

Answers 1B P3 Materials 2002

1) The moduli of metals and ceramics vary over a similar range. This is because the bond energies of ionic, metallic and co-valent bonds are similar, and the variation with distance is also similar (ionic bonds are the softest being due to the inverse square law).

Most ceramics are covalent compounds which means that the bonds are directional. This makes it very difficult for dislocations to move so most ceramics have a very high yield stress. Ionic ceramics are held together by non-directional electrostatic forces and have a much lower yield stress. However this can only occur for the small range of angles for which oppositely charged ions are not brought into contact so in practice ionic ceramics can only undergo plastic flow in very carefully aligned single crystals. In general ionic ceramics are not used in structures.

Metals have a very wide range of yield stresses. Since the metallic bond is non directional dislocations can move easily, so in pure metals the yield stress is very low. Adding pinning centres can increase the yield stress to something close to that of ceramics.

A high yield stress causes low fracture toughness since cracks are not blunted by plastic flow, so ceramics are much more brittle than metals. This is made worse by the fact that ceramics are normally made by sintering solid powders, rather than casting, so there is always a number of internal flaws and residual stresses.

Ceramics are made by hot sintering of powders and although there is not much we can do to improve fracture toughness (unless we use composites), we can improve the fracture stress by minimising the flaw size. This is done by using fine powders and careful mixing to remove holes and air pockets, and avoiding high temperature annealing which increase the grain size.

From the expression for Weibull statistics the probability of survival of the test samples is related to the stress by $\ln(P) = -(\sigma/\sigma_0)^m$. Therefore the graph of $\ln(-\ln(P))$ against σ gives a straight line from which $m=5$ and $\sigma_0=125\text{MPa}$.

For the sample shown the two end sections can be treated as a single unit of volume 2.28 cm^3 under a stress 84.2 MPa . The probability of survival is 0.96 . The central section is of volume 0.302 cm^3 and is under stress 159MPa so the probability of survival is 0.881 . The probability of the sample surviving is the product of the two probabilities, 0.85 .

The sharp internal corners should be rounded to remove stress concentrations. Proof testing will eliminate weaker samples.

Comments

On the whole this was pretty well answered with an average of 11.8. The Weibull question always attracts a good many answers as it has a large numerical component. Many candidates did the numerical part accurately and efficiently, sometimes using approximate short cuts rather than plotting a graph. This is to be encouraged although the points were made much more accurately linear than would be the case for a real test. Quite a few said the pillar would fail in the central section because of the 'weakest link' effect. Some others used $1 -$ (the probability of survival), and added the probabilities, but most who got this far did the statistics correctly. Nearly 100% said that the sharp corners should be rounded in the last section so at least they have learnt something useful in the year.

In the first section there were a few answers on the lines of 'ceramics are more brittle than metals and therefore have a lower fracture toughness', (nul points). However a large proportion knew that the difference in yield stress was due to the directional nature of co-valent bonds and quite a lot could point out the difference between this and the ionic repulsion in ionic ceramics, even bringing in the number of slip planes. However almost nobody pointed out the enormous range of the yield stress in metals compared with ceramics.

The main faults were omitting any reference to dislocations or plastic flow at crack tips, they relied on vague arguments about bond breaking. They also tended to lump the modulus in with the yield stress with the same explanation. Another common error was to confuse yield stress with failure stress so that many said that the yield stress of ceramics in tension was much less than in compression.

2)

The unmarked regions are two phase regions containing the phases on the edges, e.g. $\beta+\gamma$ between β and γ . There is an ambiguity in the low temperature field on the right hand edge caused by the fact that the diagram, originally from Ashby and Jones, has cut off a horizontal line at 600. The last field is in fact Liquid plus A

A is an intermetallic compound $TiAl_3$.

There are no eutectics or eutectoids.

There are peritectics at 1460C, 52 At% Ti and 1350C and 75% Ti

There is a peritectoid at 1250C and 45 at% Ti.

Cooling 30% Al :-

At 1620C a slightly Ti rich precipitate appears leaving a slightly Al liquid.

At 1600C the whole system is solid β at 30% Al.

At 1180C precipitates of slightly Al α phase appear.

At 1150C These redissolve to give a uniform single phase solid solution of α Ti at 30% Al. The material 100% α .

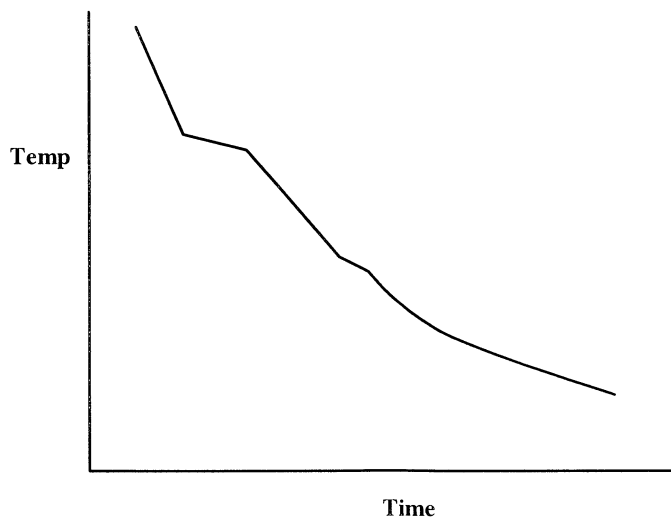
Cooling 75% Al :-

At 1370C Precipitates of γ appear in the liquid leaving an Al rich liquid.

At 1350C the γ solid and Al rich liquid combine to form the solid compound $TiAl_3$ in a peritectic reaction. No further changes take place on cooling so the composition is 100% $TiAl_3$.

Rapid cooling prevents the whole system from coming to equilibrium since phase changes normally require diffusion of atoms, which takes time. Therefore if two phases have to combine to make a single equilibrium phase it is likely that this will be incomplete and parts of the two high temperature phases will remain in the room temperature microstructure. Also if precipitates form, the composition in the centre will be different from the outside if there is not time for the composition to become uniform. This is called segregation. Other effects are the formation of martensite in steels (hard) and the inhibition of precipitation in Al alloys (soft).

The cooling curve:-



The basic shape is exponential. There will be short sections of reduced gradient at 1550 and 1220. They are not flat as the phase change takes place over a range of temperatures. The 1220 change will be much less noticeable as it is a solid state reaction. There will also be a change of slope at 1100 where the γ begins to precipitate but this will be very small.

Comments

The phase diagram question is always popular and well done, average 13.2. This year it used a complex diagram, but the questions were easy. All could label the phase diagram but about half the answers did not identify A as the intermetallic compound Al_3Ti . Many put in eutectics at the edges of the peritectic, for which marks were lost. The cooling was simple and nearly all got full marks in this bit. Only a small minority tried to use the lever rule in the final single phase microstructure. A few candidates got the Ti Al ratio the wrong way round. Most could produce at least two examples of the effects of fast cooling. About half had clearly not remembered the point of a cooling curve and drew a TTT diagram for steel. Of those who did remember, few described the basic exponential cooling and nearly all put in horizontal sections although the phase changes take place over a range of temperatures.

3) Polymers differ in four main respects

a) Degree of Cross Linking.

This is the main determining factor of the modulus above the glass temperature. The more cross links the stiffer, and the higher the yield stress. If the cross linkage is high the extra van der Waals bonds at low temperatures add little, so there is not much change in properties on cooling to low temperatures.

b) Nature and Complexity of Side chains

Large side chains inhibit plastic flow and so reduce creep and increase the yield stress. Polar side chains lead to stronger van der Waals interactions than simple atoms, like Hydrogen, and so give a higher glass temperature. They also inhibit crystallisation so tend to lead to transparent polymers.

c) Degree of crystallinity.

The main effect of crystals in the polymer is to make them translucent, amorphous polymers are transparent.

d) Chain alignment

If the polymer is stretched before cooling this makes the polymer chains straight and the covalent bonds determine the modulus. The material has a much higher modulus and yield stress in the aligned direction.

E depends mainly on the degree of cross linking. However large side groups also will also stiffen it. At low temperatures polymers with little cross linking become much stiffer due to van der Waals forces. The viscosity behaves in a similar way. However it is thermally activated so varies rapidly with temperature, changing suddenly at the glass temperature.

$$\sigma = E(\epsilon - y) \quad \text{and} \quad \dot{\sigma} = \eta \, dy/dt.$$

$$\text{For } \sigma = \sigma_0 \exp(j\omega t)$$

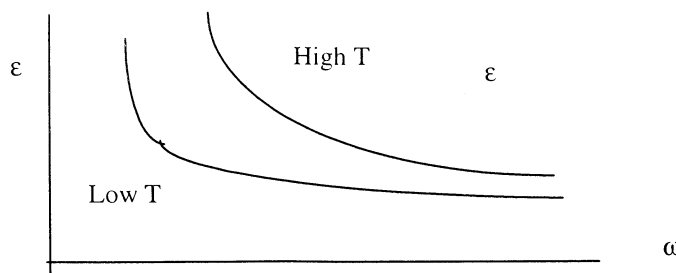
$$\sigma_0 = E(\epsilon_0 - y_0) \quad \text{and} \quad \dot{\sigma}_0 = \eta \, j\omega y_0$$

Eliminating y_0 :-

$$\sigma_0 = E_2(\epsilon_0 - \sigma_0/E_1) + j\omega\eta(\epsilon_0 - \sigma_0/E_1)$$

$$\epsilon_0 = (\sigma_0/E)(1 + E/j\omega\eta). \quad |\epsilon_0| = (|\sigma_0|/E)(1 + (E/\omega\eta)^2)$$

Since E varies relatively slowly with temperature a plot of ϵ_0 against $\omega \exp(-U/kT)$ should give a universal graph. The behaviour at low frequencies and low temperatures is the same as at high frequencies and high temperatures.



The instantaneous power is $\sigma \, d\epsilon/dt$. Therefore by analogy with electrical loss the mean loss in watts/vol is the real part of $\frac{1}{2} j\omega \sigma_0^* \epsilon_0 = \frac{1}{2} \sigma_0^2 / \eta$.

Comments

The average was 7.1

The first part of this question provided some easy marks based on the notes and the practical. The second bit was a simple spring and dashpot model which would have been trivial if it had appeared on a mechanics paper but seemed to cause considerable difficulties in a materials paper. The low average can be partly explained by candidates running out of time in what was admittedly a long paper but very few candidates could apply phasor techniques to this very simple mechanism. Even with very generous marking the average remained low.

4) Elastic strain energy is $\frac{1}{2}$ stress \times strain (per unit volume), which in tension leads directly to a maximum value at the elastic limit of $=\sigma_f^2/2E$. The same combination of properties holds for other modes of loading such as bending, which can be shown in various ways. For example, consider a beam $b \times w \times L$ under a pure moment. The stress varies linearly from 0 at the neutral axis to σ_f at the surface so at position y from the axis $\sigma=2\sigma_f y/b$. The local elastic strain energy per unit volume is $\frac{1}{2}$ stress \times strain $=2\sigma_f^2 y^2/b^2 E$. Integrating a volume element $w \times L \, dy$ over the full depth gives the total energy as $\sigma_f^2 b w L/6E$. Hence energy per unit volume $=\sigma_f^2/6E$ (so only a numerical change – the merit index is the same as in tension: σ_f^2/E). Alternatively, the stored energy in pure bending is moment \times curvature \times length, which can be derived easily from the Structures databook formulae.

The index for elastic strain energy per unit mass is then $\sigma_f^2/E\rho$. The modulus-strength selection chart in the databook can be used for the index on energy per unit volume (guide lines are marked, with the best materials towards the bottom right). The highest energy densities are in ceramics followed by elastomers and (high strength) steels. Ceramics are too brittle, while elastomers would need too large a volume for a watch, so steel is best.

A selection chart was not provided for the index $\sigma_f^2/E\rho$. (need specific modulus E/ρ – specific strength σ_f/ρ for this) but can consider the materials with high σ_f^2/E which also have low density (elastomers, fibre composites) – or substitute in a few selected values. Energy storage at low mass is required where weight is paramount and size is not such an issue – e.g. for model aeroplanes or space applications. Other important considerations are: availability in required size, ease of manufacturing and fixing to the rest of the equipment, fatigue and corrosion resistance, cost.

In a shaped section the centre of the spring is not stressed, so greater stiffness is achieved at a given weight (shape factor indicates how much stiffer than a solid cylinder). Hence we expect to get a higher energy density with a hollow bar or an I beam. This saves weight or volume, but need to beware of problems such as buckling in the thinner-walled shaped section on the compressive side. In the case of the watch, weight is not important and the volume saved is probably unusable by any other part of the watch, and so shaping is not very useful.

The dimensions of the spring are determined by the force required to drive the mechanism, and must therefore be optimised for the selected material, subject to the space restrictions.

Comments

This was not a very popular question, the average was: 9.8.

Rather few could derive the index, though credit was given for giving the result for pure tension (as on the examples paper). Many were unable to apply the index to the chart correctly, automatically going to the top left (and choosing tin or cast iron !) rather than bottom right – this in spite of the fact that the chart is even labelled “max. energy storage per unit volume” in the bottom right. It was apparent that those who struggled were unfamiliar with the problem on the examples paper and the selection charts in the databook, often just guessing $E^{1/2}/\rho$ for the mass-limited case.

5) Hot forging:- fast, less mechanical energy needed, complex shapes, but heating needs energy and the final product is relatively soft.

Cold forging:- no heating required, work hardened product, more mechanical work needed, less complex shapes possible.

Consider a sample in tension, cross sectional area A , tensile force F just about to yield. The shear stress across a plane at an angle θ to the axis is $(F \cos\theta)/(A/\sin(\theta))$. This is a maximum at 45 degrees

when the shear stress is $\sigma_y/2$. Hence $k=\sigma_y/2$. Alternatively sketch Mohr's circle for pure tension or pure shear, with diameter = σ_y and radius = k .

The Tresca yield criterion says that a sample under multi-axial loading will yield when the maximum difference in the principal stresses reaches σ_y . (This can be used to prove $k=\sigma_y/2$).

Take unit depth of material, and consider a slice x from the centre, width δx with compressive stress (positive) in x direction. Stress at x is σ_x while stress at $x+\delta x$ is $\sigma_x+\delta\sigma_x$. The force/depth due to shear on the (two) surfaces is $2k\delta x$.

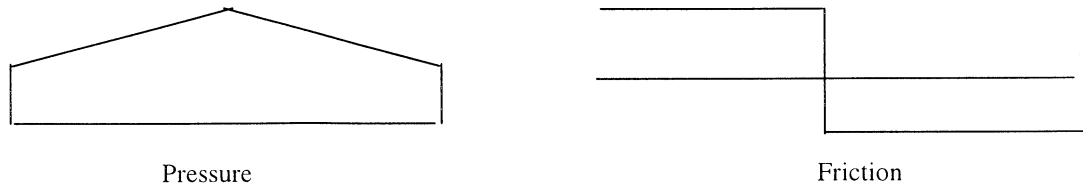
Hence $h \delta\sigma_x = -2k \delta x$, so $d\sigma_x/dx = -\sigma_y/h$. Integrating with boundary condition $\sigma_x = 0$ at $x = w/2$, $\sigma_x = \sigma_y(w/2-x)/h$

From Tresca at yield $p-\sigma_x = \sigma_y$ so $p/\sigma_y = 1+(w/2-x)/h$.

Alternatively, from Tresca: $dp - d\sigma_x = 0$, so change to differential in p : $dp/dx = -\sigma_y/h$ and use boundary condition $p = \sigma_y$ at $x = w/2$.

The mean pressure is $\sigma_y(1+w/4h)$.

Friction stress constant ($=k$), and reverses direction at neutral axis.
Linear friction hill.



Used as a model for rolling, the net force from the friction hill is offset from the axis of the rolls, hence a torque is required. The friction is necessary for the slab to be pulled into the roll bite. The usual analogy with rolling considers Coulomb friction rather than sticking friction, but the consequences are the same.

Neglect the difference in speed between input and output, and neglect friction (so compressive stress is σ_y). Effective vertical displacement is $10^{-7} = 3\text{mm}$.

Work done per metre = stress \times width \times displacement = $60 \times 10^6 \times 0.4 \times 0.003 = 72\text{kJ/m}$

Power = work done per metre \times speed = $72 \times 10^3 \times 4 = 288\text{ kW}$

Power per roll = $288/2 = 144\text{ kW}$

Angular velocity $\omega = v/r = 4/0.06 = 67\text{ rad/s}$.

Torque on each roller = Power/ $\omega = 144 \times 10^3 / 67 = 2.15\text{ kNm}$.

Comments

Not a very popular question, average 10.2.

Some candidates confused τ_y for single crystal shear with k , the shear yield stress. Most could set up the differential equation for σ_x but then made other errors: introducing μp for the shear stress, failing to exchange k for σ_y , or using incorrect boundary conditions. Many attempts at the last part were over-elaborate, trying to use the result from part (b) even though the question said the compressive stress could be taken as σ_y .

6) Hardening steels requires a rapid quench to form martensite. In simple carbon steels the quench must be very rapid which makes it impossible to harden anything except the surface. Therefore alloying elements are added to delay the transition and allow larger samples to be hardened uniformly. The lower the quench rate required to form martensite the greater the 'hardenability'. 'Weldability' is a measure of the ability of a material to be welded without losing its strength or toughness. Rapid cooling near a weld can form martensite which is hard, but brittle, and can therefore initiate failure at the weld. The greater the hardenability the lower the cooling rate to form martensite, and the more likely it is that martensite will form. The only hardening mechanism that can survive welding without change is solution hardening, as in stainless steel. Some precipitation hardening can also occur in alloy steels, but it is likely that the alloy

additions will also increase hardenability (e.g. Ti, W). Work hardening is no good, since the HAZ is likely to recrystallise, and the welded component cannot subsequently be worked. Similarly a complete heat treatment such as normalising or quench/temper after welding is unlikely to be practical. One possible solution to the problem in steels is to post-weld heat treat (to temper any martensite formed). In some other classes of alloys, precipitation hardening can occur at room temperature after welding to restore some strength (e.g. natural ageing of 7000 series aluminium alloys).

Fig.4 shows a low hardenability material since a very rapid quench is needed to produce martensite. Ac3 is the point at which austenite becomes unstable and ferrite begins to form.

Ac1 is the eutectoid temperature below which it is also possible to form iron carbide (either within pearlite or bainite).

Ao is the carbide line which shows the time at a given temperature for ferrite formation to switch to ferrite+carbide.

In general the rate at which a phase transformation proceeds depends on the undercooling and the diffusion rate. The further from the transition temperature the greater the driving force, but the lower the temperature the slower the diffusion. There is a maximum rate at some temperature below the transition temperature which is reflected in the 'nose' of the TTT diagram. Both nucleation and growth depend on the same two temperature-dependent effects, and these combine into a single transformation rate curve with a peak at a critical undercooling – hence the shape of the C-curves when plotted against time to a given fraction transformed.

Ac1 and Ac3 can be determined from the phase diagram (not Ao, which depends on transformation kinetics, or Ms which is for a metastable phase transformation).

Since Ac3 occurs at 800C, Fig. 4.7 in the data book shows that there is about 0.5 wt% carbon in it (the hyper-eutectoid composition with 800C at the bottom of the austenite field is not correct since ferrite is forming first on the TTT diagram).

The Ao line tends asymptotically to Ac1, since the rate at which carbide appears depends on the undercooling below the Ac1 line. If we are just on the eutectoid temperature, the driving force for carbide falls to zero and we will need an infinite time for it to appear. Note that this is *not* the case for the equilibrium fraction of ferrite, since there is a driving force to form this ferrite provided by the undercooling below Ac3 (i.e. this would also form just above Ac1, when there would certainly be no carbide).

Austenitising means holding the sample in γ phase region of the phase diagram. Provided the temperature is above the Ac3 line and below the melting point the only effect of the temperature is to determine the rate at which the initial α iron/iron carbide changes to γ and dissolves the carbon. The hotter the faster, but heating costs will be greater – though grain growth may also occur if too hot, which increases the hardenability.

Ac3 defines an equilibrium phase boundary, whereas Ms relates to a metastable transformation. The equilibrium transformations below Ac3 depend on temperature and time (being diffusive), whereas the martensite transformation depends on undercooling only – it is a displacive shear transformation occurring effectively instantaneously (about the speed of sound in steel). Further undercooling is required below Ms to complete the reaction, since more driving force is needed to overcome the strain energy induced in the distorted bcc lattice by the supersaturation of carbon.

- a) 100% martensite (with perhaps a few % α iron, as this would occur in one second at the nose of the TTT diagram).
- b) 90% bainite, 10% martensite
- c) 40% α iron, 10% pearlite, 50% martensite

All samples will have much the same modulus.

- a) Will have a very high yield stress and low toughness.
- b) Will have a fairly high yield stress and good toughness.
- c) With 50% martensite this will be hard and brittle, but not to the same extent as a.

Samples containing martensite could be tempered which reduces the yield strength somewhat, but greatly increases the toughness.

Comments

A very popular question and done well, average: 12.8

Full marks could be obtained without all of the points above, since the question was over-long. The commonest errors, all made by a significant number of students were:

- suggesting surface hardening techniques in (a), which are irrelevant to the strength of a weld
- confusing temperature dependences of “nucleation and growth” with “driving force and rate of diffusion”
- describing austenite as “melt” (even FCC melt in one case), and confusing eutectoid and eutectic
- thinking that a hold just below Ac1 would eventually lead to 100% ferrite (as opposed to the equilibrium fraction)
- thinking that the modulus would follow the yield strength in the last part of the question, when it will be unchanged.

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2002