CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

EXAMINATION PAPERS

2001

ENGINEERING TRIPOS: 1B

ENGINEERING TRIPOS PART IB

Monday 4 June 2001 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 The quick-return mechanism shown diagrammatically in Fig. 1 is driven by the crank OB . The slider C is constrained to move in a horizontal straight line. In the position shown in Fig. 1, the slider C has a speed of 800 mms^{-1} and an acceleration of 4000 mms^{-2} , both to the right. Your answers are to be **drawn on the sheet provided and attached to your script**.

(a) Locate the instantaneous centres of the rigid link ABC and the slider at B on the space diagram on side 1 of the sheet provided. [6]

(b) Draw a velocity diagram on side 1 of the sheet provided. By taking scale measurements from the diagrams determine the relative sliding velocities at A and B , and the angular velocities of the links ABC and OB . A suitable scale for the velocity diagram is 1 mm to represent 10 mms^{-1} . [6]

(c) Draw an acceleration diagram on side 2 of the sheet provided and by taking scale measurements from the diagrams determine the relative sliding acceleration at B , and the angular accelerations of the links ABC and OB . A suitable scale for the acceleration diagram is 1mm to represent 50 mms^{-2} . [8]

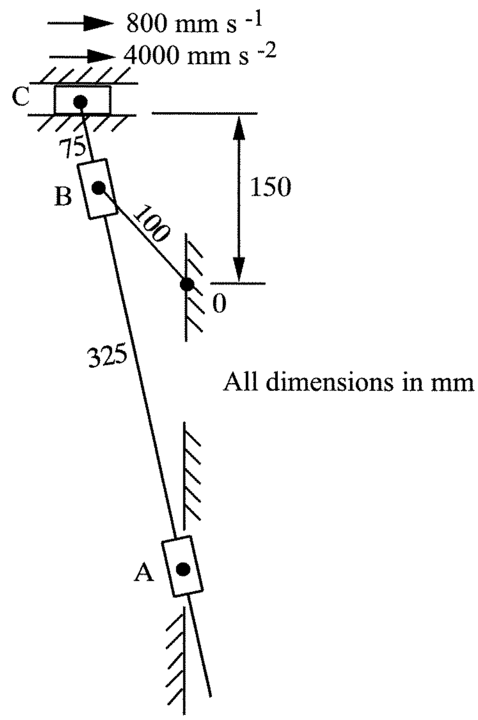

Fig. 1

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2 Figure 2 shows a thin disc of radius a that can spin in a vertical plane at the end of an arm OA of radius R. The arm rotates in a horizontal plane about a vertical axis through O. Three mutually perpendicular unit vectors are shown in Fig. 2. Both $\mathbf{e}_r$ and $\mathbf{e}_\theta$ lie in a horizontal plane, with $\mathbf{e}_r$ always pointing in the direction $\underline{\mathrm{OA}}$. Unit vector $\mathbf{k}$ always points vertically upwards. The arm rotates about its vertical axis with an angular velocity of $\dot{\theta}\mathbf{k}$ and an angular acceleration of $\ddot{\theta}\,\mathbf{k}$.

(a) The unit vectors rotate with an angular velocity of $\dot{\theta}\mathbf{k}$. By considering a small change in position of the unit vectors in a short interval of time, prove that $\dfrac{d\mathbf{e}_r}{dt} = \dot{\theta}\,\mathbf{e}_\theta$. [3]

(b) If the disc is held stationary *relative* to the rotating arm, determine expressions for the velocity and acceleration of a point B on the rim of the disc, positioned at an angle of ϕ from the vertical as shown in Fig. 2. [3]

(c) If the disc is now spun *relative* to the rotating arm with an angular velocity of $\dot{\phi}\,\mathbf{e}_\theta$ and an angular acceleration of $\ddot{\phi}\,\mathbf{e}_\theta$, determine expressions for the *absolute* angular velocity and the *absolute* angular acceleration of the disc. [4]

(d) Using the expressions on page 2 of the Mechanics Data Book, or otherwise, determine expressions for the absolute velocity and absolute acceleration of point B in the position shown in Fig. 2, with the arm rotating and the disc spinning as shown in Fig. 2. [10]

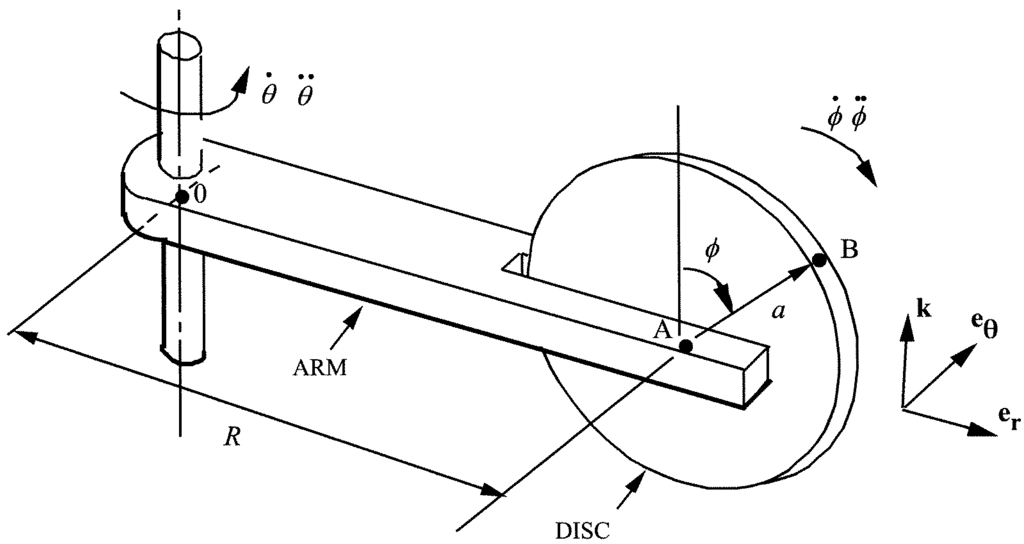

Fig. 2

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3 The mechanism shown in Fig. 3 lies in a *vertical* plane, so gravitational forces must be included. The vertical crank AB of length l is driven by a torque T such that its angular velocity ω is constant in the direction shown. The coupler BC of length $2l$ is horizontal. The link DC makes an angle of 45° to the horizontal and has a mass m attached to its mid point E.

(a) By drawing a velocity and acceleration diagram for the mechanism, determine expressions in terms of l and ω for the angular velocities of BC and DC, and the velocity and acceleration of point E. Suggested scales are $l\omega$ = 50 mm for the velocity diagram, and $l\omega^2$ = 50 mm for the acceleration diagram. [6]

(b) If all the bars are light and all the joints are frictionless, determine an expression, in terms of m, l, ω and gravitational acceleration, g, for the torque T required to drive the crank at the instant shown in Fig. 3 [4]

(c) The bar BC is now replaced by a uniform bar with a mass of m. Determine an expression for the *increase* in the torque required to drive the crank. [6]

(d) Friction couples of Q are introduced at joints B, C and D. Determine an expression for the further *increase* in the torque required to drive the crank. [4]

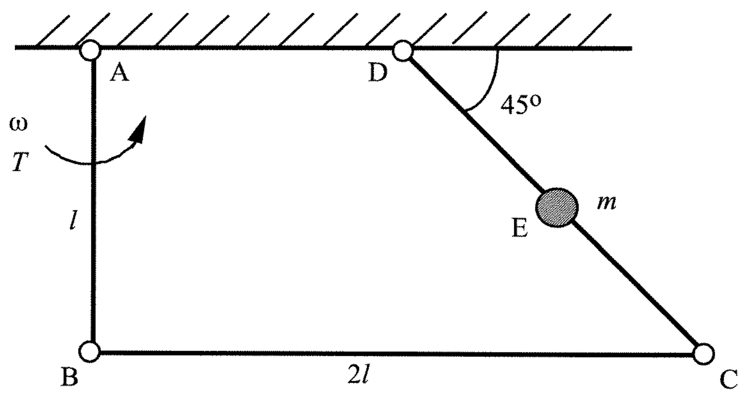

Fig. 3

SECTION B

4 A thin, uniform beam BC of length $2l$ and mass m is suspended in equilibrium by two inextensible light wires AB and CD as shown in Fig. 4. Wire CD breaks suddenly.

(a) Just after the wire breaks, the initial vertical and horizontal accelerations of the centre of gravity of the beam G are defined by a_v and a_h respectively, and the initial angular acceleration of the beam is defined by β . By considering the initial acceleration of the beam, show that $a_v = a_h + l\,\beta$. [3]

(b) Determine expressions for β and the tension in the wire AB just after the break in terms of m , l and the gravitational acceleration g . [6]

(c) Are there shear forces and bending moments in the beam at B and C just after the wire breaks? If there are, what are their magnitudes? [3]

(d) By considering the loading on a small element of the beam at a distance x from G , determine expressions for the shear force and bending moment along the beam just after the wire breaks. Use the sign conventions for shear force and bending moment specified in the Structural Mechanics Data Book (p. 4). [8]

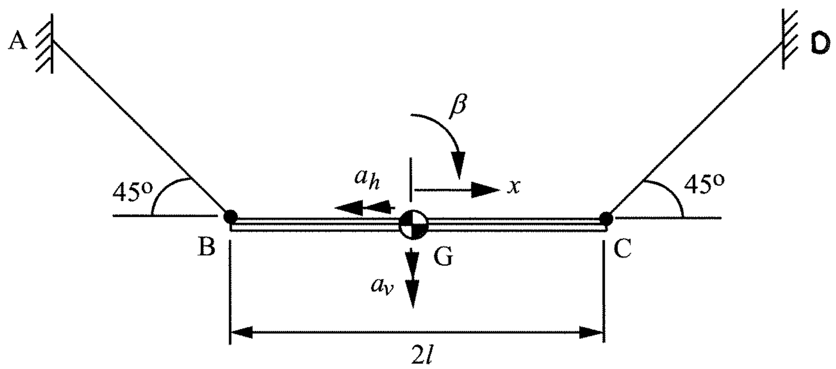

Fig. 4

5 (a) Explain what you understand by a linear impulse. Explain the principle of the conservation of the moment of momentum. [4]

(b) Two solid uniform discs A and B are mounted on thin, collinear, light shafts, which can turn freely in frictionless bearings, and are shown diagrammatically in Fig. 5. These two shafts can be connected by a light clutch C . Disc A has a radius of 30 mm and mass of 0.5 kg, and disc B has a radius of 20 mm and a mass of 0.25 kg. With the clutch disengaged, disc A is spun up to an angular speed of 2000 rpm while disc B remains stationary. The clutch is then engaged rapidly and the two discs eventually reach the same angular speed. What is the magnitude in rpm of this common angular speed and how much energy is lost? [4]

(c) Would your answers to part (b) be different if the clutch had been engaged slowly? Explain the reason for your answer. [2]

(d) A third, solid uniform disc D , mounted on a thin, light shaft that can turn freely in frictionless bearings, is brought into frictional contact with disc B , as shown diagrammatically in Fig. 5. Disc D has a radius of 50 mm and mass of 1.25 kg. Disc A is again spun up to an angular speed of 2000 rpm while discs B and D remain stationary. The clutch is then engaged. When slipping of the clutch ceases and there is no further slipping between discs B and D , what is the angular speed of disc D in rpm? [10]

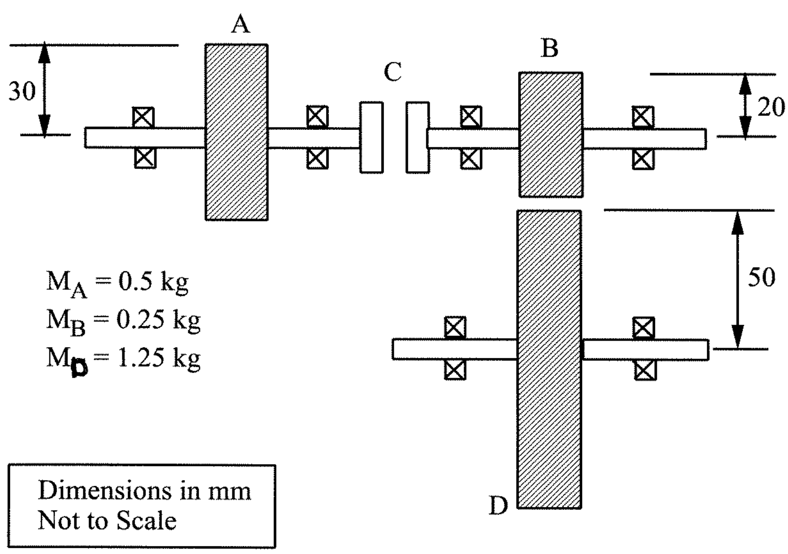

Fig. 5

6 A shaft of 1.2 m is supported in two bearings, P and Q, 1 m apart, see Fig. 6. The shaft carries three pulleys, which are fixed to the shaft, one at each end and one 0.4 m from the left hand bearing, P . The end pulleys, A and C , are out of balance to the extent of 0.018 kg m and 0.030 kg m respectively, and the middle pulley B is out of balance by 0.024 kg m.

(a) Distinguish briefly between statically and dynamically balanced rotating shafts. [2]

(b) Find the angular positions, relative to pulley A, between the out of balance masses of pulleys B and C so as to provide static balance. [6]

(c) Find the dynamic load on each of the bearings when the statically balanced shaft rotates at 500 rpm. [6]

(d) You are given two balance masses of magnitude 0.1 kg that are to be added to pulleys A and C to give dynamic balance while maintaining the conditions of static balance. Find the radius and the angular positions where each balance weight should be added. [6]

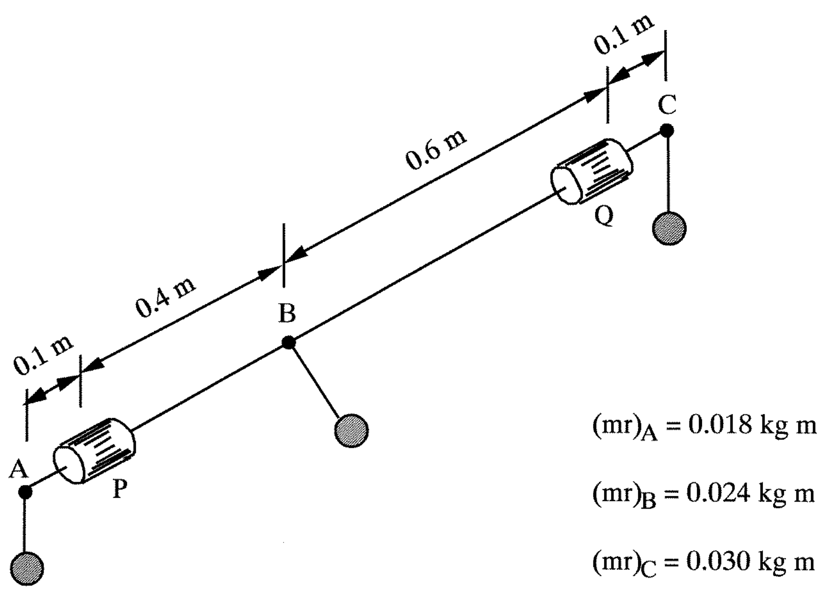

Fig. 6

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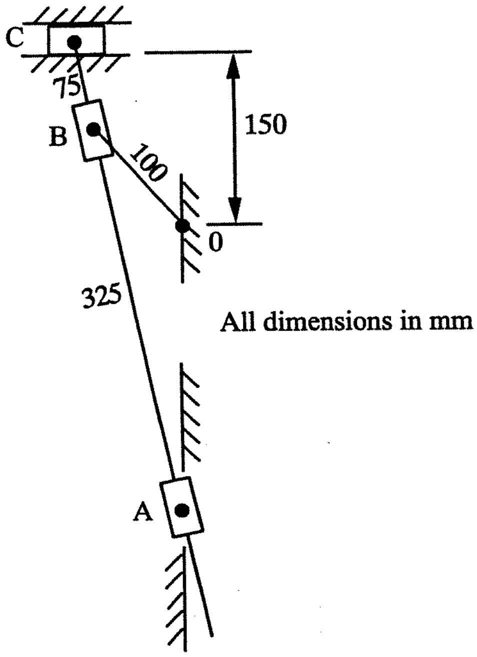
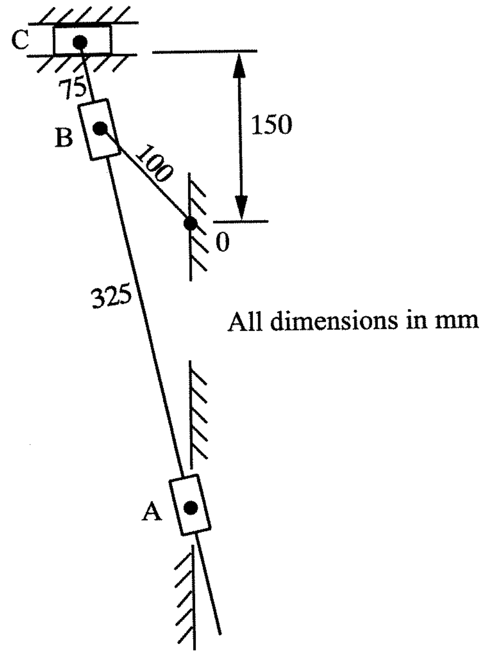
ENGINEERING TRIPOS PART IB Paper 1 MECHANICS Question 1

This sheet is to be attached to your script.



VELOCITY DIAGRAM



ACCELERATION DIAGRAM

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