IB ANSWERS

ENGINEERING TRIPOS Part IB 2002

Paper 1 – MECHANICS

Answers

1. (a) (i) 0.006 kg m^2 (ii) 31.4N
   (b) (i) discs A, B and C at 0°, 66° and 237° respectively (ii) BCDA (iii) 191N
2. (b) $-4a\omega^2\mathbf{i} + 2a\omega^2\mathbf{j}$ (d) $-3ma^2\omega^2$
3. (a) $\Omega = \frac{r_a}{r_b}\omega$ (b) $\omega_2 = 2\omega_1$, $\Omega_2 = \omega_1$ (c) 60kJ
4. (c) $\frac{2}{7}$ (d) $\frac{2}{7}u$
5. (a) $x\omega\mathbf{e}_1 + (a\omega - \dot{x})\mathbf{e}_2$.(c) $x\omega^2 - \ddot{x} = 0$ (d) $\frac{\sqrt{5}}{2}a$.
6. (a) $a\ddot{\theta}\,\mathbf{i} - a(\ddot{\theta} + \ddot{\phi})\,\mathbf{j}$ (b) $\frac{mg\sqrt{2}}{5}$ (c) $-\frac{mga}{20}$

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Paper 2 – STRUCTURES

1 b) (i) 6
(ii) $\delta_Q = 1500/EA$ horizontally, 0 vertically;
Forces BQ = 2.5 kN, CQ = $2.5/\sqrt{2}$ kN, DQ = 0, rest by symmetry

2 a) (i) 200 MPa, 100 MPa, 0 MPa
(ii) 2386, 486, −1414 με

b) 603 kNm

c) 507 is incorrect

3 a) 2

b) (i) $M = 3EI\alpha T/L$ at B and F
(ii) $\delta_D = 5\alpha LT/2$ vertically, $2\alpha LT$ horizontally; $\Theta_D = 3\alpha LT/2$

4 a) UB 254 × 102 × 25 (or others with $Z_p \geq 300\ cm^3$)

b) UB 305 × 102 × 25

c) No effect

d) No

5 a) $F_1 = \dfrac{20kb}{(\sqrt{3}+\alpha)}$

b) $F_2 = \dfrac{12.6kb}{(1+2\alpha)}$

c) $\alpha < 0.064$

6 a) $M_A = 240$ kNm, $M_B = M_C = 90$ kNm, $M_D = 0$ kNm
$S_A = S_B = S_C = S_D = 30$ kN
$T_A = T_B = T_C = T_D = 50$ kNm

b) $GJ_1 = 55.7\times 10^{12}$ Nmm^2, $GJ_2 = 20.4\times 10^{12}$ Nmm^2

c) $\Theta = 0.68°$

d) At A: $\sigma_L = 220.4$ N/mm^2; $\tau = 18.2$ N/mm^2;
At G: $\sigma_L = 45.5$ N/mm^2; $\tau = 12.7$ N/mm^2

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Paper 3 – MATERIALS – NUMERICAL ANSWERS

1) m=5 σ_o=125 MPa Survival probability = 0.85

3) $|\varepsilon_o|=(|\sigma_o|/E)(1+(E/\omega\eta)^2)$ Loss==$\frac{1}{2}\sigma_o^2/\eta$.

5) $p/\sigma_y = 1+(w/2-x)/h$ Torque=2.15 kNm

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Paper 4 – FLUID MECHANICS AND HEAT TRANSFER

1. (a) 723K, 583 J/K
   (b) 998K, 2.58 bar, 186 J/K
   (c) 102 kJ

2. (a) -
   (b) 46.5 W/m^2K
   (c) 28.4°C, 715 m
   (d) –

3. (a) 37%
   (b) 40%
   (c) –

4. (a) -
   (b) $u = (g \sin\theta/\nu)\,[hy - y^2/2]$
   $\tau = \rho g \sin\theta\,[h-y]$
   $Q = g \sin\theta\, h^3/3\nu$
   (c) –

5. (a) -
   (b) -
   (c) 2×10^6 N, 65×10^6 W

6. (a) $L^{1/2}$, $\mu^{1/2}$
   (b) -
   (c) frequency = constant x (V/d)

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Paper 5 – ELECTRICAL ENGINEERING

Answers

1. (b) Gain = - 49.6, = 33.9 dB

(c) $f_{3dB} = 26.53$ kHz.

2. (b) This is a unity-gain buffer

$$\text{Gain} = \frac{1}{1+\dfrac{R_i}{R_o + AR_i}}$$

Input impedance $= R_o + R_i(A+1)$

$$\text{Input impedance} = \frac{R_o}{1+A+\dfrac{R_o}{R_i}}$$

(c) $$\text{Gain} = -\frac{R_2}{R_1}\left[1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right]$$

(d) Gain = –100.

3. (c) (i) power factor = 0.917 lagging, (ii) $I_l = 88.3$ A, (iii) $C = 1.42$ μF

4. (b) Curent = 8287 A, E = 20.3 kV => $E_{line} = 35.16$ kV, $\delta = 35.5^o$

(c) New $\delta = 50.74^o$, I = 10488 A, Power factor ~ 1

5. (b) 300.8 Nm; $i_f = 3.3$ A.

(c) 1790 rpm

6. (b) $(0.642 + j0.766)V_+$

(c) ~ j32.7 Ohm

(d) ~ j150 Ohm

7. (b) $E_x = E_o e^{j(\omega t - \beta z)}$

(c) $r_{max} = 63$ km

(d) $\theta_{max} = 37^o$

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Paper 6 – INFORMATION ENGINEERING

Answers

1. b) OLTF poles at s=-4 and 2. Unstable
CLTF poles at s=-1 (twice). Asymptotically stable

c) Initial value=0
Final value=-18
Initial slope=9

2. b) T_v=5, T_f=10
c) 0.01

3. b) PM=28°, GM=15dB
c) PM=38°, GM=32.5dB

4. b) -1.72<=K<=3.57, 1.484rad/s
c) GM=3.57 (K=1)
K=2.38 (GM=1.5)
Period of oscillation=4.5s

5. a) 4800Hz
c) 3200bits/s

6. b)

$$\text{power efficiency} = \frac{m_A^2}{2 + m_A^2}$$

max efficiency=1/3

c) term proportional to message is

$$\frac{a_x}{\pi}\cos\omega_m t$$

RC<=31.8μs

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Paper 7: Mathematical Methods

Answers

1. (a) $1/120$.
   (b) $\pi/3 - 13/24$.

2. (b) (i) $n = -1$, (ii) $\phi = \ln r + c$, for any constant c, (iii) $\ln\sqrt{5}$.
   (c) 4π.

3. (b) $\lambda_n = n\pi\sqrt{\alpha}/L$.
   (c) $A_0 = 150$, $A_1 = 200/\pi$, $A_2 = 0$.

4. (a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 & 4 \\ 0 & 4 & -7 \end{bmatrix} = \mathbf{LU}$, $\text{rank}(\mathbf{A}) = 2$.
   (c) Column space spanned by $[\,3\;\;6\,]^T$ and $[\,-1\;\;2\,]^T$; row space spanned by $[\,3\;\;-1\;\;4\,]^T$ and $[\,0\;\;4\;\;-7\,]^T$; nullspace is $[\,-3\;\;7\;\;4\,]^T$; left nullspace is $[\,0\;\;0\,]^T$.
   (d) $\mathbf{A}^T\bar{\mathbf{x}} = [\,0\,0\,0\,]^T$, $\bar{\mathbf{x}} = [\,0\;\;0\,]^T$.

5. (b) (i) $\mathbf{A} = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix}$, (iii) $\lambda_1 = 1$, $\mathbf{u}_1 = [\,0.3162\;\;0.9487\,]^T$, $\lambda_2 = 0.6$,
   $\mathbf{u}_2 = [\,-0.7071\;\;0.7071\,]^T$, $\lim_{k\to\infty}\mathbf{x}_k = [\,0.75\;\;2.25\,]^T$.
   (c) Unless $\mathbf{x}_0 = \mathbf{0}$, $|\mathbf{x_k}| \to \infty$.

6. (a) π/ω_m.
   (b) $H(\omega) = \begin{cases} T & \text{for } |\omega| \le \pi/T \\ 0 & \text{otherwise.} \end{cases}$

7. (a) (i) $X(\omega) = \dfrac{Te^{-i\omega T/2}}{2}\left[\text{sinc}\left(\dfrac{(\omega-\omega_0)T}{2}\right) + \text{sinc}\left(\dfrac{(\omega+\omega_0)T}{2}\right)\right]$.
   (b) (i) $f(z) = \begin{cases} 1/2\,(2+z) & -2 \le z \le -1 \\ 1/2 & -1 \le z \le 0 \\ 1/2\,(1-z) & 0 \le z \le 1 \end{cases}$, (ii) $\text{E}(Z) = -0.5$.

8. (a) (ii) 0.9035, (iii) 0.9044.
   (b) (ii) 0.286, (iii) 0.899.

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Paper 8 – SELECTED TOPICS

Q1 (b) d = 3.25 m (c) FoS = 1.92

Q2 (a) Stability ratio = 8.7 (for tunnel at 12.5m depth), so closed faced tunnelling should be used.
(b) Stability ratio = 3.7 (for tunnel at 35m depth), so open faced tunnelling can be used.

Q3 (a) Max sagging moment = 2700 kNm
Max hogging moment = 4800 kNm
(b) d = 670.8 m so use 700 m with cover
A_s= 12509 mm^2
(c) A'_s = 8424 mm^2 and A_s = 21255 mm^2

4 b) i) $\frac{bc^2}{4L}\sigma_Y^c + \frac{btc}{L}\sigma_Y^f$ ii) $\frac{bt^2}{2L}\sigma_Y^f + bc\tau_Y^c$ c) $2L\sqrt{\frac{P^*\rho_f\rho_c Lb}{\sigma_Y^f}}$

5 b) i) $\frac{3\pi}{4}\left(\frac{d}{h}\right)^2$ and $\frac{3\pi d}{h^2}$ ii) 1529 m^2/m^3; yes.

Q7 (a) 75.9 kPa 305.5 K
(c) 385 m/s
(c) 1087 K 389 kPa
(d) 633 K 43.8 kPa 377 m/s
(e) 309 kg/s

Q9 (b) 67.7 kN
(c) 192.5 kg/s
(d) 197 kN

Q.10 a) $\sigma = e\mu N$
b) $\mu = 6.7 \times 10^{-3} m^2/V.S$
c) $\sigma = 0.21\ ohm^{-1}m^{-1}$
d) $L = 3.1 \times 10^{-6}$m; $V_{DS} = 5V$

Q.11 b) 4.95×10^{-19} Joule = 3.09eV

c) $E = n^2 \frac{\hbar^2\pi^2}{2mL^2}$

d) $E = 1.5 \times 10^{-24}$ Joule = 9.4×10^{-6} eV

Q.12 c) $x = 1.6\ \mu m$