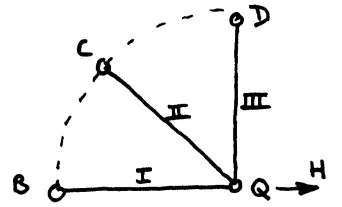
ENGINEERING TRIPOS PART IB JUNE 2002

PAPER 2 : STRUCTURES

SECTION A

Q1 a) i)

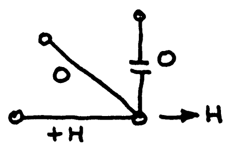


1 redundancy

Solve by Virtual Work

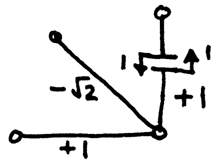
Cut DQ

System ⓪ : Particular solution in equilibrium with applied loads



$$\underline{t}_0 = \begin{bmatrix} H \\ 0 \\ 0 \end{bmatrix}$$

System ① : State of self-stress



$$\underline{s} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$$

General solution : Real system = ⓪ + x ①

$$\text{Bar forces } \underline{t} = \underline{t}_0 + \underbrace{x}_{\text{Release force in DQ}} \underline{s} = \begin{bmatrix} H \\ 0 \\ 0 \end{bmatrix} + x \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} H+x \\ -\sqrt{2}x \\ x \end{bmatrix}$$

VIRTUAL WORK :

$$\underbrace{W . \overbrace{\delta}^{\text{EXTERNAL}}}_{\text{VIRT. EQUIL.}} = \sum P . \overbrace{e}^{\text{INTERNAL}}$$

REAL COMPAT.

PICK System ① (State of self-stress) ← for which there are NO external forces $\therefore W = 0 \quad \therefore \text{LHS} = 0$

$$\therefore \text{ V.W: } \quad \overset{\text{LHS}}{0} = \overset{\text{RHS}}{\underline{s} . \underline{e}} \quad \text{and} \quad \underline{e} = \underline{\underline{F}} . \underline{t} \quad \text{(Real extensions)}$$

$$\therefore \underline{e} = \frac{L}{EA} \begin{bmatrix} 1 & & \\ & 1/2 & \\ & & 1 \end{bmatrix} . \begin{bmatrix} H+x \\ -\sqrt{2}x \\ x \end{bmatrix} = \frac{L}{EA} \begin{bmatrix} H+x \\ -x/\sqrt{2} \\ x \end{bmatrix}$$

$$0 = \underline{s} . \underline{e} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} . \begin{bmatrix} H+x \\ -x/\sqrt{2} \\ x \end{bmatrix} \frac{L}{EA}$$

$$\therefore \quad H + x + x + x = 0 \qquad \therefore \quad x = -H/3$$

$$\therefore \underline{t} = \begin{bmatrix} H+x \\ -\sqrt{2}x \\ x \end{bmatrix} = \begin{bmatrix} H - H/3 \\ +\sqrt{2}H/3 \\ -H/3 \end{bmatrix} = \begin{bmatrix} 2H/3 \\ \sqrt{2}H/3 \\ -H/3 \end{bmatrix} \qquad \text{Q.E.D.}$$

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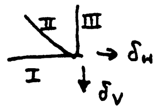
Q1. a) ii

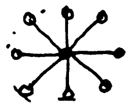


By inspection

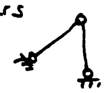
$$\left.\begin{array}{l}\delta_H = e_{I} \\ \delta_V = e_{III}\end{array}\right\} \quad \underline{e} = \frac{L}{EA}\begin{bmatrix} H + x \\ -x/\sqrt{2} \\ x \end{bmatrix} = \frac{L}{EA}\begin{bmatrix} 2H/3 \\ H/3\sqrt{2} \\ -H/3 \end{bmatrix}$$

$$\therefore \quad \delta_H = \frac{2}{3}\frac{HL}{EA} \quad \text{and} \quad \delta_V = \frac{-HL}{3EA} \qquad \text{Q.E.D.}$$

b) i)



i) Six redundancies (delete six bars to get



)

[Alternatively: Maxwell's rule for pin-jointed assembly (Data Book)

$$s - m = b + r - Dj$$

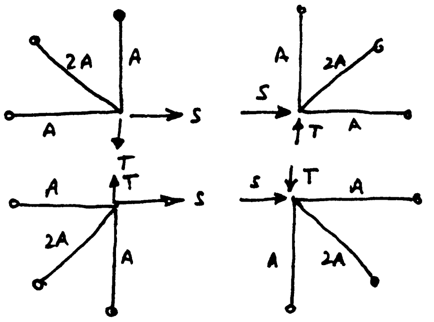
m = mechanisms = 0 ; b = bars = 8 ; r = restraints on joints = $2 \times 8 = 16$;
D = dimensions = 2 ; j = joints = 9 ;

$$\therefore \quad s = 8 + 16 - 2(9) = 6$$

]

b)ii) Use superposition:

Bar areas equal 2A
$S = 2500$ kN, $T = ?$



Consider top left quadrant:
Deflection of central node

$$= \uparrow \frac{1}{3}\frac{SL}{EA} \quad \text{PLUS} \quad \downarrow \frac{2}{3}\frac{TL}{EA}$$

$$\rightarrow \frac{2}{3}\frac{SL}{EA} \qquad \leftarrow \frac{1}{3}\frac{TL}{EA}$$

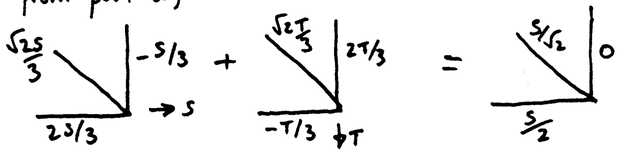
(using results from part a)).

Now vertical deflection = 0 $\therefore \frac{1}{3}\frac{SL}{EA} = \frac{2}{3}\frac{TL}{EA} \Rightarrow \underline{T = S/2}$

$$\text{horizontal deflection} = \frac{2}{3}\frac{SL}{EA} - \frac{1}{3}\frac{TL}{EA} = \frac{1}{3}\frac{SL}{EA}\left[2 - \frac{1}{2}\right] = \frac{1}{2}\frac{SL}{EA}$$

$$= \frac{1}{2}\frac{(2500)(1.2)}{EA} = \underline{\underline{\frac{1500}{EA}}}$$

Forces in bars:
from part a)

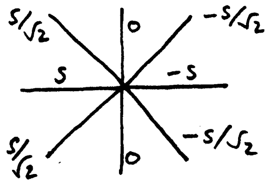


Horiz: $\frac{2S}{3} - \frac{T}{3} = \frac{S}{3}\left(2 - \frac{1}{2}\right) = \frac{S}{2}$

Diag: $\frac{\sqrt{2}S}{3} + \frac{\sqrt{2}T}{3} = \frac{\sqrt{2}S}{3}\left(1 + \frac{1}{2}\right) = \frac{S}{\sqrt{2}}$

Vert: $-\frac{S}{3} + 2T/3 = \frac{S}{3}(-1+1) = 0$

SUPERPOSE:



where $S = 2.5$ kN

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Q1 cont'd.
Extract from Examiner's Report:

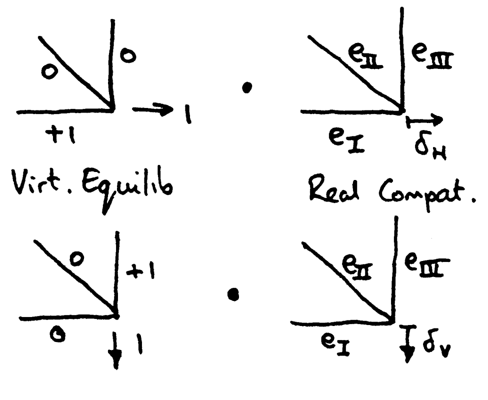
Q1. **Statically-indeterminate truss (ELASTIC)**

Attempts: 122 out of 237. Average mark: 12.5/20.

A generally pleasing response to this question (which was almost identical to a recent Part IIA question). Most students demonstrated that they had mastered the vector notation for virtual work that has been introduced by the lecturer, Dr Guest (e.g. $\mathbf{t} = \mathbf{t}_0 + x\mathbf{s}$, $\mathbf{e} = \mathbf{F}\mathbf{t}$, $0 = \mathbf{s}.\mathbf{e}$, etc). Most got full marks for part a), the three bar truss. However, to compute the displacements, many students used a second application of virtual work, dotting the full system with itself. This led to a page of calculations and some wasteful expenditure of time, when answers can just be read from the earlier results: the joint displacement components are equal to the extensions of the horizontal and vertical bars (This obvious result can also be obtained from virtual work (if so desired) by dotting the full, real (compatibility) system with virtual (equilibrium) systems having unit tensions in only the horizontal or vertical bars respectively, instantly giving $1.\delta_H = 1.e_I$ and $1.\delta_V = 1.e_{III}$). (See below)

The second part of the question caused some problems. For the number of redundancies, many just spotted that removing 6 bars left a statically-determinate truss. Most, however, used Maxwell's rule, and of these, few obtained the correct answer. Many spotted that the eight-bar truss could be obtained by superposing four of the trusses analysed in part a) and then made reasonable attempts. However none noticed that self-equilibrating vertical forces would then be necessary to ensure vertical compatibility between the trusses, and thus no model answers were obtained.

(n.b. the final part of the question should have stated that all bars had the same area, behaved elastically and were initially stress-free. However, all students made appropriate assumptions, and one student gained marks for referring to "the infinite number of possible solutions".)



EXT. INT. Real comp.

V.W. → $1.\,\delta_H = 1.\,e_I$

Virt. Eq.

V.W. → $1.\,\delta_V = 1.\,e_{III}$

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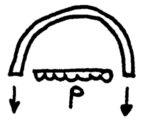
<u>QUESTION 2.</u>

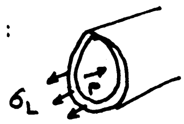
a) i) Hoop stress:



$\sigma_h t = pr \qquad \therefore \ \sigma_h = \dfrac{pr}{t}$

$\therefore \ \sigma_h = 2.5 \times 10^6 \left(\dfrac{N}{m^2}\right) \dfrac{0.4\,m}{0.005\,m} = \underline{\underline{200\ MPa}}$

Longitudinal stress:

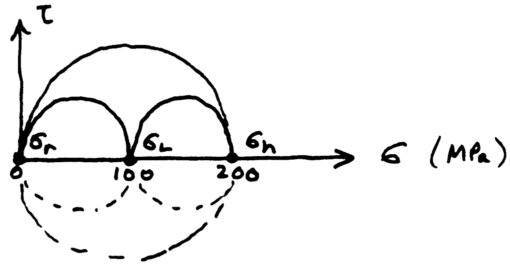


$p(\pi r^2) = \sigma_L (2\pi r t)$

$\therefore \ \sigma_L = \dfrac{pr}{2t} = \dfrac{\sigma_h}{2} = \underline{\underline{100\ MPa}}$

Through-thickness, radial stress $\underline{\underline{\sigma_r \simeq 0}}$ (Thin-walled assumption)

Mohr's circle of stress



a) ii) <u>Strains</u>: $\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}\right)$ ← Hooke's Law

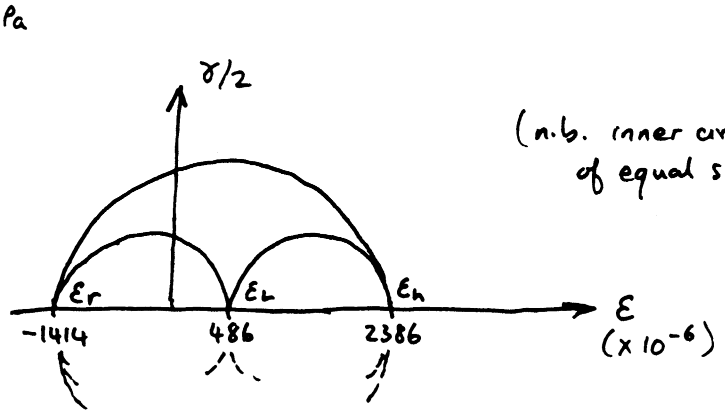
Al alloy: $E = 70\,GPa \quad \nu = 0.33$ Data book, p1

$\varepsilon_L = \dfrac{1}{70\times10^3\,MPa}\left(100 - 0.33(200)\right) MPa = 486 \times 10^{-6}$ (Dimensionless)
(= 486 "microstrain")

$\varepsilon_h = \dfrac{1}{70\times10^3\,MPa}\left(200 - 0.33(100)\right) MPa = 2386 \times 10^{-6}$

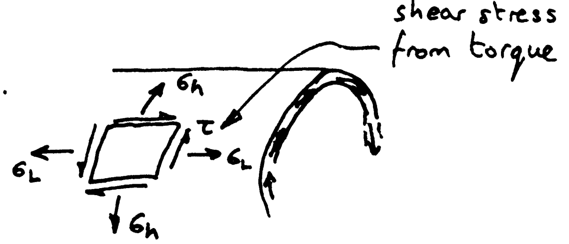
$\varepsilon_r = \dfrac{1}{70\times10^3\,MPa}(-0.33)(100+200)\,MPa = -1414 \times 10^{-6}$

Mohr's circle of Strain

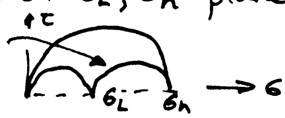


(n.b. inner circles are of equal size)

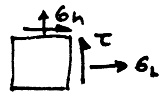
Q2 cont'd. Part b).

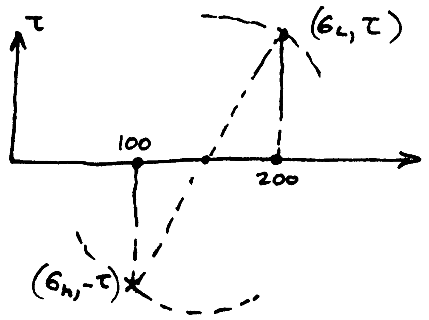


Note: shear stress from torque acts on σ_L, σ_h plane, so affects inner Mohr's circle



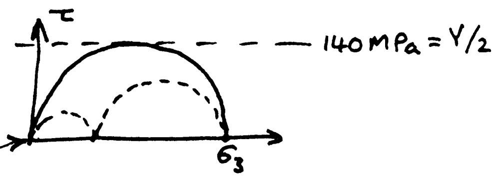
As torque is applied, the inner σ_L, σ_h circle grows





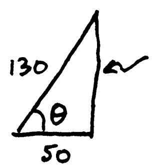
Centre of inner σ_L, σ_h circle remains at $\frac{100+200}{2} = 150\,\text{MPa}$

Yield occurs when biggest (i.e. outer) circle hits $\tau = \pm Y/2$ (Tresca) $= \pm 140\,\text{MPa}$

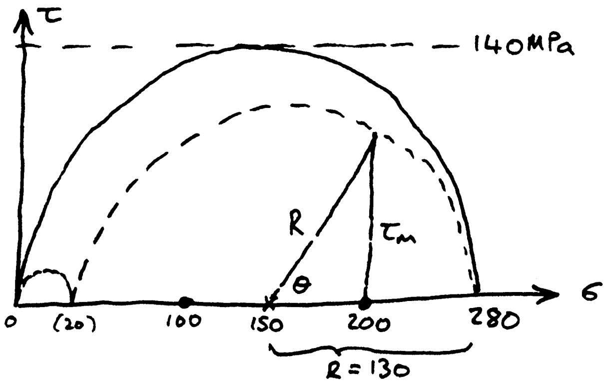


and because thru-thickness $\sigma_r = 0$ still, then right-hand end $\sigma_3 = 280\,\text{MPa}$.

$\therefore$ Radius of inner σ_L, σ_h circle $= 280 - 150 = 130\,\text{MPa}$ (and $\therefore$ inner, still)



$\therefore$ $= 120\,\text{MPa} = \tau_m$ (max shear stress due to torque).



$\therefore$ Torque at point of yielding $T = (2\pi r^2 t)\,\tau_m = 2\pi(400\text{mm})^2(5\text{mm}) \times 120\,\text{N/mm}^2$

$(T = 2A_e t\,\tau)$ $= \underline{\underline{603\,\text{kNm}}}$

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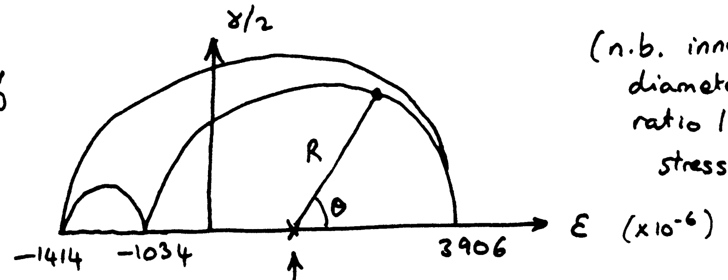
Q2 Part c). Mohr's circle of strain

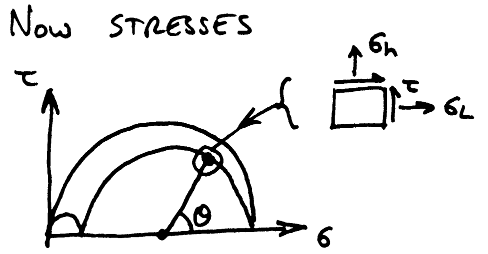


(n.b. inner circle diameters in some ratio 1:13 as for stress inner circles)

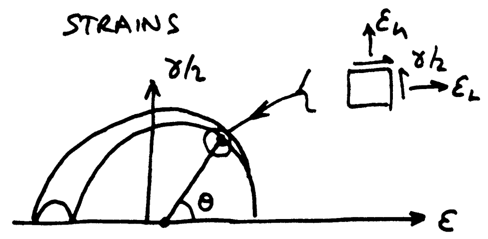
$$\text{centre} = \frac{-1034+3906}{2} = 1436$$

$$\text{radius} = \frac{1034+3906}{2} = 2470$$

Now STRESSES

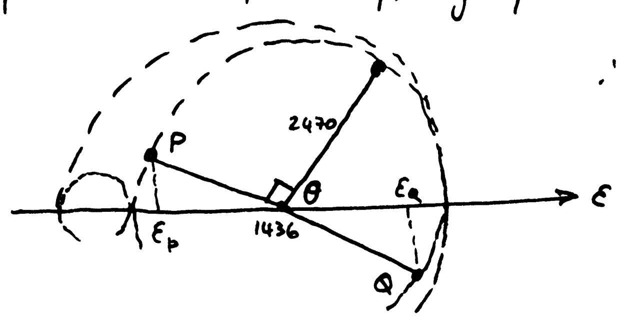


so STRAINS



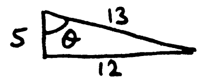
SO : strain gauges at 45° to hoop, longit. dirns correspond to points at 90° from hoop, longit points on Mohr's circle

$\therefore$



$\therefore$ Points P, Q correspond to strain gauges, and gauges read normal strains ε_P, ε_Q.

Triangles in proportion



(see stress diag., part b))

$$\therefore \quad \varepsilon_P = 1436 - \frac{12}{13}(2470) = -844 \quad \text{microstrain}$$

$$\varepsilon_Q = 1436 + \frac{12}{13}(2470) = 3716 \quad ''$$

$\therefore$ Suspect reading of 507 microstrain.

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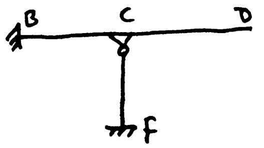
Q2: Extract from Examiners Report.

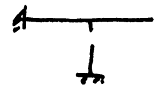
Q2. **Thin-walled structure and Mohr's circles of stress and strain (ELASTIC)**
Attempts: 220 out of 237. Average mark: 14/20.
A very popular question that was answered remarkably well. Almost all obtained the correct stresses and strains in the wall of the pressurized cylindrical shell, and could draw the appropriate Mohr's circles. There has been debate about whether introducing both stress and strain constructions into Part IB presents too much difficulty but the popularity and success at this question suggest that it does not. On thin-walled theory, a small proportion of students thought that the through-thickness strain was zero, even when they had successfully applied 3D Hooke's Law to calculate the in-plane strains. Parts b) and c) were rather challenging, requiring mastery of the two Mohr's circle constructions, yet a number of model answers were submitted. Regarding mistakes, a few thought that equal-and-opposite torques of magnitude T meant that shear stresses on the central section had to integrate to $2T$. The most common error was not to consider all three Mohr's circles when applying the shear stress, and thereby apply Tresca's criterion to an inner circle. For the strain gauges, only a few recognized that the principal axes of stress and strain coincide, such that angles to the directions of principal *strain* can be obtained from the Mohr's circle of *stress*. To find the strains along the 45 degree directions, many therefore erroneously rotated by 90 degrees from the principal strains, rather than from the hoop/longitudinal strains.

(n.b. The question should have specified "Assume linear, elastic, isotropic behaviour up until the yield point".)

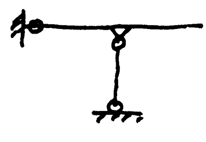
Q3.



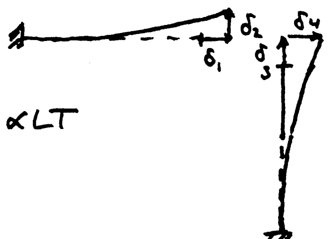
a) Number of redundancies = 2

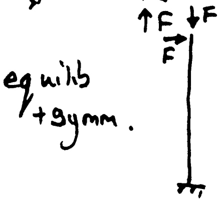
[e.g. weld up hinge → −1
make a cut  → +3 } → 2

or e.g. insert two hinges to obtain a 3-pinned arch



b) i) Ignore CD for now





$\delta_1 = \delta_2 = \delta_3 = \delta_4 = \alpha L T$
(compatibility and symmetry)

and $\delta_2 = \dfrac{FL^3}{3EI}$ (data book)

equilib + symm.

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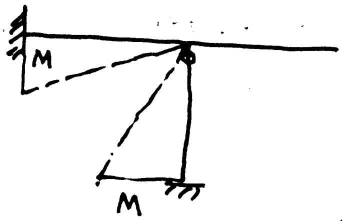
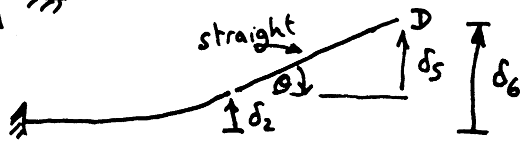
Q3 cont'd. $\therefore \dfrac{FL^3}{3EI} = \alpha LT$ $\quad\therefore F = \dfrac{3EI\alpha T}{L^2}$

BMD



$$\underline{\underline{M = FL = \frac{3EI\alpha T}{L}}}$$

bii) Displacements of D:

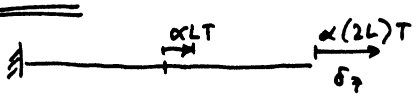


vert: $\delta_6 \uparrow = \delta_2 + \delta_5 = \delta_2 + \theta . L = \alpha LT + \left(\dfrac{FL^2}{2EI}\right) L$ ← Data Book

$= \alpha LT + \dfrac{3\cancel{EI}\alpha T}{\cancel{L^2}} \dfrac{\cancel{L^2} L}{2\cancel{EI}} = \alpha LT + \dfrac{3}{2}\alpha LT$

$= \underline{\underline{\dfrac{5}{2}\alpha LT}}$

horiz: $\underset{\rightarrow}{\delta_7}$ =



$= \underline{\underline{2\alpha LT}}$

rotation: $\theta_D = \theta_c = \theta = \frac{3}{2}\alpha LT$

c) With axial compressibility: $\delta_1 = \alpha LT - \dfrac{FL}{EA}$ (axially)

$\delta_2 = \dfrac{FL^3}{3EI}$ (transverse)

$\delta_1 = \delta_2 = \delta_3 = \delta_4$ (compatibility and symmetry)

$\therefore \dfrac{FL^3}{3EI} = \alpha LT - \dfrac{FL}{EA} \Rightarrow F\left(\dfrac{L^3}{3EI} + \dfrac{L}{EA}\right) = \alpha LT$

$\Rightarrow \dfrac{FL^3}{3EI}\left(1 + \dfrac{3I}{AL^2}\right) = \alpha LT \Rightarrow F = \underbrace{\dfrac{3EI\alpha T}{L^2}}_{\text{original value (part b)}} \Big/ \underbrace{\left(1 + \dfrac{3I}{AL^2}\right)}_{\text{reduction factor}}$

Moments $M = FL$ $\therefore$ reduced by factor → Q.E.D.

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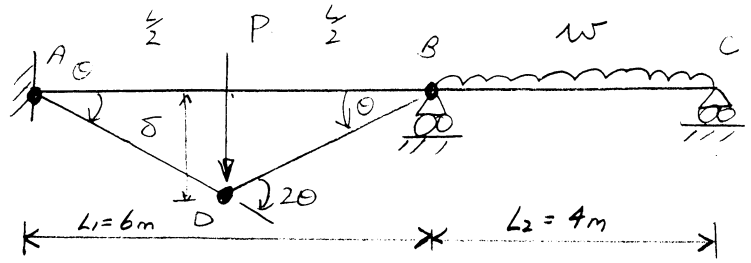
Q3. **Statically-indeterminate frame (ELASTIC)**
Attempts: 85 out of 237. Average mark: 10.6/20.
A modest number of attempts, loosely divisible into 'almost completely correct', and 'almost completely incorrect' responses. Full, correct answers often occupied only two sides of paper. Many students recognized 'by inspection' that the degree of indeterminacy was two ("weld up hinge then make a cut" or "add two hinges to get three-pinned arch"). However a large proportion applied Maxwell's rule for trusses to this frame, and all manner of unlikely predictions for the degree of indeterminacy were put forward.
All bar one attacked the problem using deflection coefficients, finding en route that, for this load case, symmetry reduced the problem to one of a single unknown. The most common mistake was to use the wrong deflection coefficients (e.g. for a cantilever with applied end-moment rather point load). One student tried to express the problem in the manner of Question 1 ($\mathbf{t} = \mathbf{t}_0 + x\mathbf{s}$, etc.) but did not get far.

QUESTION 4

(a) Upper bound solution.

Mechanism 1



Assume displacement at D, δ_D.

Work done $WD = P.\delta_D$

Energy dissipated $ED = M_p.4\theta$

$\tan\theta = \frac{\delta}{L/2} = \frac{2\delta}{L} \simeq \theta$

$WD = ED \qquad P\delta = M_p.4.\frac{2\delta}{L} \Rightarrow M_p = \frac{PL}{8}$

$\Rightarrow M_p = \frac{120\times 6}{8} = 90\ kNm \qquad \Rightarrow Z_p \geq \frac{M_p}{\sigma_y} = \frac{90\times 10^3}{300\times 10^6} = 300\times 10^{-6}\ m^3$ $(300\ cm^3)$

Databook ⇒ Require UB 254×102×25 ($Z_p = 306\ cm^3$) for span AB.

Other UB's include UB 203×133×30 ($Z_p = 314$), UB 305×102×25 ($Z_p = 342\ cm^3$)

Mechanism 2

Assume hinge in BC is at midspan.

$WD = wL.\frac{\delta_E}{2}$

$ED = M_p.(\theta_1 + 2\theta_1) = 3\theta_1 M_p \qquad \theta_1 = \frac{\delta_E}{L/2} = \frac{2\delta_E}{L}$

$WD = ED \qquad wL\frac{\delta_E}{2} = 3M_p.\frac{2\delta_E}{L}$

$M_p = \frac{wL^2}{12} = \frac{60\times 16}{12} = 80\ kNm$

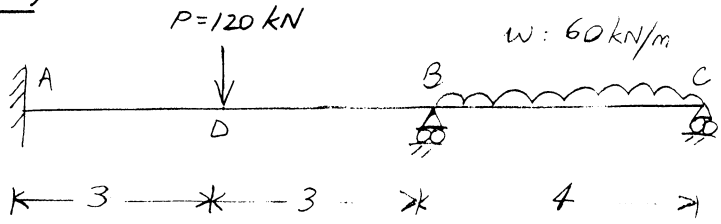
Since M_p in BC < M_p in AB require $M_p = 90\ kNm$

Want UB with cross-section for which $Z_p \geq 300\ cm^3$

From databook UB 254×102×25 has $A_{section} = 32.0\ cm^2$ $Z_p = 306$

other UB's include UB 203×133×30, UB 305×102×25

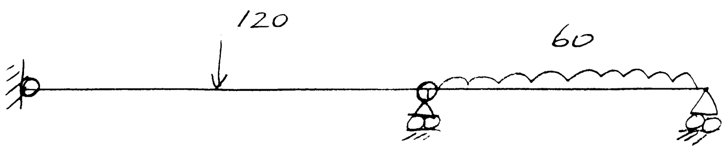
Q4 (cont.)



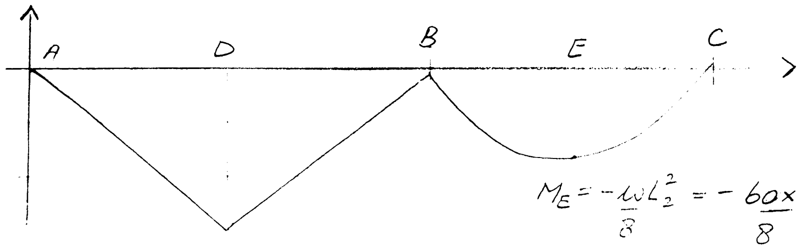
(b) Lower Bound Solution

Particular equilibrium soln/

Hold pins at A + B to make beam determinate.



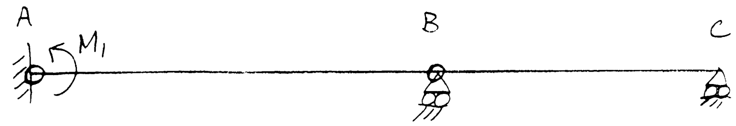
BMD



$M_E = -\frac{wL_2^2}{8} = -\frac{60\times16}{8} = -120\,kNm$

$M_D = -\frac{PL_1}{4} = -\frac{120\times6}{4} = -180\,kNm$

State of self-stress 1 (for pin at A)



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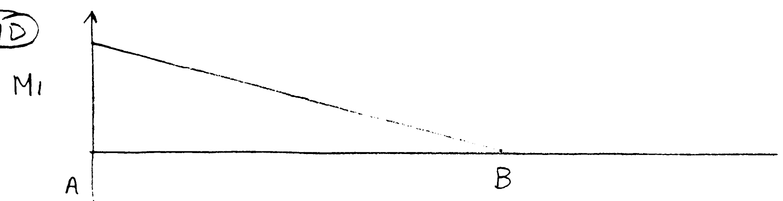
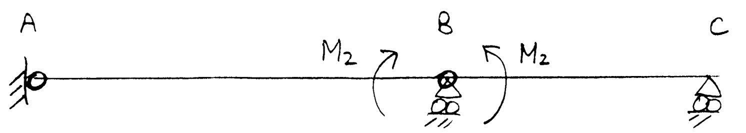


State of self-stress 2 (for pin at B)



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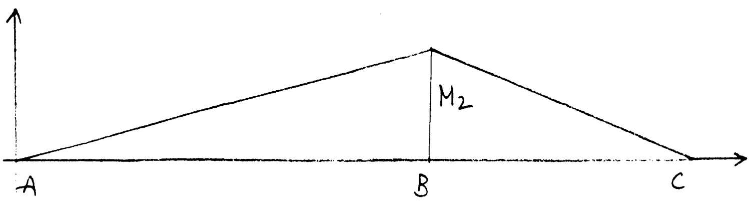
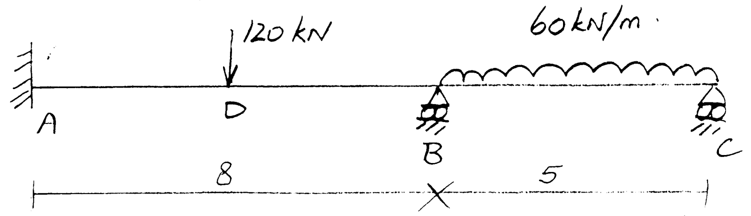
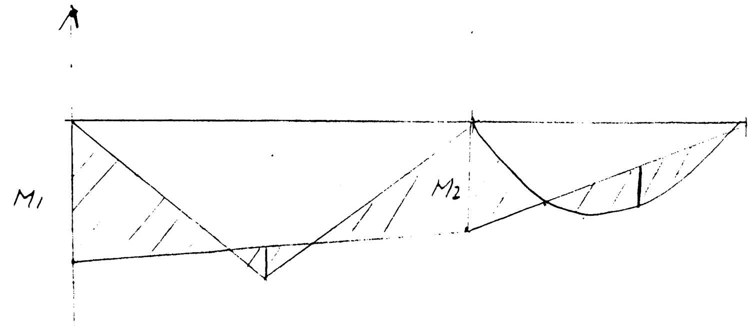
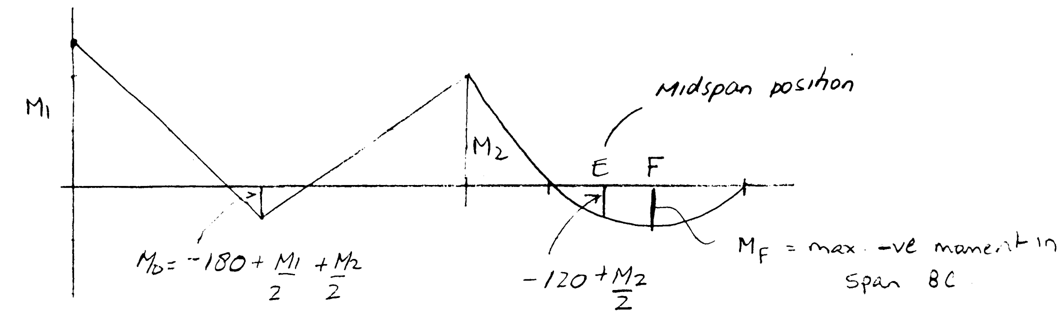


Q4 (cont.)

General bending moment diagram   (Add particular sol^n + Self-stress ($\times 2$))







For an optimum sol^n using smallest UB let $M_1 = M_2$

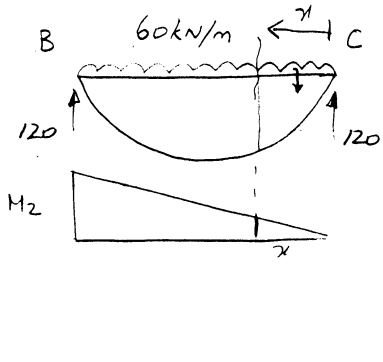
Optimise at D at midspan of AB then check other span BC.

Let $M_D = -M_1 = -M_2$

At D $M_D = -180 - \frac{M_D}{2} - \frac{M_D}{2}$ $\Rightarrow$ $M_D = -90$ kNm

$\therefore M_1 = M_2 = 90$ kNm

Check span BC - must ensure moment nowhere exceeds $|M_1| = 90$ kNm



$$M_x = 60 \times x \times \frac{x}{2} - 120 \cdot x + \frac{x}{4} \cdot M_2$$

$$= 30x^2 - 120x + \frac{90}{4} x$$

$$= 30x^2 - 120x + 22.5x = 30x^2 - 97.5x$$

Q 4 (cont.)

To find M_x maximum magnitude

$$\frac{dM_x}{dx} = 0 \Rightarrow 60x - 97.5 = 0 \qquad x = \frac{97.5}{60} = 1.625 \text{ from C.}$$

$$M_{BC\,max} = 30 \times 1.625^2 - 97.5 \times 1.625 = 79.22 - 158.44$$
$$= \underline{-79.22} \quad (<90 \text{ OK})$$

$$|M_{BC\,max}| < |M_1| \Rightarrow \text{OK.}$$

$|M| \leq 90$ kNm everywhere in span AC. Choose UB suitable for this moment.

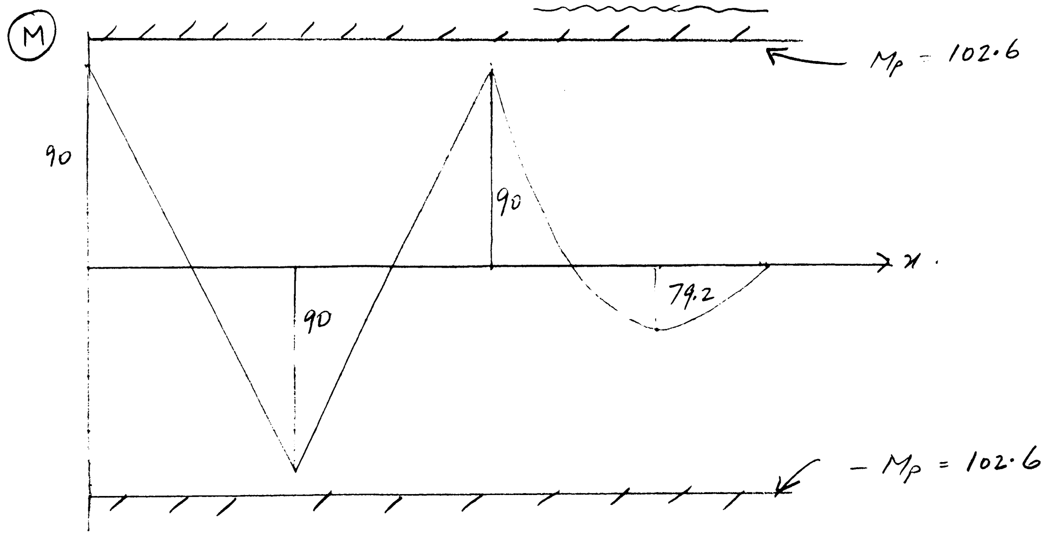
$$\sigma_y = \frac{M_p}{Z_p} \Rightarrow Z_p \geq \frac{M_p}{\sigma_y} = \frac{90 \times 10^3 \text{ Nm}}{300 \times 10^6 \text{ Nm}^{-2}} = 300 \times 10^{-6} \text{ m}^3$$

Require $\underline{Z_p \geq 300 \text{ cm}^3}$

From Structures Data Book. $\underline{\text{UB } 305 \times 102 \times 25}$ ($Z_p = 342$ cm^3, $A = 31.6$ cm^2)

This UB has the smallest cross-section for which $Z_p \geq 300$ cm^3.

$$M_p = Z_p . \sigma_y = 342 \times 10^{-6} \times 300 \times 10^6 \times 10^{-3}$$
$$= \underline{102.6 \text{ kNm}}$$



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Q4 (Cont.)

(c) If Support B settles 50mm there would be no effect on the collapse load (although the order in which plastic hinges formed may alter.) provided the steel beam was ductile.

(d) It would not be valid to perform the same upper bound plastic analysis if the beam was cast iron because cast iron is typically brittle and hence no plastic hinges could develop.

Question 4 Plastic collapse of a statically indeterminate beam (PLASTIC)
(220 attempts, average 111.6/20)

This question was very popular being attempted by 93% of candidates. Eight students obtained full marks whilst two obtained zero marks. The vast majority could identify the appropriate *upper bound* mechanisms in part (a) and calculate the required plastic modulus *Zp* for span AB under the point load however a frustratingly large number went on to assume energy was dissipated at the simply supported hinge at C in span BC and wrote $ED = 4Mp\theta$ rather than the correct value of $3Mp\theta$ for this case. Units were again often incorrectly specified or ignored altogether with some extraordinary combinations being given for bending moment in some cases. The other most common error was to choose the lower value of $Mp = 80$ kNm from span BC as the critical value rather than the higher value of $Mp = 90$ kNm from span AB when selecting the appropriate member size for the entire structure from A to C. A few used values of *Zyy* rather than *Zxx* when selecting the appropriate Universal Beam from the databook.

The *lower bound* part (b) proved more problematic and was marked very generously as a result. This was a quite difficult question although the overall methodology was well presented by a high proportion of candidates. A number of candidates failed to appreciate that this was a plastic analysis question and headed off in pursuit of a purely elastic solution using deflection coefficients from the data book. The answers to part(c) were divided quite evenly between those that seemed to have listened in lectures and those that had missed the point that residual stresses do not effect the collapse load of such a structure. Part (d) was well answered by most students who recognised that ductility is a fundamental requirement when applying plastic analysis. Understandably the *lower bound* theory is found to be somewhat difficult by many students and may need further attention in the lecture course next year.

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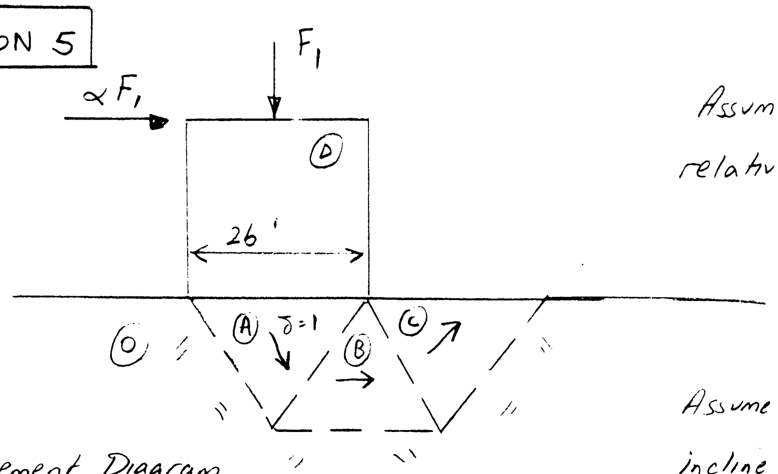
QUESTION 5

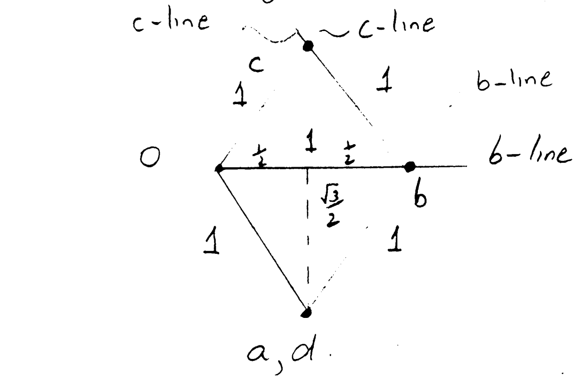
(a)



Assume (D) does not slip relative to (A)

Assume (A) moves $\delta = 1$ down incline of surface oa.

Displacement Diagram



$$WD = F_1 \frac{\sqrt{3}}{2} + \alpha F_1 \frac{1}{2}$$

$$ED = k.1.\{\underset{oa}{2b.1} + \underset{ab}{2b.1} + \underset{bo}{2b.1} + \underset{cb}{2b.1} + \underset{co}{2b.1}\}$$

$$= 10kb$$

$$WD = ED$$

$$\therefore F_1\left(\frac{\sqrt{3}+\alpha}{2}\right) = 10kb$$

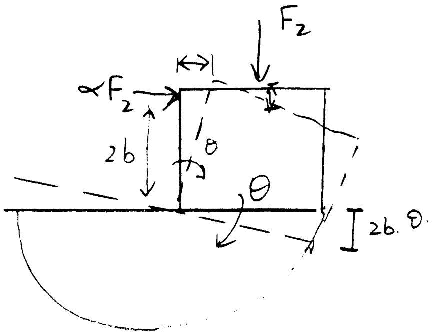
$$F_1 = \frac{20\,kb}{(\sqrt{3}+\alpha)}$$

$\alpha = 0$ $F_1 = 11.55kb$

$\alpha = 0.5$ $F_1 = 8.96kb$

$\alpha = 1$ $F_1 = 7.32\,kb$

(b)



Assume rotation is θ.

$$WD = F_2\, b.\theta + \alpha F_2\, 2b.\theta$$

$$= F_2 b\theta(1+2\alpha)$$

$$ED = k.1.\ \pi.2b.\ 2b.\ \theta.$$

$$= 4\pi\theta\, kb^2$$

$$WD = ED$$

$$F_2 b\theta(1+2\alpha) = 4\pi\theta kb^2$$

$$F_2 = \frac{4\pi kb}{(1+2\alpha)} = \frac{12.57\,kb}{(1+2\alpha)}$$

$\alpha = 0$ $F_2 = 12.57\,kb$

$\alpha = 0.5$ $F_2 = 6.29\,kb$

$\alpha = 1$ $F_2 = 4.2\,kb.$

5/2

Q5 (c) Put $\frac{20\,kb}{(\sqrt{3}+\alpha)} = \frac{4\pi\,kb}{(1+2\alpha)}$ when $F_1 = F_2$

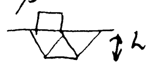
$\therefore\ \cancel{20}^{5}kb + \cancel{40}^{10}kb\alpha = \cancel{4}\sqrt{3}\,\pi kb + \cancel{4}\pi\alpha kb$

$\therefore\ \alpha = \frac{\sqrt{3}\pi - 5}{10-\pi} = \frac{0.44}{6.86} = 0.064$ $\quad F_1$ Mech ① $= F_2$ Mech ②

If $\alpha > 0.064$ Mechanism 6b is critical i.e. $F_2 < F_1$

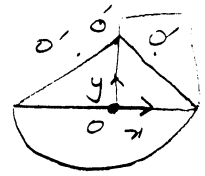
* $\alpha < 0.064$ " 6a is critical $F_1 < F_2$ *

(d) Mech. 1: Optimise geometry by finding $\frac{dF}{dh} = 0$ where h = height of triangles to find optimum h.



Mech 2: Optimise geometry by varying position of centre (O) of slip circle (ideally optimise in both $x + y$ position by moving O to O').

i.e.



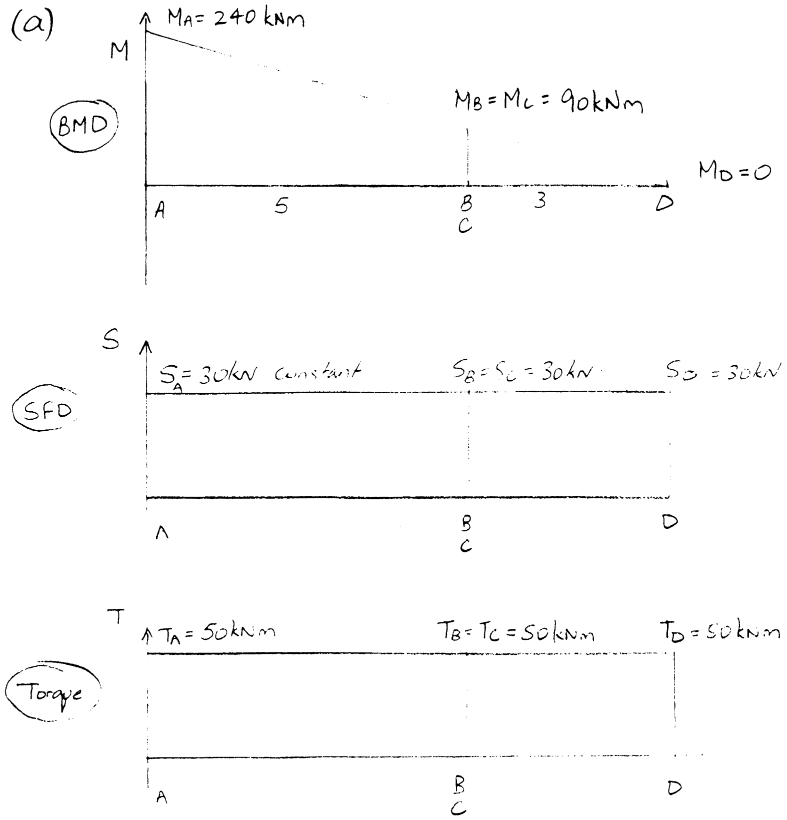
Question 5 Slip plane analysis of continua (PLASTIC)
(105 attempts, average 13.5/20)

This question was unpopular but well answered by those that attempted it. Four students obtained full marks for this question. It is likely that many students chose not to even revise this topic as it had not turned up in an exam for many years. It was a relatively straightforward slip-plane question quite similar to those asked in the examples problem sheet.

The most common mistake for Mechanism 1 was to assume the wedge underneath the block underwent a purely vertical displacement rather than sliding down the inclined surface at an angle of 60 degrees to the horizontal. As a result the displacement diagrams generated were also incorrect. Most found Mechanism 2 much easier to analyse with many getting full marks for this section. Parts (c) and (d) were well answered by most candidates.

QUESTION 6 — IB PAPER 2 - STRUCTURES 2002 — 6/1

(a)



(b) Section 1 AB — OD = 500mm, $t_1 = 7$mm, $r_1 = 250$mm.

$$J = \int r^2 \, dA = \int_0^{2\pi} r^2 \, r t \, d\theta = \left[r^3 t \theta \right]_0^{2\pi} = 2\pi r^3 t$$

$$J = I_{xx} + I_{yy} \Rightarrow I_{xx} = \frac{J}{2}.$$

$$J_1 = 2\pi r_1^3 t_1 = 2\pi \; 250^3 . 7 = 687 \times 10^6 \text{ mm}^4$$

$$I_1 = J_1/2 = 343 \times 10^6 \text{ mm}^4$$

Steel — $G = 81$ GPa — $E = 210$GPa

$$\boxed{GJ_1 = 81 \times 10^3 \times 687 \times 10^6 = 55.7 \times 10^{12} \text{ Nmm}^2.} \quad (55.7 \times 10^6 \text{ Nm}^2)$$

$$EI_1 = 210 \times 10^3 \times 343 \times 10^6 = 72.2 \times 10^{12} \text{ Nmm}^2 \quad (72.2 \times 10^6 \text{ Nm}^2)$$

Section 2 CD — OD = 400 mm, $t_2 = 5$mm, $r_2 = 200$mm.

$$J_2 = 2\pi r_2^3 t_2 = 2\pi \times 200^3 \, 5 = 251 \times 10^6 \text{ mm}^4$$

$$I_2 = J_2/2 = 126 \times 10^6 \text{ mm}^4$$

$$\boxed{GJ_2 = 20.4 \times 10^{12} \text{ Nmm}^2} \quad (20.4 \times 10^6 \text{ Nm}^2)$$

$$EI_2 = 26.4 \times 10^{12} \text{ Nmm}^2 \quad (26.4 \times 10^6 \text{ Nm}^2)$$

Q6 (cont.)

(c) $\frac{T}{\phi} = GJ \Rightarrow \phi = \frac{T}{GJ}$

for AB $\phi_1 = \frac{T}{GJ_1} = \frac{50\times10^3}{55.7\times10^6} = 897.7\times10^{-6}$ rads/m

$\theta_1 = \phi_1 . L_1 = 897.7\times10^{-6}\times 5 = 4.49\times10^{-3}$ rads (0.26 degs)

For CD: $\phi_2 = \frac{T}{GJ_2} = \frac{50\times10^3}{20.4\times10^6} = 2.45\times10^{-3}$ rads/m

$\theta_2 = \phi_2 . L_2 = 2.45\times10^{-3}\times 3 = 7.35\times10^{-3}$ rads (0.42 degs)

$\Rightarrow$ Total rotation $\theta = \theta_1 + \theta_2 = 0.26 + 0.42 = 0.68$ degrees (11.8×10^{-3} rads)

(d) (i) Location A. Axial stress $\sigma_a = \frac{P}{A_1} = \frac{500\times10^3}{2\pi r_1 . t_1} = \frac{500\times10^3}{2\pi\times250\times7} = 45.5\ N/mm^2$

Bending stress $\sigma_b = \frac{My}{I} = \frac{240\times10^3\times0.25\times10^{-6}}{343\times10^{-6}} = 174.9\ N/mm^2$

$\Rightarrow$ Max. longitudinal stress $\sigma_L = 174.9 + 45.5 = \underline{220.4\ N/mm^2}$

Shear stress due to torque $\tau_T = \frac{T}{2A_e t} = \frac{50\times10^3}{2\pi r_1^2 t_1} = \frac{50\times10^3\times10^3}{2\pi\times250^2\times7} = \underline{18.2\ N/mm^2}$

Shear stress due to shear force $\tau_s = \frac{SA_c\bar{y}}{I.t}$ where $A_c = 0 \Rightarrow \tau_s = 0$

$\Rightarrow$ Shear stress at A $= \underline{18.2\ N/mm^2}$

(ii) Location G Axial stress $\sigma_a = 45.5\ N/mm^2$ as in (i)

Bending stress $\sigma_b = 0$ at neutral axis where $y = 0$.

$\Rightarrow \underline{\sigma_L = 45.5\ N/mm^2}$

τ_T

Shear stress due to torque $\tau_T = 18.2\ N/mm^2$ as in (i)

Also $\tau_s = \frac{SA_c\bar{y}}{I.2t}$ where $\bar{y} = \frac{2r}{\pi} = \frac{2\times250}{\pi} = 159.2$ mm ; $A_c = \frac{A_1}{2} = \pi r_1 t = \pi\times250\times7$
$= 5.498\times10^3\ mm^2$

τ_s

$\therefore \tau_s = \frac{30\times10^3\times5498\times159.2}{343\times10^6\times2\times7} = 5.5\ N/mm^2 \Rightarrow \underline{\tau_G = 18.2 - 5.5 = 12.7\ N/mm^2}$

Question 6 Thin walled structure (ELASTIC)
(191 attempts, average 11.0/20)

Four students obtained full marks for this question. This was a standard problem on the response of a thin walled structure to bending, torsion and axial loading. It was very similar in principle to questions on this topic in several exams in recent years although a slight complication was introduced by adopting a circular cross-section and dividing the pole into two different sized sections. In part (a) many students failed to read the question properly. To obtain full marks both a sketch of the force resultants *and* the values at the specified points were required. Too often either just a sketch or just calculated values were given. Many still found difficulty in drawing the bending moment diagram, often incorrectly showing a step in moment at the change in section at BC. The calculation of rotation in part (c) also proved difficult with many confused over the term for the enclosed area, A_e and also confusing J with I. In part (d)(i) relatively few students recognised that the shear stress on the line of symmetry at A due to the applied load *F* was zero although most were able to obtain the uniform shear stress resulting from the torsion *T*. Another common error was to forget to include provision for the axial load on the longitudinal stress on the section.