ENGINEERING TRIPOS PART IB

Thursday 7 June 2001 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer at least **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

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SECTION A

*Answer at least **one** question from this section*

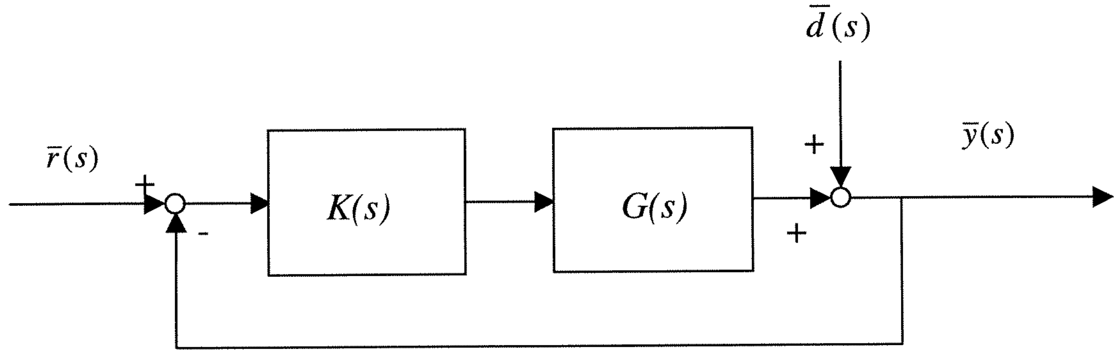

Fig. 1

1 (a) Negative feedback is applied to the linear system $G(s)$ with compensator $K(s)$, as shown in Fig. 1. Derive, in terms of $G(s)$ and $K(s)$, the closed loop transfer functions between (i) $\bar{r}(s)$ and $\bar{y}(s)$, (ii) $\bar{d}(s)$ and $\bar{y}(s)$. Explain the importance of both closed loop transfer functions for the overall design of a feedback controller for the system $G(s)$. [7]

(b) The transfer function of the linear system is given by:

$$G(s) = \frac{1}{s+3}$$

Determine, for a controller with $K(s) = 1$, the steady state response of $y(t)$ when:

(i) $r(t) = H(t)$ and $d(t) = 0$,

(ii) $r(t) = 0$ and $d(t) = \sin(\omega_0 t)$,

(iii) $r(t) = 5H(t-10)$ and $d(t) = \sin(\omega_0 t + \phi_0)$,

where $H(t)$ is the unit step function, $\omega_0 = 1$ rad/s and $\phi_0 = \pi/4$ rad. [6]

(c) The same linear system now has negative integral feedback with $K(s) = k_I/s$ applied to it. Sketch the response of the closed loop system to a unit step input in $r(t)$ when (i) $k_I = 0$, (ii) $k_I = 2.25$, (iii) $k_I = 10$. Comment on the effect of integral control on this system as k_I is increased. Suggest a suitable value for k_I if the system is to have a good response to rapid changes in the desired signal $r(t)$. [7]

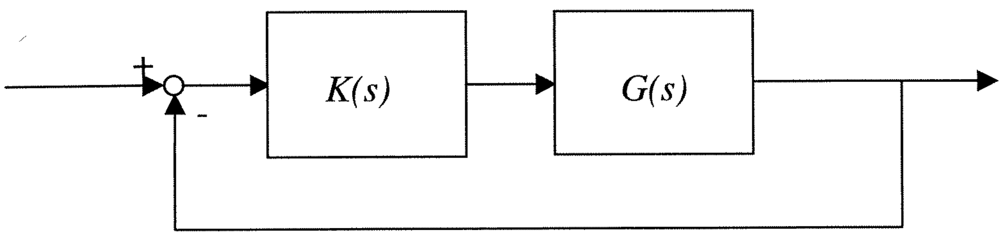

Fig. 2

2 (a) In the control system shown in Fig. 2 the transfer function $G(s)$ is defined as:

$$G(s) = \frac{200}{(s+4)(s+1)}$$

Sketch the Bode plot for $G(s)$, paying particular attention to asymptotes and corner frequencies.

Hence, or otherwise, determine the phase margin of the closed loop system when proportional feedback with $K(s) = k_p = 6$ is applied. Comment on the expected performance of the system in response to step changes in the desired input. What is the effect on the stability of the closed loop system of increasing k_p ?

[10]

(b) The proportional controller is replaced with a compensator having transfer function

$$K(s) = \frac{12(s+50)}{(s+100)}$$

in order to improve the stability margin of the closed loop system. Estimate the new phase margin and comment on how the closed loop response is likely to be improved.

[10]

Note: logarithmic graph paper is supplied for plotting the Bode diagrams in this question.

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3 (a) Explain what measurements are required in order to construct a Nyquist diagram for a system which is to be controlled by proportional negative feedback with $K(s) = k_p$ (see Fig. 3a). How can the Nyquist diagram be used to determine the closed loop stability or otherwise of such a system? You may assume that $G(s)$ is a stable system. [6]

(b) Part of the Nyquist diagram for a system having transfer function

$$G(s) = \frac{s+2}{2s(s+3)(s+1)(s+0.5)}$$

is shown in Fig. 3b. Sketch the full Nyquist diagram for this system over the range $0 < \omega < \infty$, paying particular attention to the limit $\omega \to 0$. [5]

(c) For $K(s) = 1$, determine approximately:

(i) the gain and phase margin for the closed loop system;

(ii) the closed loop gain, $H(j\omega) = \frac{G(j\omega)K(j\omega)}{1+G(j\omega)K(j\omega)}$, when $\omega = 1$;

(iii) the maximum amplitude of the sensitivity function, defined as

$$S(j\omega) = \frac{1}{1+G(j\omega)K(j\omega)};$$

(iv) the range of frequencies for which $|S(j\omega)| > 1$.

Explain the significance of your answers to (iii) and (iv) for system behaviour. [9]

Note: an extra copy of Fig. 3b is supplied at the end of this paper. This may be annotated with your constuctions and handed in with your answer to Question 3.

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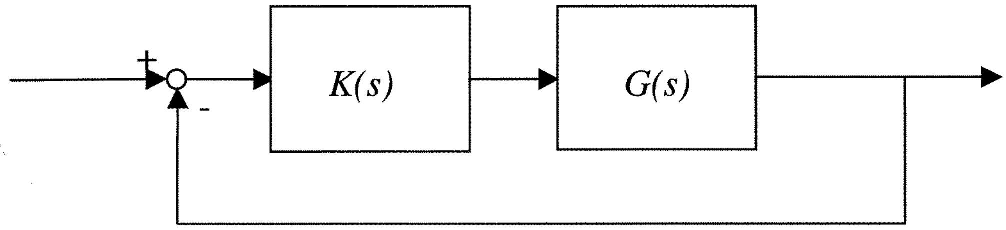

Fig. 3a

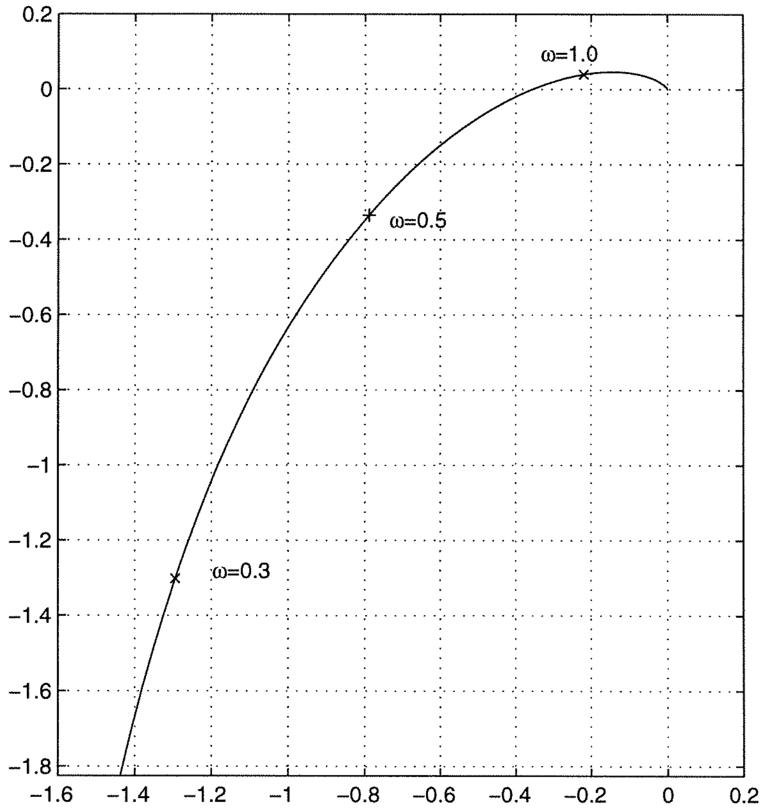

Fig. 3b

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4 Fig. 4 shows a simplified model of an active suspension system for one wheel of a car. M is one quarter of the car's mass, m is the mass of one wheel and k is the tyre stiffness. The force F is generated by a hydraulic actuator.

(a) Show that the transfer functions relating the suspension deflection $z = y - x$ to each of F and w are given by

$$\frac{(m+M)s^2+k}{Ms^2(ms^2+k)} \quad \text{and} \quad \frac{-k}{ms^2+k},$$

respectively. [7]

(b) If the force F is generated according to the rule $\overline{F}(s) = -K(s)\overline{z}(s)$, calculate the closed loop transfer function from w to z. Hence calculate the closed loop transfer function from w to y. Draw a block diagram of the closed loop system which clearly shows the action of the feedback. [6]

(c) A proportional plus derivative action controller

$$K(s) = k_p + k_d\, s$$

is used to control the active suspension system. Determine the steady-state amplitude of oscillation for both y and z if $w = A\cos(10t)$, assuming the closed loop system to be stable. Take $M = 500$ kg, $m = 10$ kg, $k = 500 \times 10^3$ N/m and $k_p = k_d = 1$. Hence determine the steady state amplitude of oscillation of x. Comment on the performance of the active suspension at this frequency. [7]

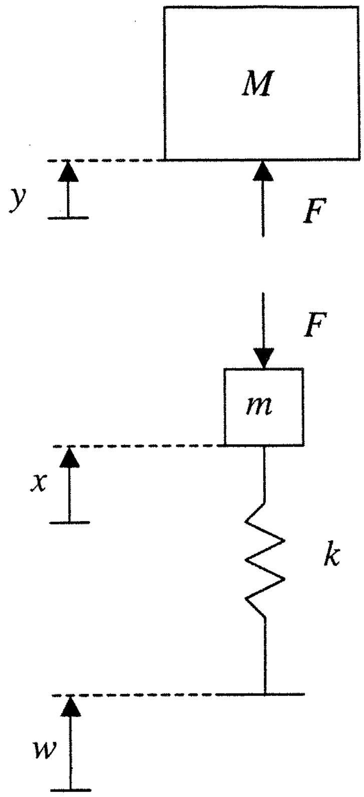

Fig. 4

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SECTION B

Answer at least ***one*** *question from this section*

5 (a) Describe the three main types of metal cable which are used in telecommunications, comparing them in terms of their cross-section, cost, bandwidth, sensitivity to electromagnetic interference, and any other relevant factors. Give an example of a typical use of each type of cable which you describe. [8]

(b) A pair cable connecting a source to a load has a loop resistance of R_{loop} and a total capacitance C between the two conductors. If the source and load impedance are both equal to R_S, the open-loop source voltage is V_S, and the output voltage across the load impedance is V_L, show that the frequency response of the circuit is

$$\frac{V_L}{V_S} = \frac{R_S}{(R_{loop} + 2R_S)} \times \frac{1}{1 + j\omega T},$$

where $T = C(R_{loop}/4 + R_S/2)$. [6]

(c) A pair cable has a loop resistance of 15 Ω /km and shunt capacitance of 30 nF/km. If the source and load resistances are 600 Ω, estimate the maximum lengths of this cable that may be used for the following signals:

(i) A 3.4 kHz bandwidth speech signal;

(ii) A 51.2 kHz bandwidth data signal.

[6]

6 (a) Double Sideband Amplitude Modulation (DSB-AM) of a carrier at frequency ω_C by an information-bearing signal $x(t)$ is defined by the equation

$$s(t) = a_0[1 + m_A x(t)] \cos(\omega_C t).$$

State the advantages and disadvantages of DSB-AM in comparison with Single Sideband Modulation (SSB) and Frequency Modulation (FM). Assuming that $x(t)$ has a maximum magnitude of 1, state the maximum value of m_A which can be used, and explain why this limit is imposed. [5]

(b) By considering the case where the information-bearing signal is a cosine wave, $x(t) = \cos(\omega_M t)$, show that the DSB-AM signal can be described as the sum of a central carrier-frequency and two sidebands. [5]

(c) One type of DSB-AM modulator is based on a "square-law" circuit with input-output characteristic

$$y(t) = v(t)^2.$$

Show that if the input to this circuit is

$$v(t) = b + a\cos(\omega_C t) + x(t),$$

where $x(t)$ is as in part (b), then the required DSB-AM signal can be obtained by applying a linear filter to the output of the square-law circuit. State the passband frequency range of the filter, and derive the relationship between b , a , a_0 , and m_A . [5]

(d) Describe and sketch a DSB-AM demodulator, and explain how it works. [5]

END OF PAPER

KEEP BUT DO
NOT PRINT.

ENGINEERING TRIPOS PART IB, Paper 6, Candidate number:

Extra copy of Fig. 3b which may be annotated and handed in with your answer to Question 3.