Thermofluid Mechanics.

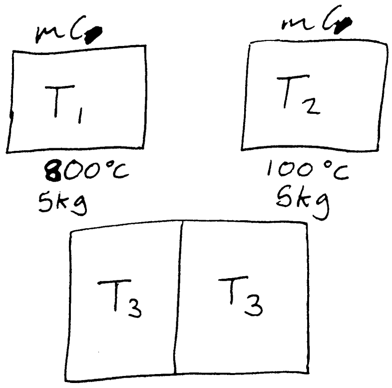
PAD

IB PAPER 4, 2001/2002

CRIB

1 (a) $\quad$ 1

$c_{IRON} = 437\ J/kgK$



$$2T_3 = T_1 + T_2$$

$$2T_3 = 2 \times 273 + 900 = 723 \times 2$$

$$\underline{\underline{T_3 = 723\,K}}$$

$$dQ = mC\,dT$$

$$dS = \frac{dQ_{rev}}{T} = mC\frac{dT}{T}$$

$$\Delta S = S_1 - S_2 = \int_1^2 \frac{dQ_{rev}}{T} = mC\ln\frac{T_2}{T_1}$$

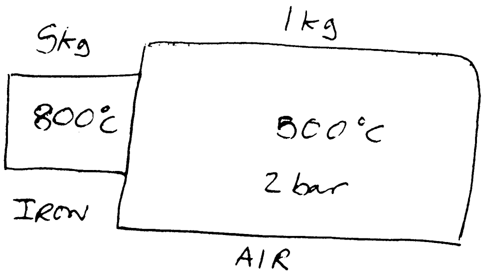
$$\Delta S_{system} = \Delta S_1 + \Delta S_2 = mC\left[\ln\left(\frac{T_3}{T_1}\right) + \ln\left(\frac{T_3}{T_1}\right)\right]$$

$$\Delta S_{sy} = 5 \times 437\left(\ln\left(\frac{723}{800+273}\right) + \ln\left(\frac{723}{100+273}\right)\right)$$

$$\Delta S_{sy} = 5 \times 437 \times 0.267$$

$$\underline{\underline{\Delta S_{sy} = 583.5\ J/K}}$$

1 (b)



$C_v = C_p - R$

$$\underbrace{M\,C\,\Delta T}_{\text{IRON}} + \underbrace{M\,C_v\,\Delta T}_{\text{AIR}} = 0$$

$$1 \times C_v \times \left(T_{FINAL} - (600+273)\right) = 5 \times 437\,(800+273-T_{FINAL}$$

$$T_{FINAL} = \frac{727 \times (500+273) + 5 \times 437 \times (800+273)}{727 + 5 \times 437}$$

$$T_{FINAL} = \underline{998.1\ k}$$

$PV = mRT$ IDEAL GAS

$$\therefore \frac{P_F}{P_I} = \frac{T_F}{T_I}$$

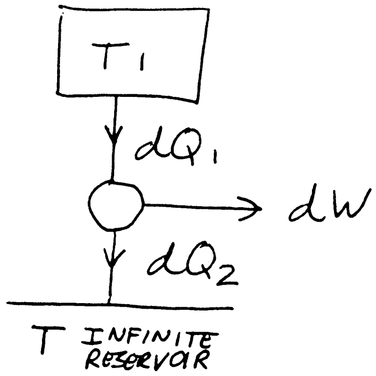
$$\underline{P_2 = 2.58\ bar}$$

$$\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Delta S = 1014 \times \ln\left(\frac{998.1}{273+500}\right) - 287 \ln\left(\frac{2.58}{2}\right)$$

$$\Delta S = \underline{\underline{186.1\ J/k}}$$

(c)



FOR MAX WORK $\dfrac{dQ_1}{T_1} = \dfrac{dQ_2}{T}$

$$dW = dQ_1 - dQ_2$$

$$\therefore \quad dW = dQ_1 - dQ_1 \frac{T}{T_1}$$

$$dQ_1 = -C\,dT_1\,m$$

$$dW = mC\left(\frac{T}{T_1} - 1\right) dT_1$$

$$W = mC \int_{800+273}^{500+273} \left(\frac{T}{T_1} - 1\right) dT_1$$

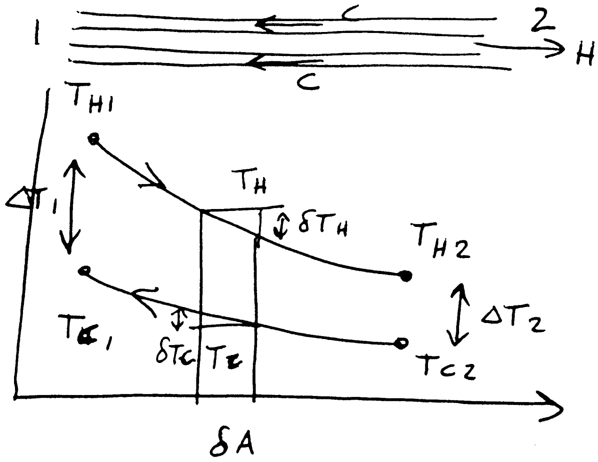
$$W = mC \left[T \ln T_1 - T_1\right]_{800+273}^{500+273}$$

$$W = 5 \times 437 \times \left[773 \ln \frac{773}{1073} - (773 - 1073)\right]$$

$$\underline{\underline{W = 101.6\,kJ}}$$

ANSWER

2 (a)



$$dQ = U\,\delta A\,(T_1 - T_2) = U\,\delta A\,\Delta T \quad *$$

$$\delta Q = -m_H C_{pH}\, dT_H = -m_C C_{pC}\, dT_C$$

$$d(\Delta T) = dT_H - dT_C = -\left(\frac{1}{m_H C_{pH}} - \frac{1}{m_C C_{pC}}\right) dQ$$

INTEGRATE

(1) $$-\left(\frac{1}{M_H C_{pH}} - \frac{1}{M_C C_{pC}}\right) Q = \Delta T_2 - \Delta T_1$$

$$-\left(\frac{1}{M_H C_{pH}} - \frac{1}{M_C C_{pC}}\right) U\,dA = \frac{d\Delta T}{\Delta T}$$

INTEGRATING ALONG LENGTH

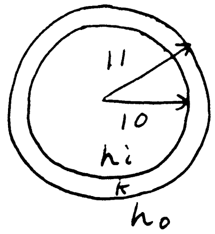
(2) $$-\left(\frac{1}{M_H C_{pH}} - \frac{1}{M_C C_{pC}}\right) UA = \ln \frac{\Delta T_2}{\Delta T_1}$$

DIVIDING (1) BY (2) GIVES [6]

$$Q = UA\, \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

ANSWER

(b)



$$Q = h_i(\Delta T_{OIL\,FILM})2\pi r_i$$

$$Q = \frac{2\pi k(\Delta T_{WALL})}{\ln \frac{r_o}{r_i}}$$

$$Q = h_o(\Delta T_{WATER\,FILM})2\pi r_o$$

$$Q = h_{TOTAL}(\Delta T_{OIL\,TO\,WATER})2\pi r_m$$

$$\frac{1}{h_{TOTAL}} = \frac{r_m}{h_i r_i} + \frac{r_m \ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{r_m}{h_o r_o}$$

$$= \frac{10.5}{50 \times 10} + \frac{10.5e^{-3} \ln\left(\frac{11}{10}\right)}{390} + \frac{10.5}{2000 \times 11}$$

$$= 0.021 + 2.57e^{-6} + 4.773e^{-4}$$

$$= 0.0215$$ [5]

$$h_{TOTAL} = 46.5\ W/m^2 K$$

(c) $Q = 2100 \times (100-50) \times 1 = 4180 \times (T_w - 20) \times 3$

$$T_w = 8.37 + 20 = \underline{\underline{28.37\,^\circ C}}$$

$\Delta T_1 = 50k - 20k$ $\Delta T_2 = 100 - 28.37 = 71.63k$

$\Delta T_1 = 30k$

$$Q = hA\frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \qquad A = \frac{Q}{h}\,\frac{\ln \frac{\Delta T_1}{\Delta T_2}}{\Delta T_1 - \Delta T_2}$$

ANSWERS

(2) (c) CONT

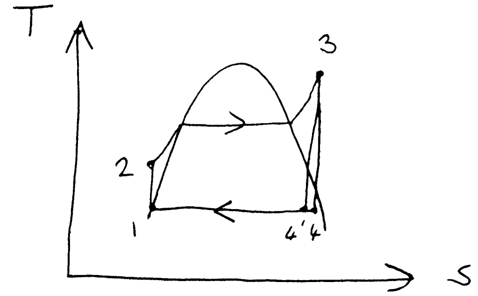
$$A = 47.2\,m^2$$

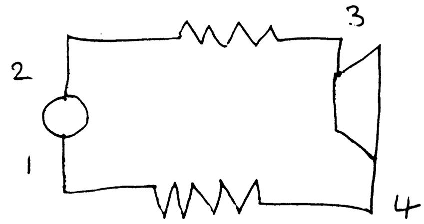
$$L = \frac{A}{2\pi r_m} = \frac{47.2}{0.066} = 715\,m$$

(d) RATE OF HEAT TRANSFER LIMITED BY THE CONVECTIVE HEAT TRANSFER ACROSS THE OIL FILM.

THIS IS BECAUSE OF LOW Re NUMBER OF OIL FLOW. THE BOUNDARY LAYER IS LAMINAR. NEED TO RAISE h FOR OIL FILM. INTERMITANT STEPS IN SURFACE. SPIRAL IN TUBE ETC.

3a





STARTING AT BOILER EXIT

$T_3 = 600°C$ $\quad$ $P_3 = 60\,bar$

$h_3 = 3656\ kJ/kg$ $\quad$ $S_3 = 7.1664$

$S_4' = S_3$ $\quad \therefore \quad h_4' = 2155$ (h-s DIAGRAM)

$$R_T = \frac{h_3 - h_4}{h_3 - h_4'} = 0.88$$

$$\boxed{\begin{aligned} h_4 &= 2335.6 \\ x_4 &= 0.91 \end{aligned}}$$

$h_1 = 121.4\ kJ/kg$ TABLES

$h_2 = h_1 + v\Delta P$

$v_1 = 1.004e^{-3}\ m^3/kg$

$\therefore \quad h_2 = 127.4\ kJ/kg$

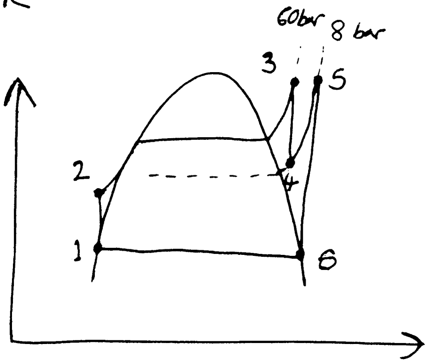
3a) CONT

$$R = \frac{W_{out}}{Q_{IN}} = \frac{h_3 - h_4 - (V\Delta P)}{h_3 - h_2}$$

$$R = \frac{3656 - 2335.6 - 6}{3656 - 127.4}$$

$$R = 0.37$$

3b)



$s_5 = s_4'$ $\quad h_4' = 3015$

$$h_4 = 3660 - 0.88 \times (3660 - 3015)$$

$$h_4 = 3092$$

$h_5 = 3698$ (h-s DIAGRAM 8bar 600°c)

$$s_5 = 8.13 = s_6'$$

$$h_6' = 2450$$

$$h_6 = 3698 - 0.88\,(3698 - 2450)$$

$$h_6 = 2600$$

SUPERHEATED

$$R = \frac{W_{TURB1} + W_{TURB2} - W_{FP}}{Q_{IN}} = \frac{(h_3 - h_4) + (h_5 - h_6) - V\Delta}{(h_3 - h_2) + (h_5 - h_4)}$$

3b CON'T

$$R = \frac{(3656 - 3092) + (3698 - 2600) - 6}{(3656 - 127.4) + (3698 - 3092)}$$

$$R = 0.40$$

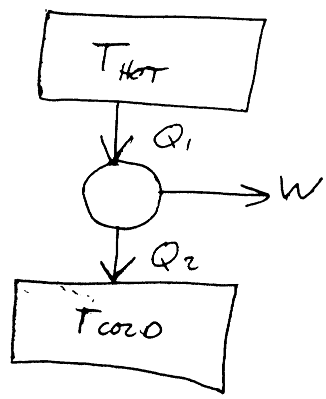
REHEAT INCREASES MEAN TEMPERATURE OF HEAT ADDITION, RAISES R.
REHEAT ALSO MEANS THAT TURBINES OPERATE WITH DRY STEAM => BETTER EFFICIENCY, LESS BLADE DAMAGE

3c

FOR A SYSTEM UNDERGOING A CYCLIC PROCESS, CLAUSUIS' INEQUALITY STATES THAT

$$\oint \frac{dQ}{T} \leq 0$$

WHERE dQ REPRESENTS HEAT TRANSFER INTO THE SYSTEM.



$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} \leq 0$$

$$\frac{Q_2}{Q_1} \geq \frac{T_2}{T_1}$$

$$R = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$R = 1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

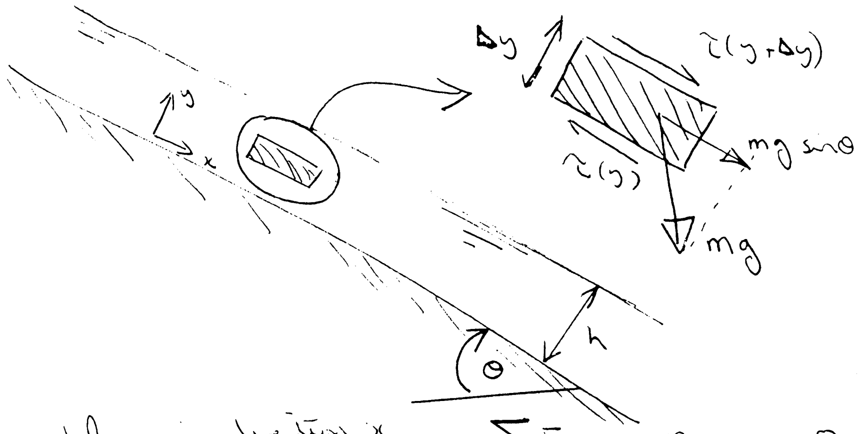
$T_2 = 32.9\,°C$
$T_1 = 600\,°C$

$$R_{MAX} = 0.6$$

RAISE AVERAGE TEMPERATURE OF HEAT INPUT - FEED WATER HEATING

SECTION B

④ (a)



Newton's second law, in direction x, $\sum F_x = m a_x = 0$

$$\Rightarrow \quad [\tau(y+\Delta y) - \tau(y)]\Delta x = -mg\sin\theta = -\rho\Delta x\Delta y\, g\sin\theta$$

$$\Rightarrow \quad \frac{\tau(y+\Delta y) - \tau(y)}{\Delta y} = \frac{\partial\tau}{\partial y} = -\rho g\sin\theta$$

$$\tau = -[\rho g\sin\theta]\,y + \text{const.}$$

$\tau = 0$ at $y = h$, so,

$$\tau = [\rho g\sin\theta][h-y]$$

substitute for τ using Newton's law of viscosity

$$\rho\nu\frac{\partial u}{\partial y} = \rho g\sin\theta(h-y)$$

$$\Rightarrow \quad \underline{\underline{\nu\frac{\partial u}{\partial y} = g\sin\theta(h-y)}} \qquad \text{—} ①$$

To get expression in 4(a) differentiate: $\underline{\nu\frac{\partial^2 u}{\partial y^2} = -g\sin\theta}$

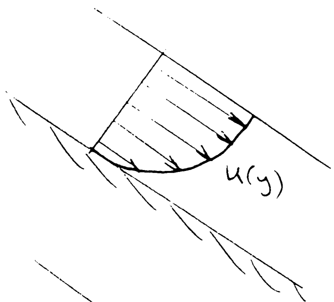
(b) To find $u(y)$ integrate ①:

$$u = \frac{g\sin\theta}{\nu}\left(hy - y^2/2\right) + \text{const.}$$

but $u = 0$ at $y = 0$ $\Rightarrow$ const. $= 0$

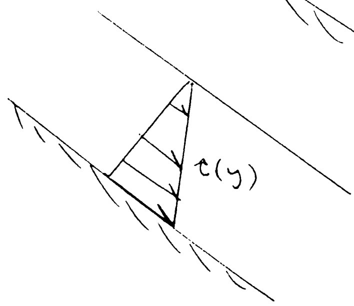
④ (b) cont.

$$u = \frac{g \sin\theta}{\nu}\left[hy - \frac{y^2}{2}\right]$$



Also from ①,

$$\tau = \rho g \sin\theta \left[h - y\right]$$



Volumetric flow rate is,

$$Q = \int_0^h u\,dy = \frac{g\sin\theta}{\nu}\int_0^h \left[hy - y^2/2\right] dy$$

$$= \frac{g \sin\theta}{\nu}\left[\frac{h^3}{2} - \frac{h^3}{6}\right]$$

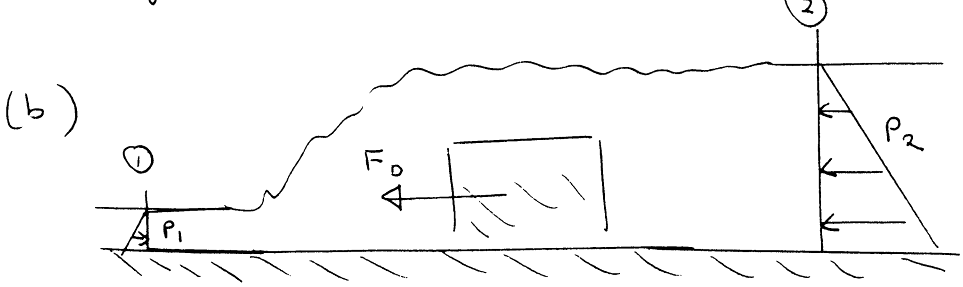
$$= \frac{g \sin\theta\, h^3}{3\nu}$$

(c) Equation ① is still valid so,

$$u = \int_0^y \frac{g \sin\theta\,(h-y)}{\nu(y)}\,dy$$

(5) (a) At 1 and 2 there is uniform horizontal motion. The vertical forces acting on a fluid element must therefore sum to zero (no vertical acceleration). Thus gravitational and pressure forces balance – i.e. we have a hydrostatic force balance.

(b)



$$\underset{\sim}{F} = \sum_{OUT} \dot{m}\,\underset{\sim}{u} - \sum_{IN} \dot{m}\,\underset{\sim}{u}$$

$$\Rightarrow \quad F_x = \dot{m}u_2 - \dot{m}u_1$$

$$\Rightarrow \quad \underbrace{\tfrac{1}{2}\rho g h_1^2}_{\uparrow} - \underbrace{\tfrac{1}{2}\rho g h_2^2} - \underset{\uparrow}{F_D} = \dot{m}u_2 - \dot{m}u_1$$

average pressure x height   force on fluid by block

$$\Rightarrow \quad F_D = \tfrac{1}{2}\rho g (h_1^2 - h_2^2) + \dot{m}(u_1 - u_2)$$

$$\Rightarrow \quad F_D = \tfrac{1}{2}\rho g (h_1^2 - h_2^2) + \rho h_1 V_1 (V_1 - V_2)$$

(c) $h_1 V_1 = h_2 V_2 \Rightarrow V_2 = 2.85\ m/s.$

$$F_D = 30\left\{\tfrac{1}{2}\, 9.81 \times 10^3 \left(1.14^2 - 7.35^2\right) + 10^3 \times 1.14 \times 18.4\,(18.4 - 2.85)\right\}$$

$$= 2.03 \times 10^6\ N$$

(4)

(5) (c) cont.

Apply extended Bernoulli along top streamline 1 → 2.

$$\Delta C = \left(\cancel{p_1} + \tfrac{1}{2}\rho V_1^2 + \rho g h_1\right) - \left(\cancel{p_2} + \tfrac{1}{2}\rho V_2^2 + \rho g h_2\right)$$

drop in Bernoulli const.

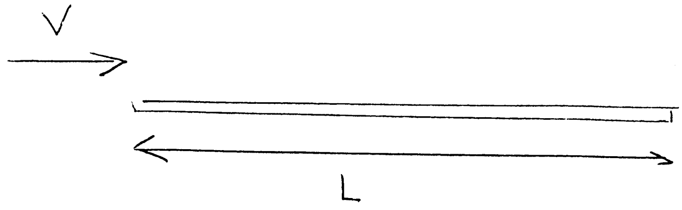
$$= \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right) + \rho g (h_1 - h_2)$$

$$= 104 \times 10^3 \ \text{N/m}^2.$$

$$\text{Power} = Q\,\Delta C = 629 \ \text{m}^3/\text{s} \times 104 \times 10^3 \ \text{N/m}^2$$

$$= 65 \times 10^6 \ \text{Watts}$$

6 (a)



$$F/w = f(\rho, \mu, V, L) \qquad \text{——— (1)}$$

No. parameters = 5
No. dimensions = 3

$\Rightarrow$ No. groups = 2.

Which groups?

(1) Form drag coefficient from F/w

F/wL has dimensions of pressure

ρV^2 has dimensions of pressure

$$\Rightarrow \quad \Pi_1 = \frac{F/w}{\rho V^2 L} \quad \text{(dependent dimensionless gp.)}$$

(2) We have μ – which suggests we form Reynolds no.

$$\Pi_2 = \frac{\rho V L}{\mu} \quad \text{(independent dimensionless gp.)}$$

Eqn. (1) becomes,

$$\Pi_1 = f(\Pi_2)$$

or

$$F/w = \rho V^2 L \, f\left(\frac{\rho V L}{\mu}\right)$$

But $F \propto V^{3/2} \Rightarrow f(x) = \dfrac{A}{x^{1/2}}$, A = constant.

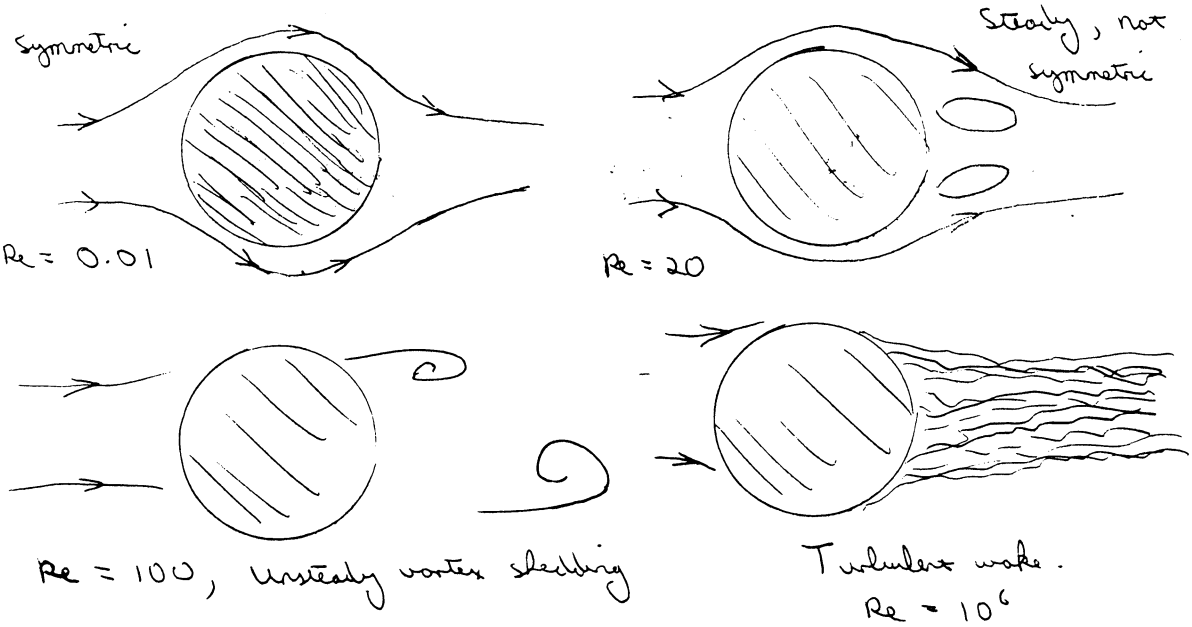
Thus,
$$F/w = \text{constant} \times \rho V^2 L \times \sqrt{\frac{\mu}{\rho V L}}$$

$$\Rightarrow F \propto \mu^{1/2}, \; V^{3/2}, \; L^{1/2}$$

(b) Same as before, except replace F/w by F (force per unit length) and L by d (diameter).

$$F = f(\rho, \mu, V, d)$$

becomes
$$\frac{F}{\frac{1}{2}\rho V^2 d} = f\left(\frac{\rho V d}{\mu}\right) \Rightarrow C_D = f(Re)$$



(c) frequency $\omega = f(V, d, \cancel{\nu})$ (viscosity not important)
$$\omega = f(V, d)$$

3 parameters, 2 dimensions $\Rightarrow$ 1 group : $\Pi = \frac{\omega d}{V}$

$$\Rightarrow \omega = \frac{V}{d} \times \text{const.}$$