H&MH

Part I B Mechanics 1 2002

This paper was pretty well done. The average mark was 62%
Section A was more popular than Section B – and generally better done.

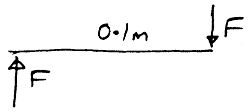
1/a/i/ $J = 4 \times \frac{1}{2} m a^2 = 4 \times \frac{1}{2} \times 1.2 \times (0.05)^2 = 0.006 \text{ kg m}^2$

ii/ Gyroscopic couple : $Q = J \omega \Omega$

$$= 0.006 \times \frac{10\,000}{60} \times 2\pi \times 0.5$$

$$= \pi \text{ Nm}$$

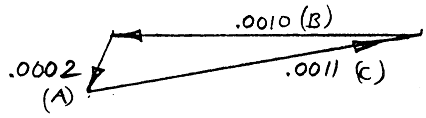
bearings are 0.1 m apart



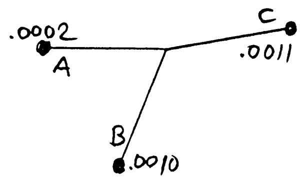
$\therefore$ $\underline{\underline{F = 10\pi \text{ N}}}$

an easy 6 marks for any one who bothered to learn up gyros

b/i/ Balancing : for static balance draw a closed force polygon



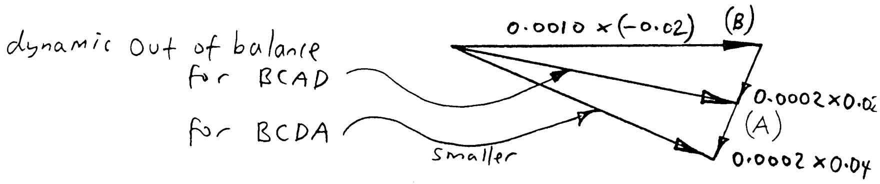
ii/ For dynamic balance take moments about disc C :



BCAD : distances "x" BC = −0.02m
CA = 0.02m

BCDA : BC = −0.02m
CA = 0.04m

Moments are $m r \omega^2 x$



iii Dynamic out of balance measured from diagram :

$$Q = 0.000092 \times \omega^2$$

$$= 0.00092 \times \left(\frac{10{,}000}{60} \times 2\pi\right)^2 \times 0.02$$

$$= 20 \text{ Nm}$$

$\therefore$ Bearing force = $\underline{\underline{200 \text{ Nm}}}$

generally well done

2/ This question was well done but too many didn't show enough working to demonstrate the "show that's" were not "fudged". It is essential to show working & construction lines. Don't rub them out!

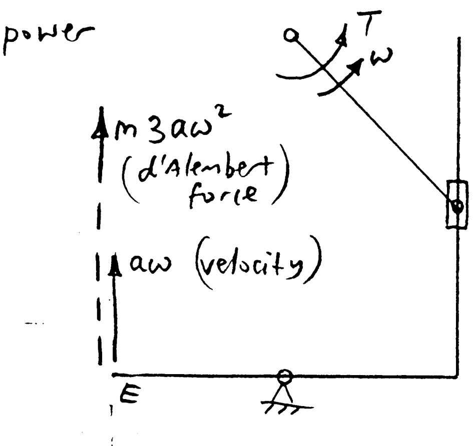
a/ Two options: ① velocity diagram (see attached)
or ② instantaneous centres see attached.

b/ see acceleration diagram attached

c/ see acceleration diagram attached

$$\underline{a}_{Brod} = -4a\omega^2 \underline{i} + 2a\omega^2 \underline{j}$$

d/ Use d'Alembert and virtual power



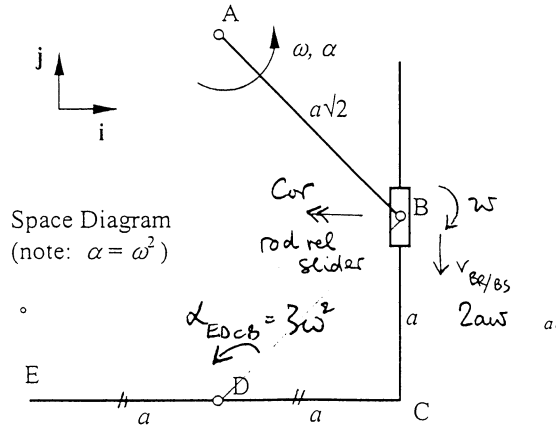
The work done by the driving torque and by the d'Alembert force at E must sum to zero

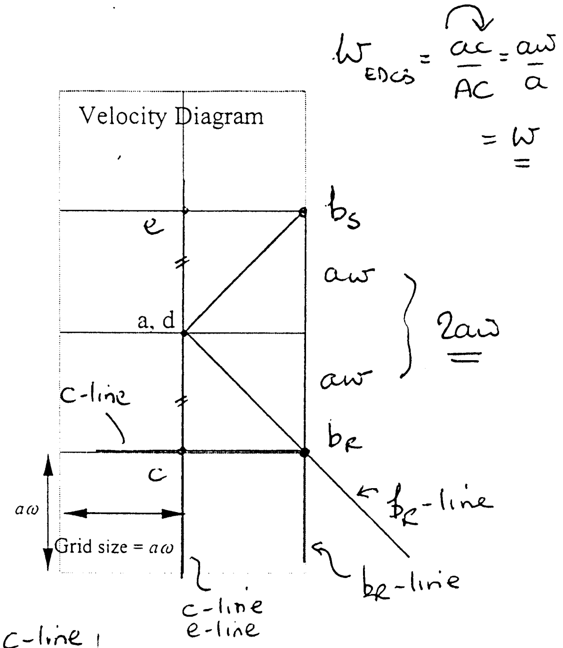
$$\therefore \quad T\omega + 3ma\omega^2 \, a\omega = 0$$

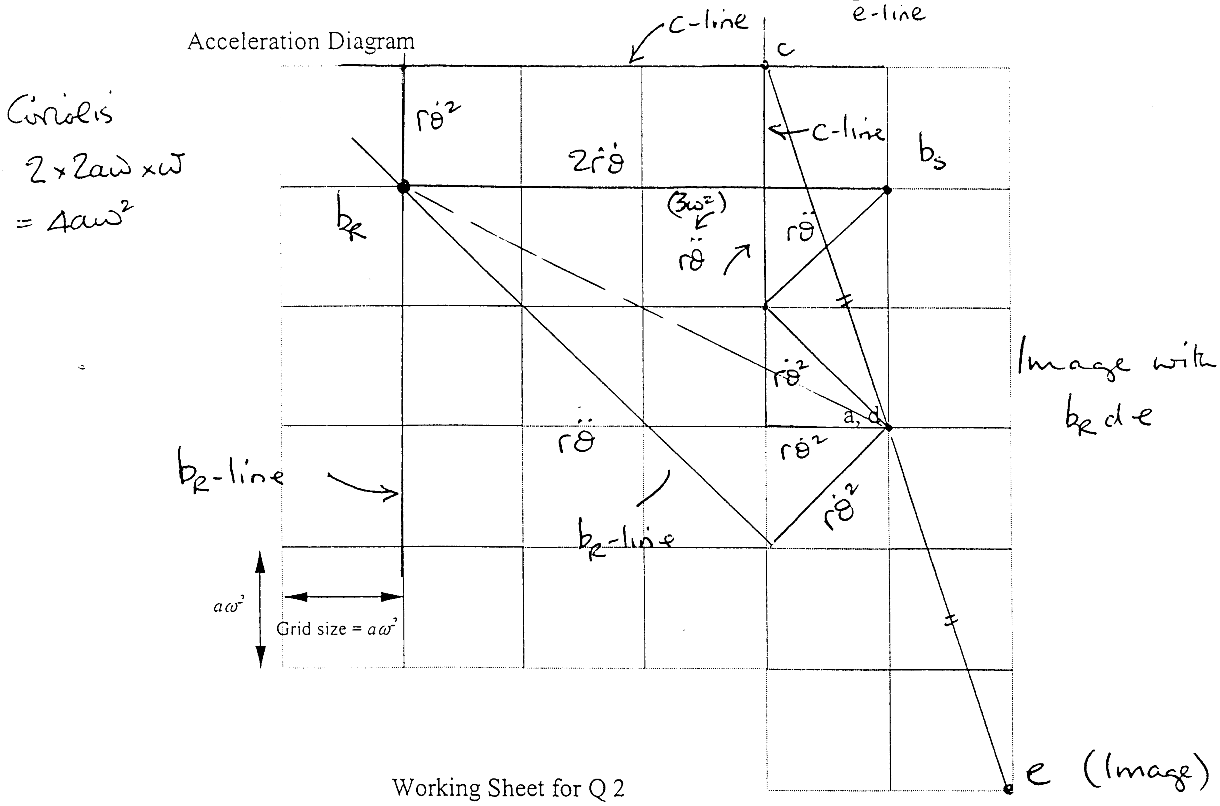
$$\therefore \quad \underline{\underline{T = -3ma^2\omega^2}}$$

the sign is important - it is a "braking" torque

ENGINEERING TRIPOS PART IB
Monday 3 June 2002 9 to 11
Paper 1
MECHANICS

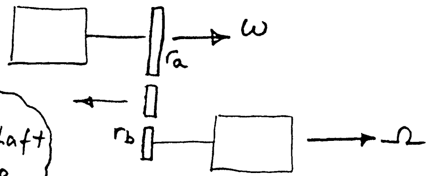






Working Sheet for Q 2
(may be handed in with your script)

3. a/



ω & Ω are in the same sense

ω for the layshaft is in the opposite sense

$\omega r_a = \Omega r_b \quad \therefore \quad \underline{\underline{\Omega = \omega \frac{r_a}{r_b}}}$

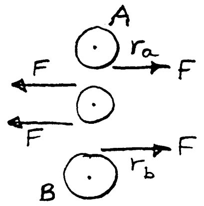
b/ <u>Before</u>

$\boxed{J} \longrightarrow \omega_1$
A

$\boxed{2J} \longrightarrow \Omega_1 = 2\omega_1$
B

<u>During</u>

$\boxed{J} \longrightarrow F r_a$
A



$F r_b \longleftarrow \boxed{2J}$
B

consider forces on layshaft to get torques on A & B

<u>After</u>

$\boxed{J} \longrightarrow \omega_2$
A

$\boxed{2J} \longrightarrow \Omega_2 = \frac{\omega_2}{2}$
B

<u>Changes in angular momentum:</u>

For A: $J(\omega_2 - \omega_1) = F r_a$ ①

For B: $2J(\Omega_2 - \Omega_1) = -F r_b$

$\therefore 2J\left(\frac{\omega_2}{2} - 2\omega_1\right) = -F r_b$ ②

note: $\frac{r_b}{r_a} = 2$ during slipping

$\frac{②}{①}: \; -\frac{r_b}{r_a} = -2 = \frac{\omega_2 - 4\omega_1}{\omega_2 - \omega_1}$

$\therefore -2(\omega_2 - \omega_1) = \omega_2 - 4\omega_1$

$\therefore 6\omega_1 = 3\omega_2 \quad \therefore \underline{\underline{\omega_2 = 2\omega_1}}$

& $\underline{\underline{\Omega_2 = \omega_1}}$

c/ $\Delta E = \frac{1}{2}(2J\Omega_2^2 + J\omega_2^2) - \frac{1}{2}(2J\Omega_1^2 + J\omega_1^2)$

$= \frac{1}{2}J\omega_1^2[(2 + 4) - (8 + 1)]$

$= \frac{1}{2}4 \times (100)^2[-3] = \underline{\underline{-60 \text{ kJ}}}$

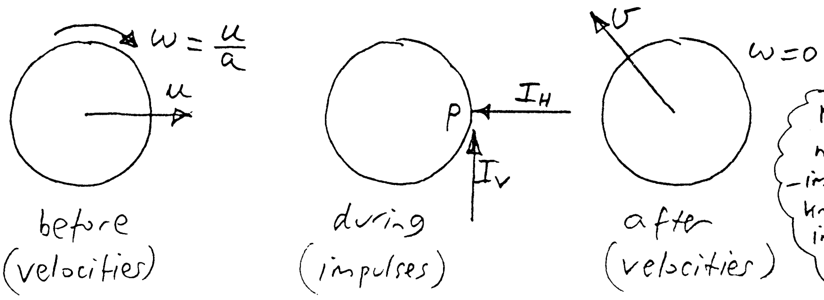
easy marks!

SECTION B

4. a/ An ideal impulse $\underset{\sim}{I}$ is an infinite force $\underset{\sim}{F}$ acting over an infinitessimal time dt such that $\underset{\sim}{I} = \int \underset{\sim}{F}\, dt$ is finite.

Gravity forces are finite and can be neglected in comparison with forces $\underset{\sim}{F}$ which are infinite.

b/



There is no impulse delivered at the point of contact with the ground — many thought there is

Many had no diagram — impossible to know where impulses apply

During the collision, all impulses pass through P hence moment of momentum about P remains unchanged

$$\therefore \quad \underset{(\text{before})}{J\omega} = \underset{(\text{after})}{\frac{mv}{\sqrt{2}}a} \qquad \left(J = \frac{2}{5}ma^2\right)$$

$$\therefore \quad \frac{2}{5}ma^2\frac{u}{a} = \frac{mv}{\sqrt{2}}a \qquad \therefore \quad \underline{\underline{v = \frac{2\sqrt{2}}{5}u}}$$

c/ $\mu \geq \dfrac{I_V}{I_H}$ at the wall.

Many misread the question and calculated a coefficient of restitution

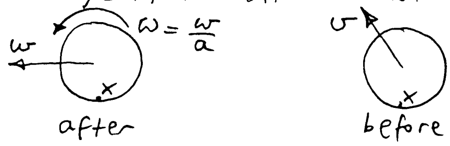
$$\text{Momentum} \leftarrow \quad I_H = m\left(\frac{v}{\sqrt{2}} + u\right) = \frac{7}{5}mu$$

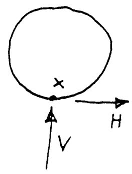
$$\text{Momentum} \uparrow \quad I_V = m\frac{v}{\sqrt{2}} = \frac{2}{5}mu$$

$$\therefore \quad \underline{\underline{\mu \geq \frac{I_V}{I_H} = \frac{2}{7}}}$$

d/ Each bounce delivers an impulse at X

The accumulated change in momentum about X must be zero.



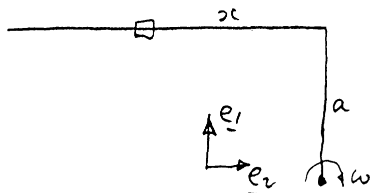


$$\text{Moment of Momentum about X} : \quad \underset{(\text{before})}{\frac{mva}{\sqrt{2}}} = \underset{(\text{after})}{mwa + \frac{2}{5}ma^2\frac{w}{a}}$$

$$\therefore \quad \frac{2}{5}u = \frac{7}{5}w \qquad \therefore \quad \underline{\underline{w = \frac{2}{7}u}}$$

Many used conservation of energy. The collisions are clearly not elastic

5 a/ $\underset{\sim}{r} = a\underline{e}_1 - x\underline{e}_2$



$\dot{\underline{r}} = a\dot{\underline{e}}_1 - \dot{x}\underline{e}_2 - x\dot{\underline{e}}_2$

$= a\omega\underline{e}_2 - \dot{x}\underline{e}_2 + x\omega\underline{e}_1$

$$\underline{\underline{\dot{\underline{r}} = x\omega\underline{e}_1 + (a\omega - \dot{x})\underline{e}_2}}$$

$\dot{\underline{e}}_1 = \omega\underline{e}_2$

$\dot{\underline{e}}_2 = -\omega\underline{e}_1$

those that used the data-book formula made mistakes & took a whole page over a/ & b/

b/ $\ddot{\underline{r}} = \dot{x}\omega\underline{e}_1 + x\omega\dot{\underline{e}}_1 - \ddot{x}\underline{e}_2 + (a\omega - \dot{x})\dot{\underline{e}}_2$ noting $\dot{\omega} = 0$

$= \dot{x}\omega\underline{e}_1 + x\omega^2\underline{e}_2 - \ddot{x}\underline{e}_2 - (a\omega - \dot{x})\omega\underline{e}_1$

$$\underline{\underline{\ddot{\underline{r}} = (-a\omega^2 + 2\dot{x}\omega)\underline{e}_1 + (x\omega^2 - \ddot{x})\underline{e}_2}}$$

c/ Frictionless rod ∴ No force in $\underline{e}_2$ direction

∴ No acceleration in $\underline{e}_2$ direction

∴ $\underline{\underline{\ddot{x} - \omega^2 x = 0}}$ (1)

Some tried to use $x = ut + \frac{1}{2}at^2$ This fails because the acceleration is not constant

Some thought this is SHM

d/ $x = a\cosh\omega t$

$\dot{x} = a\omega\sinh\omega t$

$\ddot{x} = a\omega^2\cosh\omega t$ substitute x & $\ddot{x}$ into (1)

∴ (1) is satisfied.

Most solved the de using $e^{\lambda t}$ etc & eventually found $\frac{1}{2}(e^{\omega t} + e^{-\omega t}) = \cosh\omega t$. But "show that" is a licence to shortcut

To complete the "show that", need to note that $x = a$ at $t = 0$

Force ⊥ rod = 0 ∴ $\underline{e}_1$ compt of acceleration = 0 ∴ $2\dot{x}\omega = a\omega^2$

∴ $\dot{x} = \frac{a\omega}{2}$ but $\dot{x} = a\omega\sinh\omega t$

∴ $\sinh\omega t = \frac{1}{2}$ Also $\cosh^2\omega t - \sinh^2\omega t = 1$

∴ $x = a\cosh\omega t = a\sqrt{1 + (\frac{1}{2})^2} = \underline{\underline{\frac{a\sqrt{5}}{2}}}$ done well

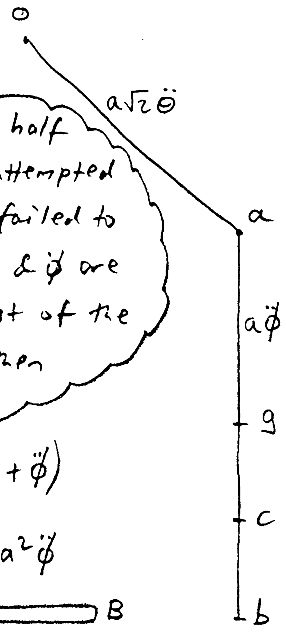
e/ $x = a\cosh\omega t$ "cosh" is even so no change to motion for −ve ω ie the mass moves out

Physical reasoning: centrifugal force acts outwards either way

Force on rod is zero when $\sinh\omega t = \frac{1}{2}$ ∴ t is −ve if ω −ve

∴ Force is never zero

6 a/ Draw an acceleration diagram noting that $\dot\theta$ & $\dot\phi$ are zero at the instant in question



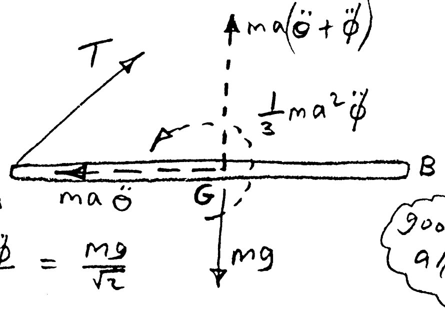
More than half those who attempted this question failed to notice that $\dot\theta$ & $\dot\phi$ are zero. The rest of the question is then impossible

$$\underline{a_G} = a\ddot\theta\,\underline{i} - a(\ddot\theta + \ddot\phi)\,\underline{j}$$

from the diagram

Most who used the Data Book formula for acceleration made a page of work for themselves & got wrong answers

b/ Use d'Alembert



$$\Sigma F \nwarrow \; : \; ma\sqrt{2}\,\ddot\theta + \frac{ma\ddot\phi}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$$

$$\therefore\; 2\ddot\theta + \ddot\phi = \frac{g}{a} \qquad (1)$$

good diagrams make all the difference

$$\Sigma M_A \; : \; \frac{1}{3}ma^2\ddot\phi + ma^2(\ddot\theta + \ddot\phi) = mga$$

$$\therefore\; \frac{4}{3}\ddot\phi + \ddot\theta = \frac{g}{a} \qquad (2)$$

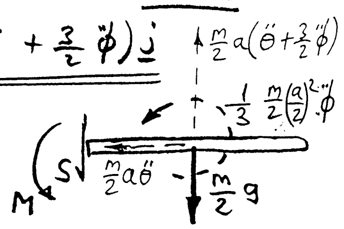
part b/ generally well done

$$2\times(2) - (1) \quad \therefore\; \frac{8}{3}\ddot\phi - \ddot\phi = \frac{2g}{a} - \frac{g}{a} \qquad \therefore\; \underline{\underline{\ddot\phi = \frac{3}{5}\frac{g}{a}}}$$

and from (2), $\ddot\theta = \frac{g}{a} - \frac{4}{3}\ddot\phi \qquad \therefore\; \underline{\underline{\ddot\theta = \frac{1}{5}\frac{g}{a}}}$

$$\Sigma F \nearrow \; \therefore\; T = \frac{mg}{\sqrt{2}} - ma\frac{\ddot\phi}{\sqrt{2}} = \frac{mg}{\sqrt{2}}\left(1 - \frac{3}{5}\right) = \underline{\underline{\frac{\sqrt{2}mg}{5}}}$$

c/ From accel diag $\quad \underline{\underline{\underline{a_c} = a\ddot\theta\,\underline{i} - a\left(\ddot\theta + \frac{3}{2}\ddot\phi\right)\underline{j}}}$



Use d'Alembert for bar segment GB

$$\Sigma M_G \; : \; M = -\frac{1}{3}\frac{m}{2}\left(\frac{a}{2}\right)^2\ddot\phi + \frac{m}{2}g\frac{a}{2} - \frac{m}{2}a\left(\ddot\theta + \frac{3}{2}\ddot\phi\right)\frac{a}{2}$$

$$= -\frac{ma^2}{8}\left(\left(\frac{1}{3}+3\right)\ddot\phi + 2\ddot\theta - \frac{2g}{a}\right)$$

$$= -\frac{ma^2}{8}\left(\frac{10}{3}\,\frac{3}{5}\frac{g}{a} + \frac{2}{5}\frac{g}{a} - 2\frac{g}{a}\right) = \underline{\underline{-\frac{mga}{20}}} \quad \text{(i.e. hogging)}$$

Lots forgot that the segment GB has mass $\frac{m}{2}$ (not mass m) & moment of inertia $\frac{ma^2}{24}$