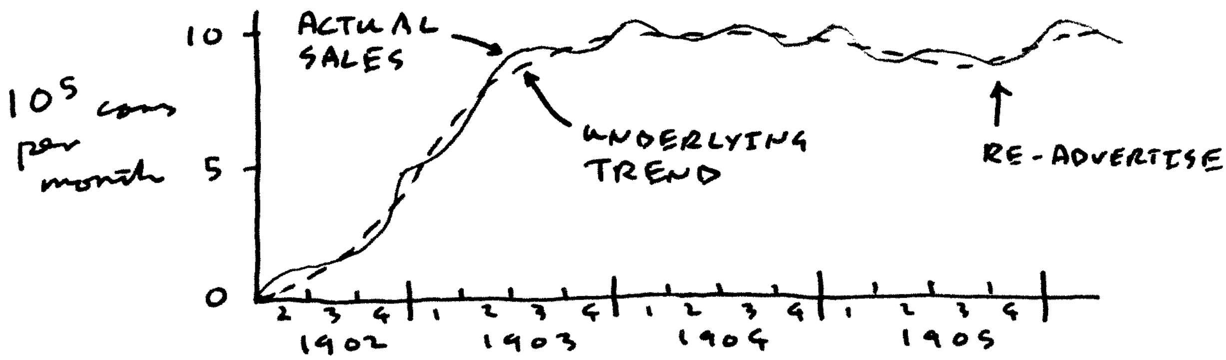
a) Following BS7000, the principal stages in a product's development cycle are:

- Initial trigger
- Product planning – to produce a project proposal
- Feasibility study – to produce a design brief
- Design – to produce a product definition
- Development – to refine the product definition
- Production
- Distribution
- Consumption (including promotion)
- Disposal (including questions of recycling)

The project has now completed the 'design' phase, and will move forward to development. Methods for production and distribution must now be planned, and the refined product definition should take into account the issues raised by these plans, as well as those of consumption and disposal / recycling. Patents should be applied for, on the new widget.

b) Sales will hopefully show a string initial growth after launch, which will weaken as the product reaches maturity. Eventual sales may reach the level of the non-widget beer (though we must expect the sales of that product to weaken). The sales graph will also show seasonal variations, with higher sales in summer and around Christmas. A final decline in sales should not occur for perhaps a decade, but further advertising campaigns may be necessary to prevent sales from weakening when new competitors are marketed.

Predicted sales pattern:



c) Eight important design specifications (excluding production aspects) are:

Geometry:	Must fit inside existing cans
Materials:	All materials must be food-approved
Safety:	Must be safe to open under all conditions Must not contain parts which could be swallowed
Cost:	Additional cost should not exceed about 12p per can
Operation:	Beer must taste as good as a draught beer
Legal:	Widget must not infringe existing patents
Life:	Cans should have the same shelf life as non-widget cans

6. d) Six important issues which must be addressed when planning the Manufacturing Strategy are:

1 Will the widgets be produced in-house, or bought out?

2 Where will the new can-filling line(s) be? How will the raw materials (beer, cans, widgets, nitrogen &c.) be transported to this site, and how will the product be distributed afterwards? What space, machinery and personnel will be needed, and will the operators need special training?

3 Is the proposed production method likely to infringe existing patents? Will licenses need to be negotiated? How will this affect profit margins?

4 What kind of inspection and quality control will be needed?

5 How will the new widget be inserted into the cans? How will the cans and widgets be sterilised, and kept sterile until the beer has been added and the can has been closed?

6 How will the product be marketed? What 'image' should it be given in the advertising campaign?

3)

Section C.

(Q7) a) Turbofan engine produces thrust through a small velocity change to a large mass flow. This gives a high propulsive efficiency as the waste exit K.E. is lower.

Higher bypass ratio results in a higher propulsive efficiency and a smaller high velocity core exit flow so can be much quieter.

Bypass ratio is limited by blade stress, tip speed being fixed. Also do not want too low shaft speed as this makes the LP Turbine design difficult.

b) $P_{O23} = 1.65 \times P_{O2} = 1.65 \times 31\ \text{kPa} = \underline{\underline{51.15\ \text{kPa}}}$

$T_{O23}^{ISEN} = T_{O2}\left(\frac{P_{O23}}{P_{O2}}\right)^{\frac{\gamma-1}{\gamma}} = 245\ (1.65)^{1/3.5} = 282.69\ \text{K}$

$\eta_{ISEN} = \frac{\Delta T_{O\ IDEAL}}{\Delta T_{O\ ACTUAL}} \Rightarrow T_{O23} - T_{O2} = (T_{O23}^{ISEN} - T_{O2})/\eta$

$T_{O23} = T_{O2} + (T_{O23}^{ISEN} - T_{O2})/\eta = 245 + (282.69 - 245)/0.88 = \underline{\underline{287.83\ \text{K}}}$

c) $T_{jb} = T_{O23}\left(\frac{P_{jb}}{P_{O23}}\right)^{\frac{\gamma-1}{\gamma}} = 287.83\left(\frac{20}{51.15}\right)^{1/3.5} = 220.10\ \text{K}$

$V_{jb}^2 = 2C_p(T_{O23} - T_{jb}) = 2 \times 1010\,(287.83 - 220.10)$ $\quad \underline{\underline{V_{jb} = 369.9\ \text{m/s}}}$

$\left[M_{jb} = V_{jb}/\sqrt{\gamma R T_{jb}} = 369.9/\sqrt{1.4 \cdot 287 \cdot 220.10} = 1.244 \right]$

d) $T_{O23} = 287.83\ \text{K}$ $\quad T_{O3} = 751\ \text{K}$

$P_{O23} = 51.15\ \text{kPa}$ $\quad P_{O3} = 22 \times P_{O23} = 22 \times 51.15 = 1125.3\ \text{kPa}$

$T_{O4} = 1500\ \text{K}$ $\quad P_{O4} = P_{O3} = 1125.3\ \text{kPa}$

HP TURBINE WORK = CORE COMPRESSOR WORK $\Rightarrow C_p\Delta T_o = C_p\Delta T_o \Rightarrow \Delta T_o = \Delta T_o$

$T_{O4} - T_{O45} = T_{O3} - T_{O23}$

$T_{O45} = T_{O4} - (T_{O3} - T_{O23}) = 1500 - (751 - 287.83) = \underline{\underline{1036.83\ \text{K}}}$

$\eta_{ISEN} = \frac{\Delta T_{O\ ACTUAL}}{\Delta T_{O\ IDEAL}} \Rightarrow T_{O4} - T_{O45}^{ISEN} = (T_{O4} - T_{O45})/\eta$

$T_{O45}^{ISEN} = T_{O4} - (T_{O4} - T_{O45})/\eta = 1500 - (1500 - 1036.83)/0.90 = 985.37\ \text{K}$

$P_{O45} = P_{O4}\left(\frac{T_{O45}^{ISEN}}{T_{O4}}\right)^{\frac{\gamma}{\gamma-1}} = 1125.3\left(\frac{985.37}{1500}\right)^{3.5} = \underline{\underline{258.63\ \text{kPa}}}$

e) WORK BALANCE $\quad \dot{m}_{TOTAL}\, C_p\, \Delta T_{o\,FAN} = \dot{m}_{CORE}\, C_p\, \Delta T_{o\,LPT}$

$\Delta T_{O\,LPT} = \frac{\dot{m}_{TOTAL}}{\dot{m}_{CORE}} \Delta T_{O\,FAN} = (1 + BPR)(T_{O23} - T_{O2})$

$T_{O45} - T_{O5} = (1 + BPR)(T_{O23} - T_{O2})$

$T_{O5} = T_{O45} - (1 + BPR)(T_{O23} - T_{O2})$

$T_{O5} = 1036.83 - (1 + 8.5)(287.83 - 245) = \underline{\underline{629.9\ \text{K}}}$

(Q7 cont e....)

$$\eta_{ISEN} = \frac{\Delta T_{0\,ACTUAL}}{\Delta T_{0\,IDEAL}} \Rightarrow T_{045} - T_{05}^{ISEN} = (T_{045} - T_{05})/\eta$$

$$T_{05}^{ISEN} = T_{045} - (T_{045} - T_{05})/\eta = 1036.83 - (1036.83 - 631.95)/0.92$$

$$T_{05}^{ISEN} = 594.5 \text{ K}$$

$$P_{05} = P_{045}\left(\frac{T_{05}^{ISEN}}{T_{045}}\right)^{\frac{\gamma}{\gamma-1}} = 260.60\left(\frac{594.5}{1038.83}\right)^{3.5} = \underline{\underline{36.90 \text{ kPa}}}$$

f) $$T_5 = T_{05}\left(\frac{P_5}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}} = 631.95\left(\frac{20}{36.9}\right)^{1/3.5} = 528.69 \text{ K}$$

$$V_5^2 = 2c_p(T_{05} - T_5) = 2 \times 1010\,(631.95 - 528.69)$$

$$\underline{\underline{V_5 = 452 \text{ m/s}}}$$

$$\left[M_5 = V_5/\sqrt{\gamma R T_5} = 452 \,/\sqrt{1.4 \times 287 \times 528.69} = 0.98 \right]$$

(15)

Q8 a) As the fan has a much larger diameter its rotational speed must be much lower to maintain a similar tip mach number. Hence a two-shaft arrangement allows both fan and core compressor to run at similar tip mach numbers.

b) The stage loading in the compressor must be lower (≈ 0.4) than in the turbine (≈ 2.0). Hence need more stages to achieve the necessary work input. Stage loading must be lower in the compression process as the blade boundary layers must flow against an adverse pressure gradient across the stage.

c) $V_\theta^{REL} = V_\theta - U \Rightarrow \dfrac{V_\theta^{REL}}{V_x} = \dfrac{V_\theta}{V_x} - \dfrac{U}{V_x} \Rightarrow \tan\alpha^{REL} = \tan\alpha - \dfrac{1}{\phi}$

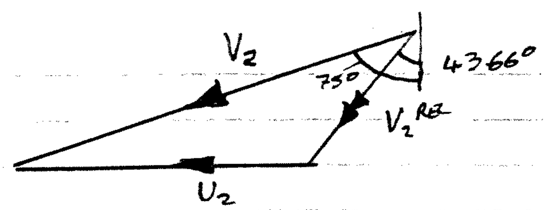
ROTOR INLET $\alpha_2 = 75^\circ$

$\tan\alpha_2^{REL} = \tan 75^\circ - \dfrac{1}{0.36} \Rightarrow \underline{\underline{\alpha_2^{REL} = 43.66^\circ}}$ (+ve ⇒ CO-ROT)

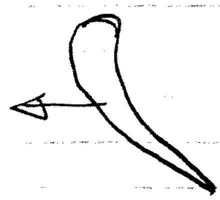
ROTOR EXIT $\alpha_3^{REL} = -73^\circ$

$\tan\alpha_3 = \tan(-73^\circ) + \dfrac{1}{0.36} \Rightarrow \underline{\underline{\alpha_3 = -26.25^\circ}}$ (-ve ⇒ COUNTER-ROT)

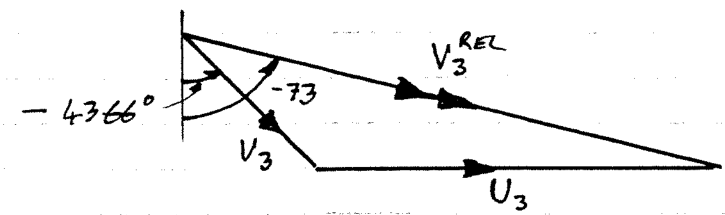
INLET :



ROTOR



ROTOR OUTLET :



d) $V_x = 210$ m/s

$V_{\theta 2} = V_x \tan\alpha_2 = 210 \tan 75^\circ = \underline{783.73 \text{ m/s}}$

$V_{\theta 3} = V_x \tan\alpha_3 = 210 \tan(-26.25^\circ) = \underline{-103.56 \text{ m/s}}$

$U = 210/0.36 = 583.33$ m/s

$W_x = U(V_{\theta 2} - V_{\theta 3}) = 583.33(783.73 - -103.56) = \underline{\underline{517.583 \text{ kJ/kg}}}$

$\Delta T_0 = W_x / c_p = 517.583/1.010 = 512.46$ K

$T_{03} = T_{02} - \Delta T_0 = 1850 - 512.46 = \underline{\underline{1337.54 \text{ K}}}$

(Q8 cont)

e) $\eta = \dfrac{\Delta T_{0\,ACTUAL}}{\Delta T_{0\,IDEAL}}$ $\qquad \Delta T_{0\,IDEAL} = \Delta T_{0\,ACTUAL} / \eta$

$T_{02} - T_{03}^{ISEN} = 512.46 / 0.88 = 582.34 \text{ K}$

$T_{03}^{ISEN} = 1850 - 512.46/0.88 = 1267.66 \text{ K}$

$P_{03} = P_{01} \left(\dfrac{T_{03}^{ISEN}}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} = 1 \times 10^5 \left(\dfrac{1267.66}{1850} \right)^{3.5} = \underline{\underline{266.323 \text{ kPa}}}$

f) $V_{x3} = 210 \text{ m/s}$
$V_{\theta 3} = -103.56 \text{ m/s}$ $\Big\} \; V_3 = \sqrt{V_{x3}^2 + V_{\theta 3}^2} = 234.15 \text{ m/s}$

$T_3 = T_{03} - V_3^2/2c_p = 1337.54 - 234.15^2/2 \times 1010 = \underline{\underline{1310.40 \text{ K}}}$

$P_3 = P_{03} \left(\dfrac{T_3}{T_{03}} \right)^{\frac{\gamma}{\gamma-1}} = 266.323 \left(\dfrac{1310.40}{1337.54} \right)^{3.5} = \underline{\underline{247.88 \text{ kPa}}}$

$\rho_3 = \dfrac{P_3}{RT_3} = \dfrac{247.88 \times 10^3}{287 \times 1310.40} = 0.6591 \text{ kg/m}^3$

$\dot{m} = \rho_3 v_{x3} A_3$

$A_3 = \dfrac{\dot{m}}{\rho_3 v_{x3}} = \dfrac{50}{0.6591 \times 210} = \underline{\underline{0.3612 \text{ m}^2}}$

Q9

a) $T_1 = T_{01} / (1 + \frac{\gamma-1}{2} M_1^2) = 248 / (1 + 0.2 \cdot 0.85^2) = \underline{\underline{216.69\ K}}$

$P_1 = P_{01} / (1 + \frac{\gamma-1}{2} M_1^2)^{\frac{\gamma}{\gamma-1}} = 31 / (1 + 0.2 \cdot 0.85^2)^{3.5} = \underline{\underline{19.33\ kPa}}$

$v_1 = M_1 \sqrt{\gamma R T_1} = 0.85 \sqrt{1.4 \times 287 \times 216.69} = \underline{\underline{250.81\ m/s}}$

} CRUISE

b) $\frac{\dot{m} \sqrt{c_p T_{01}}}{P_{01} A_{REF}} = f_n \left(\gamma, \frac{T_{04}}{T_{01}} \right)$ & $\frac{P_{03}}{P_{01}} = f_n \left(\gamma, \frac{T_{04}}{T_{01}} \right)$ ASSUME CHOKE NOZZLE.

SAME P_{03}/P_{01} $\Rightarrow$ $\left.\frac{T_{04}}{T_{01}}\right|_{STAT} = \left.\frac{T_{04}}{T_{01}}\right|_{CRUISE}$

STAT: $T_{01} = 288\ K$, $P_{01} = 101\ kPa$, $\dot{m} = 10\ kg/s$, $T_{04} = 1800\ K$

$T_{04}|_{CRU} = T_{01}|_{CRU} \left.\frac{T_{04}}{T_{01}}\right|_{STAT} = 248 \cdot \frac{1800}{288} = \underline{\underline{1550\ K}}$ @CRUISE

SAME CONDITIONS $\Rightarrow$ $\left.\frac{\dot{m} \sqrt{c_p T_{01}}}{P_{01} A_{REF}}\right|_{STAT} = \left.\frac{\dot{m} \sqrt{c_p T_{01}}}{P_{01} A_{REF}}\right|_{CRUISE}$

$\Rightarrow \dot{m}|_{CRU} = \dot{m}|_{STAT} \left.\frac{\sqrt{T_{01}}}{P_{01}}\right|_{STAT} \times \left.\frac{P_{01}}{\sqrt{T_{01}}}\right|_{CRU} = 10 \sqrt{\frac{288}{248}} \times \frac{31}{101} = \underline{\underline{3.308\ kg/s}}$ @CRUISE

c) $\frac{T_{04}}{T_{05}} = 1.27$

$T_{05}|_{CRU} = T_{04}|_{CRU} \times \frac{1}{1.27} = \frac{1550}{1.27} = \underline{\underline{1220.47\ K}}$ @CRUISE

CHOKED NOZZLE $\Rightarrow$ $\frac{\dot{m} \sqrt{c_p T_{05}}}{A_5 P_{05}} = 1.281$

$P_{05}|_{CRUISE} = \left.\frac{\dot{m} \sqrt{c_p T_{05}}}{A_5 \cdot 1.281}\right|_{CRUISE} = \frac{3.308 \sqrt{1010 \cdot 1220.47}}{0.016 \cdot 1.281} = \underline{\underline{179.19\ kPa}}$ @CRUISE

$M_{NOZZLE} = 1.0$

$T_{NOZZLE} = T_{05} / (1 + \frac{\gamma-1}{2} M_{NOZZ}^2) = 1220.47 / (1 + 0.2 \cdot 1^2) = 1017.06\ K$ @CRUISE

$V_{NOZZLE} = M \sqrt{\gamma R T} = 1.0 \sqrt{1.4 \cdot 287 \times 1017.06} = \underline{\underline{639.26\ m/s}}$ @CRUISE

$P_{NOZZLE} = P_{05} / (1 + \frac{\gamma-1}{2} M_{NOZZ}^2)^{3.5} = 179.19 / (1 + 0.2 \times 1^2)^{3.5} = \underline{\underline{94.66\ kPa}}$ @CRUISE

d) Simple convergent nozzle so $M_{NOZZLE} = 1$ (as choked). However then have un-contained expansion downstream to a higher Mach.

e) GROSS THRUST $= \dot{m} V_{NOZZLE} + (P_{NOZZLE} - P_{AMB}) A_{NOZZLE}$

$$\frac{\text{GROSS THRUST}}{P_{01}} = \underbrace{\left(\frac{\dot{m} V_{NOZZLE}}{P_{01}} + \frac{P_{NOZZLE} \cdot A_{NOZZLE}}{P_{01}} \right)}_{f_n(\gamma,\ T_{04}/T_{01})} - \underbrace{\left(\frac{P_{AMB}}{P_{01}} \cdot A_{NOZZLE} \right)}_{f_n(\text{flight speed})}$$

(Q9 cont)

e) cont)

$$\text{GROSS THRUST} = \dot{m} V_{NOZZLE} + (P_{NOZZ} - P_{AMB}) A_{NOZZ}$$
$$= 3.308 \times 639.26 + (94.66 - 19.33) \times 10^3 \times 0.016$$
$$= 3319.95 \text{ N}$$

GROSS THRUST @ CRUISE = 3.320 kN

f) $$\frac{\text{GROSS THRUST}}{P_{01}} = \underbrace{\frac{\dot{m} V_{NOZZ} + P_{NOZZ} \cdot A_{NOZZ}}{P_{01}}} - \frac{P_{AMB}}{P_{01}} \cdot A_{NOZZLE}$$

$$\frac{\text{IMPULSE}}{P_{01}} = \frac{3.308 \times 639.26 + 94.66 \times 10^3 \times 0.016}{31 \times 10^3}$$

$$\frac{\text{IMPULSE}}{P_{01}} = 0.11707 \quad \text{@ CRUISE \& STATIONARY}$$

$$\text{GROSS THRUST}\Big|_{STAT} = P_{01}\Big|_{STAT} \cdot \left(\frac{\text{IMPULSE}}{P_{01}}\right) - P_{AMB} \cdot A_{NOZZLE}$$
$$= 101 \times 10^3 \times 0.11707 - 101 \times 10^3 \cdot 0.016$$

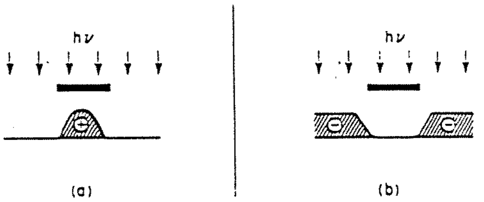
GROSS THRUST = 10.208 kN @ STATIONARY

9) Solution Q10

(a)

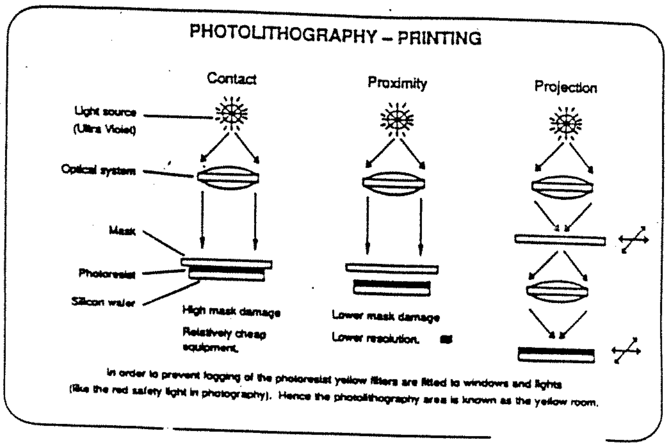
POSITIVE AND NEGATIVE RESIST

As mentioned previously photoresist is a radiation sensistive compound. They can be calssified as **positive** or **negative,** depending upon how they respond to radiation.



cross section of the resist image after development
(a) Positive photoresist. (b) Negative photoresist.

For positive resists the exposed regions become more soluble and thus more easily removed in the development process. The net result is that the patterns formed in the positive resist are the same as those on the mask. For negative resists the exposed regions become less soluble and the patterns formed in the negative resist are the reverse of the mask patterns (see above figures).



PROXIMITY/CONTACT PRINTING

is characterised by getting the mask as close as possible to the surface of the wafer to minimise errors due to dispersion of the light beam and vibration.

In some cases the mask may actually touch the wafer (contact printing). Such machines are relatively cheap but both wafer and mask will suffer some mechanical damage during exposure.

(b)

TYPES OF LITHOGRAPHY AND THEIR CAPABILITIES

FOUR PARAMETERS DETERMINE THE LITHOGRAPHIC PERFORMANCE

RESOLUTION---- THE MINIMUM FEATURE SIZE THAT CAN BE PRODUCED REPEATABLY

REGISTRATION---- HOW CLOSELY SUCCESSIVE MASK LEVELS CAN BE OVERLAID

THROUGHPUT---- NUMBER OF WAFERS THAT CAN BE EXPOSED IN 1 HOUR.

DEPTH-OF-FOCUS (FIELD)---- THE REGION OVER WHICH THE IMAGE REMAINS IN FOCUS. IF VARIATIONS IN HEIGHT OF A SURFACE ARE GREATER THAN THE DEPTH OF FIELD THEN GET DEGRADATION OF THE PATTERN.

THE MAJORITY OF IC PRODUCTION IS CARRIED OUT USING OPTICAL SYSTEMS THAT OPERATE IN THE UV

PROJECTION PRINTING

This process removes the mask from the surface of the wafer and projects the image from the mask onto the wafer. In order to provide high quality optics the mask and wafer are moved together through a well collimated beam or spot of light. The machine cost is therefore higher than that of a proximity type. The wafer and mask are not damaged because they are not close to one another and this obviously improves mask life.

The optical elements in most modern projection printers are so perfect that their imaging characteristics are dominated by diffraction effects rather than by lens aberration - diffraction limited systems. The resolution of a diffraction limited printer is roughly 0.5 (λ/NA) where NA is the numerical aperture of the projection optics and λ is the wavelength. The depth of focus of the system is approx. given by $\lambda/(NA)^2$. i.e. a high resolution (large NA) is achieved at the expense of depth of focus.

Current production line systems are capable of 0.3 micron resolution, 0.1 micron registration and a throughput of approx. 20 off 150 mm diameter wafers per hour.

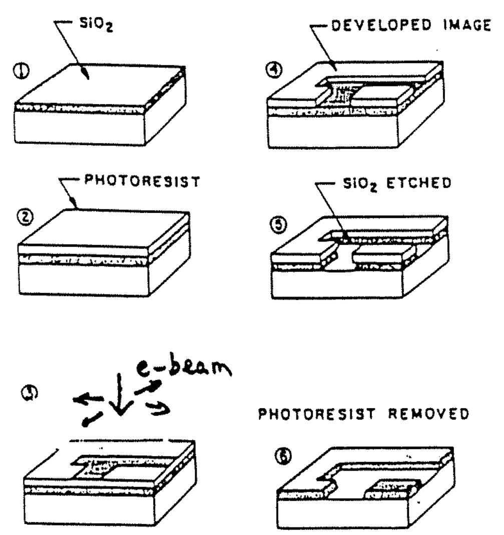
Far UV research systems are capable of approx. 0.25 micron resolution and registration of approx. 0.1 microns.

Electron beam systems - they are currently able to operate at resolutions down to 20 nm but will only have a minimal throughput and therefore these would only be used for very specialist applications. For sytems with resolutions of approx. 0.25 micron a throughput of approx 0.5 wafer per hour would be possible. E-beam systems are generally used to produce the photomasks but can be used directly write on wafers.

X-ray lithography systems have approx. 0.13 mircrons resolution and 0.13 microns registration but are still a long way from the production line.

ELECTRON BEAM WRITING:

Electron beam direct writing of a wafer is a technique where the photoresist is exposed by a beam of electrons that is steered to produce the pattern needed. No mask is used. Consequently the machine is very versatile and multiple designs can be mixed on one wafer. Used for fast turn round of new designs etc but no good for mass production.

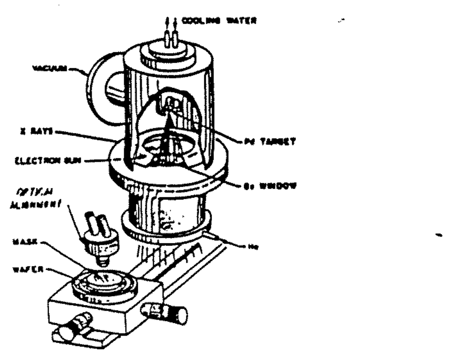


Details of the lithographic transfer process.

by dirt with low atomic number, dirt on the mask does not print as a defective pattern on the resist. Masks however are still a problem.

X-RAY LITHOGRAPHY:

This is an extension of optical proximity printing in which the exposing wavelength is in the 4 - 50 A range. Because x-ray optical elements are not yet available, x-ray lithography is limited to shadow printing. System is as shown below.



The mask and wafer are separated by about 40 microns. The full wafer is exposed in about 1 minute. The primary reason for developing x-ray lithography is the possibility of achieving high resolution and high through put at the same time. Also since x-rays are not absorbed

(21) c Step + Repeat

The step and repeat camera projects the reticle onto a portion of the plate's surface and shrinks the image to 1/10 the size of the pattern on the reticle. The plate is moved so that another exposure is made adjacent to the first one, and the process is repeated, stepping across and down the plate until the whole surface is covered. Usually a rectangular area is completely covered with the pattern (but see the section on process control and plug dice), but in some cases the array will be made circular (like a wafer).

~~Also~~ depends on resolution and depth of focus available etc.

d) Assuming a vector scan:

The max area of exposure / sec

$= 40 \times 10^6 \times 0.05 \times 10^{-6} \times 0.05 \times 10^{-6}$

$= 10^{-7}\ m^2/s$

Total area exposed $= 20\%\ \pi r^2$

$= \frac{20}{100} \times \pi \times (0.1)^2 = 0.0063 m^2$

$\Rightarrow$ Time taken $= \frac{0.0063}{10^{-7}} = 63000 s$

$\Rightarrow$ no hrs $= \frac{63000}{3600} = 17.4$ hrs.

This is a very long time to maintain stable environment and demonstrates the impracticality of e-beam writing for most applications.

(22)

PAPER 8 SECTION D 2001

Crib

Q 11. [a] Show tube of charge h high and area A. Enclosed charge is
Q = N.e.Ah

Gauss theorem.
D.A = N.e.Ah

E = Neh/ε

$V_g = -\int E.dh$

$\mathbf{V_g = Neh^2/[2\varepsilon]}$

[b] $N = 2V_g\varepsilon/eh^2 = 2x0.5x0.8.10^{-10}/1.6.10^{-19}\ x10^{-6}\ x10^{-6} = \mathbf{5.10^{20}\ m^{-3}}$ (donors)

[c] $v = \mu E$, $v = L/t$. $V_d = E.L$ $\quad \therefore \mathbf{L^2 = \mu V_d t}$

$L^2 = 0.4x2x5.10^{-12} = 4.10^{-12}$. $\mathbf{L = 2.10^{-6}\ m.}$

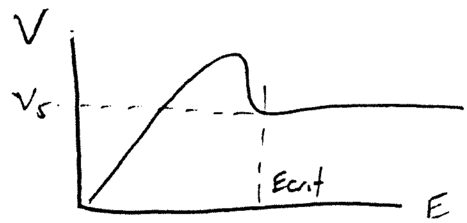
[d] $W = 50.L = 10^{-4}$ m.

$R = \rho L/A = L/[Ne\mu Wh] = 2.10^{-6} / [5.10^{20}x1.6.10^{-19}\ x\ 0.4\ x\ 10^{-4}\ x\ 10^{-6}] = \mathbf{625\ ohm.m}$

[e] $C' = Q'/V_g = Neh/V_g = Neh/[Neh^2/2\varepsilon] = 2\varepsilon/h$

$C' = 2\ x0.8.10^{-10}/10^{-6} = 1.6.10^{-4}\ F.m^{-2}$.

Q 12/



GaAs.

b] Scattering limited velocity is the maximum carrier velocity. It is caused by intense scattering by phonons which stops carriers accelerating to larger volecities.

C] $U = 0.5m^*v_s^2 = 0.5x\ 0.1.\ 9.11.10^{-31}\ 10^5\ .10^5 = 4.55.10^{-22}$ J , $4.55.10^{-22}/1.6.10^{-19} =$
$\mathbf{2.85\ .10^{-3}\ eV}$

D] $m^*_e v_e^2/m^*_h v_h^2 = 1$. $V_h = v_e/\sqrt{3} = \mathbf{5.77 10^4\ m.s^{-1}}$.

E] $\mathbf{v_s} = \mu E \qquad E = 10^5/0.1 = 10^6$ V/m. $L = V/E = 1.5/10^6 = \mathbf{1.5.10^{-6}\ m.}$

F] $t = L/v_s = 1.5.10^{-6}/10^5 = \mathbf{15.10^{-12}\ s.}$

G] $J = Nev_s = 2.10^{21}\ x\ 1.6.10^{-19}\ x\ 10^5 = \mathbf{3.2.10^7\ A/m^2}$.

23

Section E

Part I Elective: Information Engineering
"An Architecture for Wearable Computing"
Answer Sheet (2 Questions Set by PMW)

Q13 **1.** (a) *Total of 7 marks.*

(i) The final part (and first mark) is easy. The mechanism used is the *system call* or *software trap* (either wording is acceptable).

For the other two marks, they should have said something sensible about the differences between user mode and supervisor mode, but should include something about the level of access to system resources. For example:

> User Mode: Mode in which the users application code runs. As there may be multiple user applications running as separate processes, user mode is constrained to prevent interference with other processes. This means it is limited to not being able to access hardware resources, or to have unlimited access to memory space.
>
> Supervisor Mode: Mode in which the operating system runs. Low-level access to system hardware is allowed as have full access to memory space.

Clearly, some of the features of supervisor mode could also be worded as limitations of user mode.

(ii) One mark each (up to a maximum of four marks) for any of the following:

1. Controls/limits usage of system resources by user processes (allocates resources to user processes and reclaims them when finished)
2. Responds to events associated with these resources on behalf of the user to maximise system performance.
3. Hides complexity (controlling devices too complex for the user). Provides useful services to the user applications
4. Abstraction: Provides independence from specific device characteristics
5. Want to create virtual high-level devices (files etc) on top of raw device
6. Prevent interference when more than one user (protection)
7. Ensure fairness when more than one user (scheduling of devices)

As before, they do not have to get the exact wording (which is why a number of alternatives have been given). If they show an understanding of the general idea of any of these, then they should get the mark.

(ii) Marks are approximately one mark for identifying the mechanism and one mark for the explanation (giving two marks per peripheral device):

1. Keyboard entry: An interrupt peripheral. User input is orders-of-magnitude slower than the CPU speed, so polling would be very wasteful. Similarly, as each key press generally generates an action, DMA is not appropriate at all.
2. Sampling an A/D converter: The clue is in the question with the word 'sampling'. As you generally want to just check the instantaneous value, this is best implemented as a polled peripheral. DMA is only appropriate if you wanted to autonomously log the value over time (and given the slow changes, you probably wouldn't even choose DMA then).
3. Disk Interface: We are dealing with blocks of data from a fairly fast peripheral. Therefore, the most sensible option is DMA. A polled mechanism would be a heavy drain on CPU resources, while interrupt driven mechanism would generate a lot of interrupt traffic and may not be able to provide low enough latency.
4. RS232 Serial Interface: The peripheral is quite slow (but faster than a keyboard). DMA isn't really appropriate as it is a character-based device. Polling may be used, but may miss characters and will waste a lot of CPU resources in waiting for device status to change. Interrupts are the best mechanism.
   If they do suggest polling, you could deduct one mark (unless they are a bit clever and suggest putting a FIFO in the peripheral – though this isn't covered by the course).

An example of where the more creative candidate may differ from my suggestion is the battery voltage. They may say that you can provide a threshold you could generate an interrupt when it crosses the threshold. If anyone does do this, I'd even be tempted to go as far as to give a bonus mark!

~~4~~. (a) *Total of 11 marks.*

14

For each of the following OS features, I've put down key phrases. If they mention enough of them, then they should get the marks.

(i) Process is a program plus the resources it needs to run it
Multiprocessing system has many processes active simultaneously
Need mechanism to spawn new processes and switch between them
Can cooperatively yield to another processor (wait()).
Otherwise, switch when time-slice has expired (pre-emptive)
Switch when blocked waiting for IO to complete (better CPU usage)
Scheduling can be round-robin or based on priorities
Higher priority to IO bound than CPU bound (better throughput)
(Up to 3 marks)

(ii) Provide protection between independent process memory spaces
Want to appear that each process has entire address space available
Don't want entire program in memory at the same time (swapping)
Allow process to use more memory than physically available (paging)
Implements page table to translate logical to physical addresses
Pages also have protection bits (read/write, read only etc)
Need to handle page replacement (various replacement mechanisms)
Dirty bit used to mark when page written to and needs storing back
Change amount of pages allocated per process depending on miss rate
(Up to 3 marks)

(iii) Independence from specific hardware device characteristics
Simpler interface for the user (direct access complex for users)
Create virtual higher-level devices (streams, files etc)
Prevent access interference if multiple users
Provide fair access to resources when competing processes
(Up to 2 marks)

Again, if they get three or more of the hardware features listed below, the get the three remaining marks.

The features provided by the processor include:

Interrupts to respond to peripheral events
User/supervisor modes to control resource access
Software traps/system calls to switch between modes
Hardware IO registers for peripheral control

Supporting hardware includes:

Timer interrupts are required for pre-emptive scheduling
Memory Management Unit (MMU) for memory protection
MMU also handles logical to physical address translation
Translation Look-aside Buffer (TLB) speeds up page translation

(c) *Total of 3 marks.*

This is covered at the end of the course, so should be fresh in their minds. If they mention three or more of them, they get the marks.

Power features:

Switch CPU off when waiting for peripheral interrupt or DMA
Shutdown sections of the CPU not being used (e.g. multiply unit)
Peripherals powered down when not in use (device management)
Shutdown device when not in use for a while (wakeup button)

Reduce IO bandwidth or transmit power
Limit access to power intensive operations (e.g. FLASH writing)

Scale processor speed based on current load

Not dealt with in the course, but they may mention if they have used a laptop:

Reduce screen brightness when not on mains
Step CPU speed down when not on mains

I guess these are equally valid, but I would suggest either of them counts as one mark only (they shouldn't get two marks for mentioning both of these).

(27)

Q15

a Infra-red does not go through walls but is vulnerable to interference from light sources (sunlight, fluorescent). Cheap transmission and reception devices are available.
Radio penetrates walls but can work quite fast, vulnerable to radio interference, cheap devices not yet available.
Ultrasonic intrinsically slow.

b To minimise energy usage pulse position modulation is best. Pulse can be shaped, cheap timing devices are available to measure time to next pulse, many encodings of the meaning of the time differences between pulses possible, binary probably best.

c Collision avoidance by synchronisation, collision resolution by protocol. Synchronisation by using a unique marker in the modulation, otherwise a special/unique bit pattern. Use synchronisation to arrange transmissions are not at the same time.
Resolution depends on whether system is master-slave or peer-to-peer. For master-slave master receives bids and gives out transmission permissions. For peer-to-peer a distributed algorithm is required with each unit observing passing traffic and making a decision when to transmit.
A simple version is to randomise the (re)transmission interval, for example by modifying it depending on how much light is falling on the device.

d Packets can contain a source and destination address in which case the system is connectionless. May take longer to process but is self-contained. Otherwise the packet can have solely a connection identifier where each communication path has been set up in advance. This is called connection-oriented and can be good for making fast decisions when handling packets.

e Using infra-red is not likely to give high speeds, particularly if the system is diffuse and the signal needs to be decoded even after it bounces from a wall. So maximum effective data rate will be a few kilobits/sec.

f If receivers are associated with rooms it is extremely unlikely that a receiver will pick up transmissions from another room. Hence there will be very few false positives and the quality of the location information will be high. However clothes can obscure the transmitter.