ENGINEERING TRIPOS PART IB

Monday 4 June 2001 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 The steel cantilever beam AB shown in Fig. 1(a) is 30 m long and fixed rigidly to a wall at A. The cross-section is constant over its entire length and forms an hexagonal hollow box-beam as shown in Fig. 1(b). A couple T = 5000 kNm acting about the longitudinal axis of the cantilever and a vertical concentrated load P = 500 kN are applied at the end section, B. The self-weight of the cantilever beam must also be included in any calculations.

(a) Calculate the flexural rigidity, EI, (i.e. bending stiffness) of this cantilever about the minor axis of bending, XX. [3]

(b) Plot the bending moment, shear force and torque along the cantilever AB and calculate the value of each at the support, A, and at C, half-way along the cantilever. [6]

(c) Compute the maximum longitudinal bending stress in the cantilever at the support at A and state where this will be located on the cross-section. [2]

(d) At the support, A, what would be the maximum longitudinal shear stress at R, located in the centre of the top flange of the beam? [4]

(e) Calculate the rotation angle of the end section at B about its longitudinal axis. Hence find the vertical displacement of point S due to the combined effects of self-weight, concentrated load P, and couple T. [5]

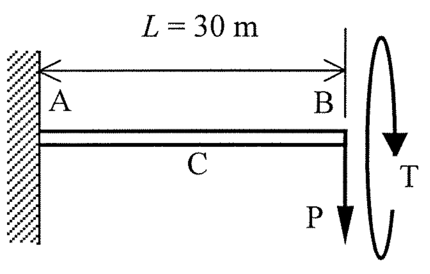


Fig. 1(a)

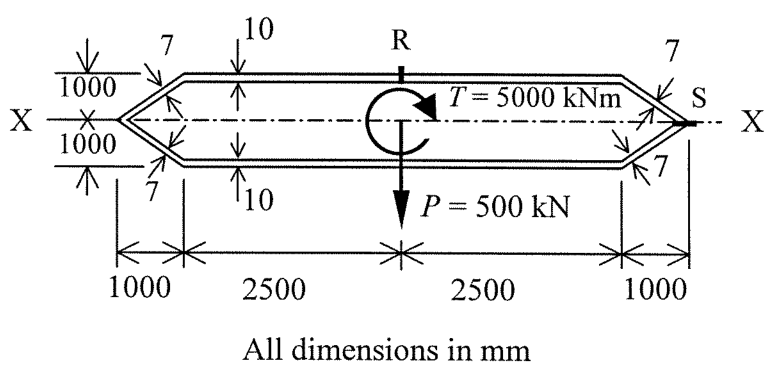


Fig. 1(b)

2 The rectangular reinforced concrete balcony slab ABCD shown in Fig. 2(a) has fully fixed supports along two sides AB and CD; is simply supported along BC; and has a free edge along AD. The balcony is required to carry a concentrated load of magnitude P, at E, midway between A and D. The slab is reinforced so that its moment capacity in sagging and hogging anywhere in the slab is m per unit width. The self-weight of the slab may be ignored. Two compatible failure mechanisms have been postulated as shown in Fig. 2(a) and Fig. 2(b).

(a) For the failure mechanism shown in Fig. 2(a) show that the collapse load of the slab in terms of the moment capacity m is given by $P = 6m$. [4]

(b) The alternative failure mechanism shown in Fig. 2(b) is now considered. The position of the load remains fixed at E whereas the positions of nodes F and G are defined by the parameter α. Find the least upper bound estimate of collapse load P in terms of moment capacity m for this mechanism. [6]

(c) Based on (a) and (b) above what is your best estimate of the collapse load, P of this slab? [2]

(d) By consideration of your result in part (b) for the failure mechanism shown in Fig. 2(b), what would be the effect on the collapse load if the length of the freely supported edge BC was varied from $4L$? [5]

(e) The reinforced concrete slab is replaced by a steel plate of uniform thickness, t = 10 mm. The yield stress of the steel is σ_y = 350 N/mm^2. What would be your best estimate of the collapse load (in kN)? [3]

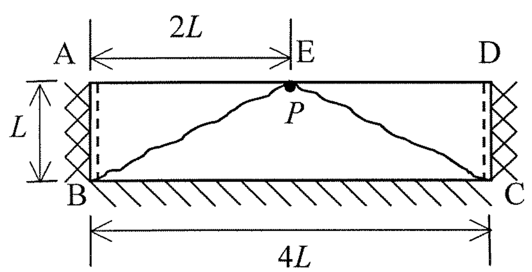


Fig. 2(a)

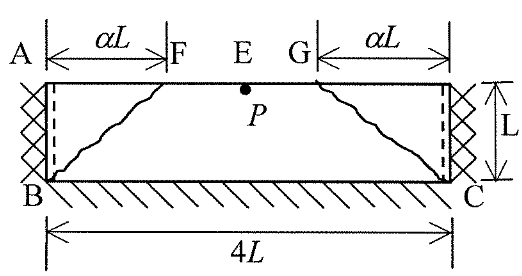
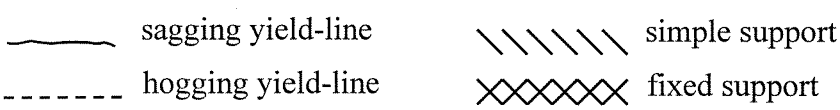


Fig. 2(b)



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3 The structure shown in Fig. 3(a) is constructed using a steel beam which runs horizontally over a span, L, from A to B. The beam is made from Grade 43 steel ($\sigma_y = 245$ N/mm^2) and has the cross-section shown in Fig. 3(b). The plastic moment capacity of the section is assumed to be constant and equal to M_p along its entire length from A to B. A point load of magnitude W is applied vertically at C where distance $a > b$. Self-weight may be ignored.

(a) Calculate M_p for this beam. [4]

(b) Find the optimum *lower bound* estimate for the collapse load of the beam. [6]

(c) A factor of safety of 2 is required under normal working conditions. Find the maximum value of concentrated load, W, that can be safely applied if the span length $L = 6$ m and $a = 4$ m. [2]

(d) Perform an *upper bound* analysis using the configuration of span length and load position given in part (c) above to provide an alternative estimate of collapse load. Comment on the relationship between your lower and upper bound estimates for the collapse load of this beam. [5]

(e) It is found after an inspection that the support at A has settled vertically by 50 mm relative to the support at B. Without calculation explain the effect this would have on the estimated collapse load of this beam. [3]

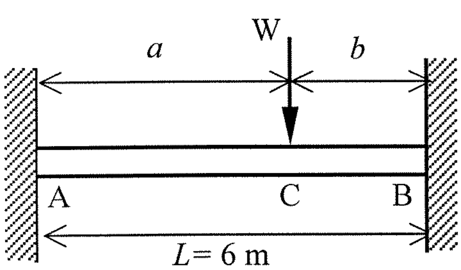
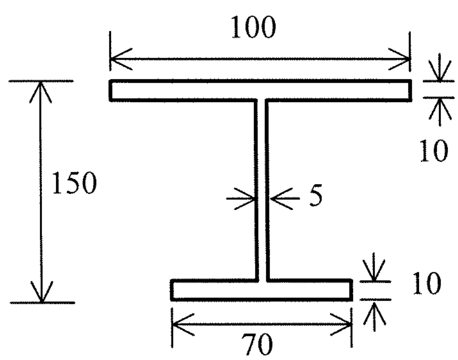


Fig. 3(a)



All dimensions in mm

Fig. 3(b)

SECTION B

4 Fig. 4 shows a two-dimensional, square frame consisting of four straight linear-elastic bars with bending stiffness, EI, and coefficient of thermal expansion, α, *rigidly jointed* at the corners. The frame is initially unstressed.

(a) What is the number of redundancies of this structure? [2]

(b) A uniform, inward "pressure" loading, w, is applied to the outside of the frame as shown in Fig. 4.

(i) Assuming that the elastic axial deformation of the bars is negligible, explain, using the concept of symmetry, why the corners of the frame do not move. [3]

(ii) Determine the bending moments at the corners of the frame. [5]

(iii) Find the change of distance between points E and F on the frame due to the application of this load. [6]

(c) The pressure loading, w, is now removed from the structure so there are no external loads. Sketch the deformation of the frame if:

(i) The temperature of the frame is raised uniformly by ΔT. [2]

(ii) The temperature of the outer surface of the structure is *increased* by $\Delta T/2$ and that of the inner surface is *decreased* by $\Delta T/2$, where the temperature variation through the thickness, t of the bars is linear. [2]

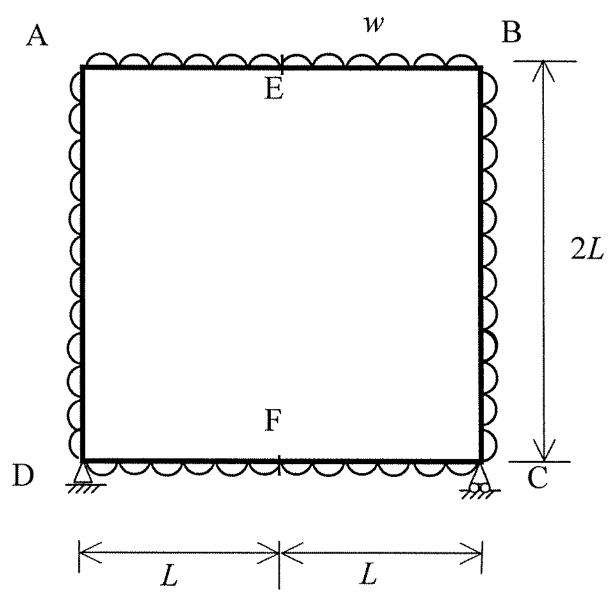


Fig. 4

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5 (a) A rosette consisting of three strain gauges at 45° measures the strains $\varepsilon_a, \varepsilon_b, \varepsilon_c$. Show that the shear strain at the same point is given by: [4]

$$\gamma_{ac} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$$

(b) A hollow steel drilling shaft, with outer diameter 200 mm and wall thickness of 10 mm, is subject to *axial loading* and *lateral pressure*, and to a *torque, T.*

Under certain conditions a rosette of the type described in part (a) above, glued to the inner surface of the shaft as shown in Fig. 5, measures the following strains:

$$\varepsilon_a = -320 \times 10^{-6} \qquad \varepsilon_b = -320 \times 10^{-6} \qquad \varepsilon_c = -12 \times 10^{-6}$$

Determine the corresponding principal strains and sketch the principal directions on the shaft. [8]

(c) The torque T is increased while the other loads remain unchanged until the material near the rosette starts to yield. Determine the corresponding value of T, assuming a yield stress $\sigma_y = 250$ N/mm² and the Tresca yield condition. End effects can be neglected. [8]

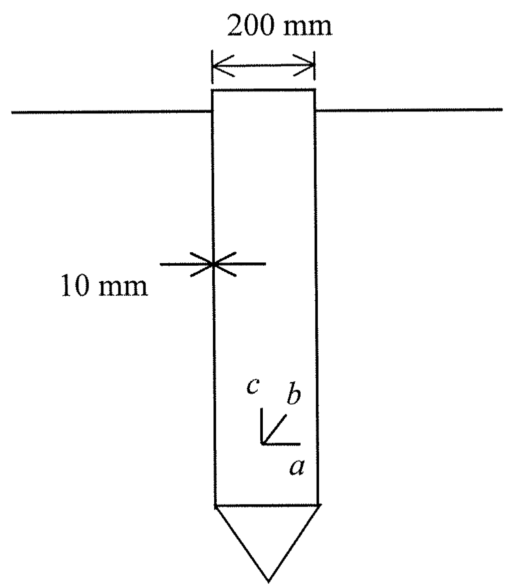

Fig. 5

6 (a) Determine the number of redundancies of the two-dimensional pin-jointed structures shown in Fig. 6(a). [4]

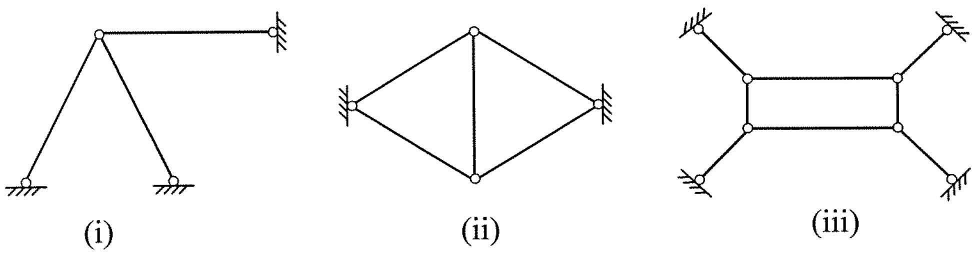

Fig. 6(a)

(b) The two-dimensional, pin-jointed structure shown in Fig. 6(b) consists of linear-elastic members with axial stiffness, AE. The structure is initially unstressed. Determine the axial forces in each of the bars due to the force, F shown. [8]

(c) A turn-buckle inserted in member PR is extended until this member is subject to a compressive force of $F/2$, while the external loads on the structure remain unchanged. Determine the forces in the other members. [8]

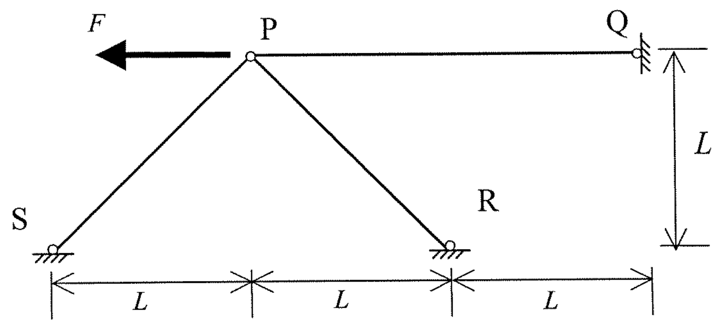

Fig. 6(b)

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