ENGINEERING TRIPOS, Part 1B 2002

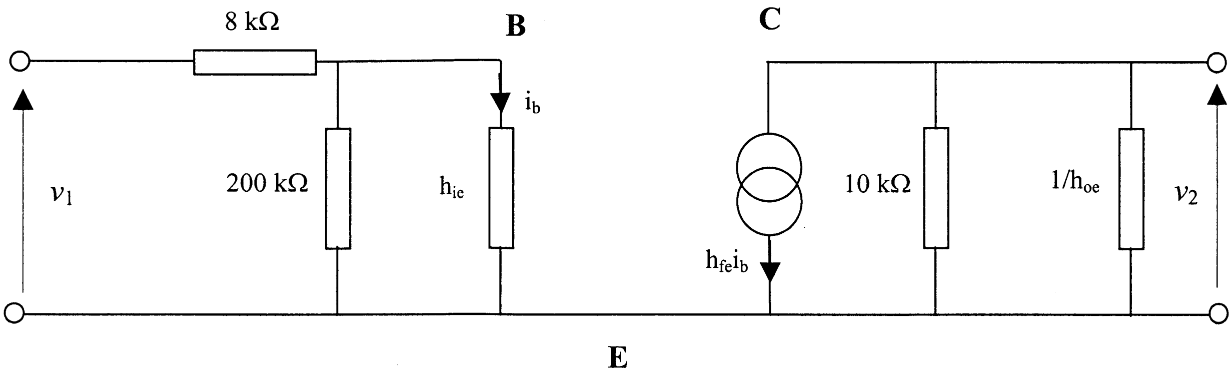
Paper 5 – ELECTRICAL ENGINEERING

Solutions

1.

(a) This is called "common emitter" because the emitter is connected to ground or "common", and is the ground common to both the input and the output.

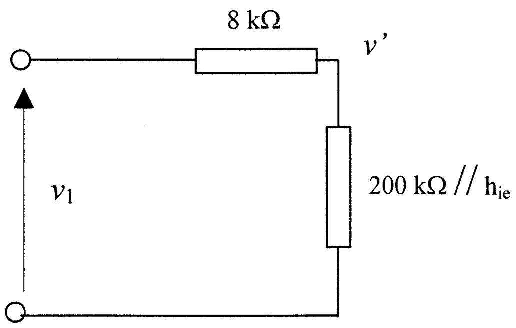
(b) Small-signal equivalent circuit:



Now, gain = v_2/v_1 $v_2 = -h_{fe}i_b \times (10\ k\Omega\ //\ 1/h_{oe}) = -7143\ h_{fe}i_b$

What is v1?

Well, as seen from the input side, equivalent circuit is:



The voltage across h_{ie} is $i_b h_{ie}$, which we will call v'. By potential divider formula,

$$v' = \frac{v_1 \times (200k\Omega\ //\ h_{ie})}{8k\Omega + (200k\Omega\ //\ h_{ie})}$$

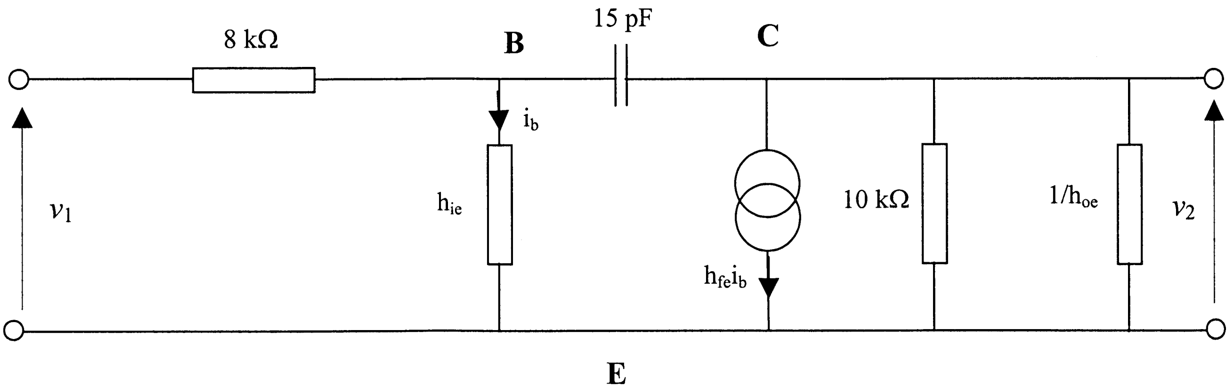
i.e., $v' = v_1/5.04$

Therefore, $v_1 = 5.04 i_b h_{ie}$

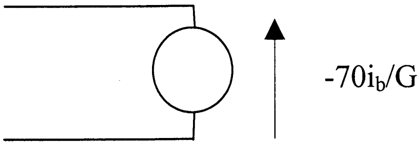
Gain = $v_2/v_1 = -7143 h_{fe} i_b/(5.04 h_{ie} i_b) =$ **- 49.6**

Gain in dB = $20\log_{10}(v_2/v_1) =$ **33.9**

(c) At high frequencies, the equivalent circuit becomes:



The output side is equivalent to:



Where G = 1/(10 kΩ // $1/h_{oe}$)

Therefore, summing currents at the base, we get:

$$\frac{v_1 - 2000i_b}{8000} - j\omega \times 15 \times 10^{-12} \times \left(2000i_b + \frac{70i_b}{G}\right) - i_b = 0$$

$$\Rightarrow \frac{v_1 - 2000i_b - 8000i_b - j\omega \times 0.06 i_b}{8000} = 0$$

$$\Rightarrow i_b = -\frac{v_1}{10000 + 0.06 j\omega}$$

i.e., when the frequency increases, the base current decreases, as current leaks across the base-collector capacitance. This reduces the current on the output side, and hence the gain.

Now, the 3dB point is when the gain is 0.707 times it's mid-band value. This happens when the real part of the denominator in the expresssion above equals the imaginary part.

i.e. ω = 166.68 kRad.s^{-1} => **f_{3dB} = 26.53 kHz.**

2.

(a) Using negative feedback:

(i) Gain is stabilised, although it is reduced.

(ii) Bandwidth is increased.

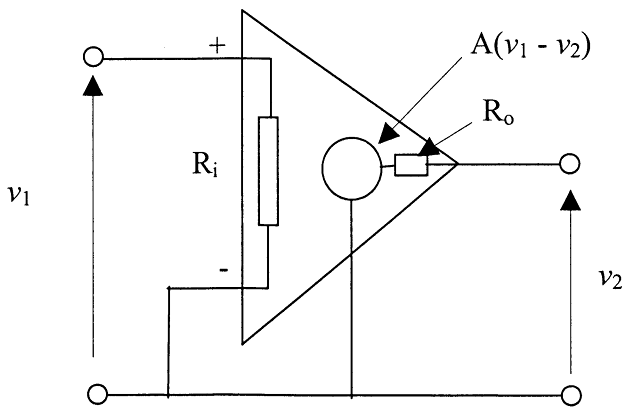
(iii) Input impedance is increased for voltage amplifiers and reduced for current amplifiers.

(iv) Output impedance is reduced for voltage amplifiers and increased for current amplifiers.

All of the above quantities are changed from their values in the absence of feedback by the factor (1 + AB), where A is the open-loop gain, B is the feedback factor and AB is the loop gain.

(b) This is a unity-gain buffer – used as a buffer between a high impedance source and a low impedance load (eg loudspeaker).

The small-signal equivalent circuit of this amplifier is:



Summing currents at the output, we can find the gain, v_2/v_1

$$\frac{v_1 - v_2}{R_i} + \frac{A(v_1 - v_2)}{R_o} - v2 = 0$$

$$\Rightarrow \frac{v_1 R_o - v_2 R_o + AR_i v_1 - AR_i v_2 - R_i v_2}{R_i R_o} = 0$$

$$\Rightarrow v_1(R_o + AR_i) - v_2(R_o + AR_i + R_i) = 0$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{R_o + AR_i}{R_o + AR_i + R_i} = \frac{1}{1 + \frac{R_i}{R_o + AR_i}}$$

i.e., Gain ~ 1, and loop gain ~ A.

Input impedance = input voltage/input current

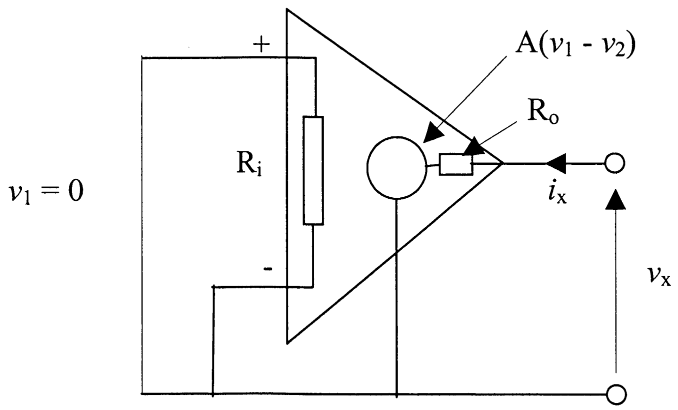
$$= \frac{v_1}{(v_1 - v_2)/R_i} = \frac{R_i}{1 - \frac{v_2}{v_1}}$$

Using expression above for v_2/v_1, we obtain for the imput impedance:

$$\mathbf{R_{in} = R_o + Ri(A+1)}$$

Output impedance?

Well, the standard routine here is to apply a test voltage v_x, to the output while shorting the input, and find the resultant current, i_x.



Summing currents at the output, we find that:

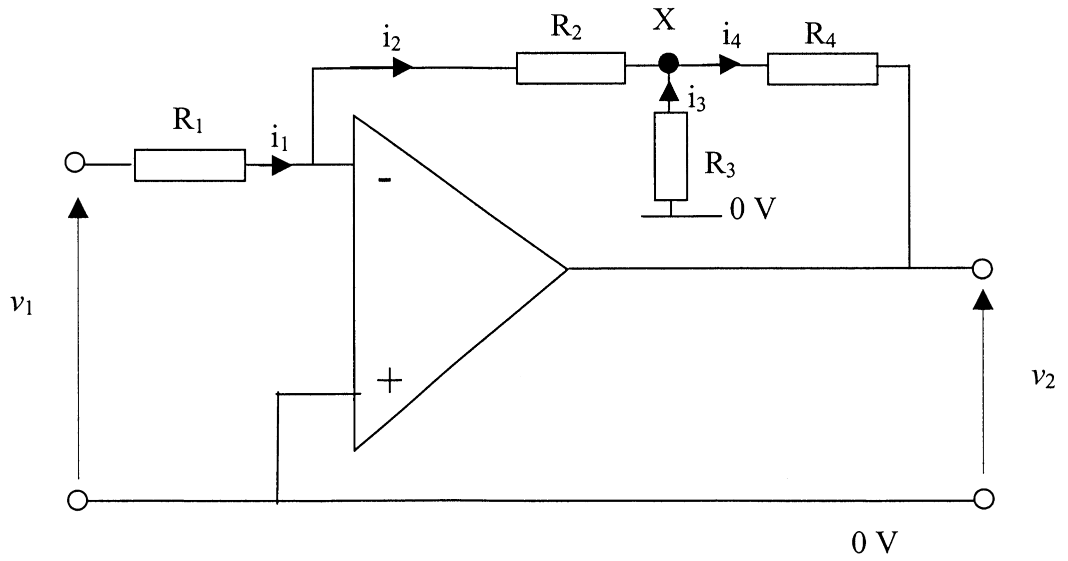
$$\frac{A(0 - v_x) - v_x}{R_o} + \frac{(0 - v_x)}{R_i} + i_x = 0$$

$$\Rightarrow i_x = v_x \left[\frac{A+1}{R_o} + \frac{1}{R_i} \right]$$

$$\Rightarrow R_{out} = \frac{R_o}{1 + A + \frac{R_o}{R_i}}$$

(c)

Assume that Op-Amp is ideal: makes analysis much simpler.



Now, (i) Current $i_1 = v_1/R_1 = i_2$ (Op-Amp is ideal, so draws no current)

(ii) Inverting terminal is at virtual earth

Voltage at node "X", $v_x = 0 - i_2R_2 = -v_1R_2/R_1$

Current $i_3 = (0 - v_x)/R_3 = R_2V_1/R_1R_3$

Now, $i_4 = i_2 + i_3 = v_1/R_1 + v_1R_2/R_1R_3$, and $v_2 = v_x - i_4R_4$

$$v_2 = -v_1\frac{R_2}{R_1} - v_1\frac{R_4}{R_1}\left(1+\frac{R_2}{R_3}\right)$$

Therefore,

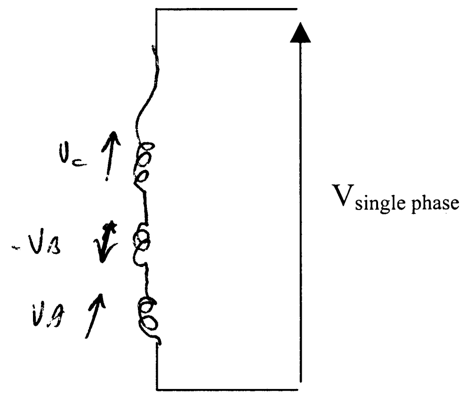
$$\Rightarrow \frac{v_2}{v_1} = -\frac{R_2}{R_1}\left[1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right]$$

(d) Gain for $R_1 = R_2 = R_4 = 1\ \text{M}\Omega$, $R_3 = 10.2\ \text{k}\Omega$ is **–100**.

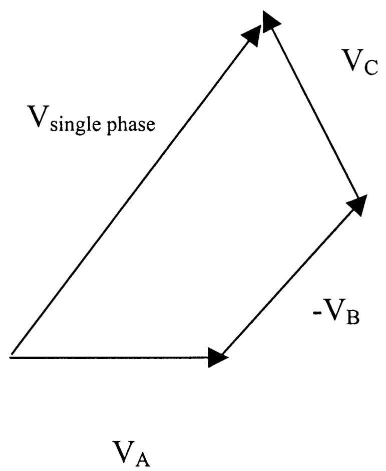
3.

(a) ac is used because then voltages can be altered arbitrarily up or down by means of transformers. This is important, as many appliances operate at different voltages.
Three-phase ac is the most efficient – more phases only represent a nominal improvement in power generation, which is overshadowed by more (expensive) cabling.
The phase difference between the phase voltages is 120^0, or $2\pi/3$ rads.

(b) Well, power per phase is $V_{ph}I_{max}$, where V_{ph} is the phase voltage.
Therefore, for three phases, the total maximum power output is $3V_{ph}I_{max}$.
If we combine the phases as $V_A - V_B + V_C$, we get the following:



i.e., as phasors :



Geometrically, $V_{single\ phase} = 2V_A$
Alternatively, if we write $V_A = V$, $V_B = Ve^{-j2\pi/3}$, $V_C = Ve^{j2\pi/3}$
We can combine these to find $V_A - V_B + V_C = V(1 + j\surd 3) = 2V$
Therefore, the power is reduced from $3V_{ph}I_{max}$ to $2V_{ph}I_{max}$, which is a factor of 1/3.

(c) Procedure: find P & Q for each load separately, and therefore find S.
Convert Delta load (load 1) to a Star : $Z_l = j\omega L/3 + R/3 = 33.3 + j104.7$
Therefore, $|Z_l| = 109.86\ \Omega$
$I_l = V_l/Z_l = 11\ kV/\surd 3 x109.86 = 57.8$ A……..$V_{ph} = V_{line}/\surd 3$
Thus, $P_l = 3xI_l^2 xRe[Z_l]$
$= 3x57.8^2x100 = 333.75$ kW

$Q_1 = I_1^2 x Re[Z_1]$

$= 57.8^2 x 104.7 = 1049.36$ kVAR

Now, for star load (load 2)

$Z_2 = 10\ \mu F \,||\, 100\ \Omega$. Resistor & capacitor in parallel => both have V_{ph} across them.

$P_2 = 3xV_{ph}^2/R = 1210$ kW

$Q_2 = -3xV_{ph}^2\omega C = -380.121$ kVAR

For whole load, total $P = P_1 + P_2 = 1543.75$ kW

Likewise, total $Q = Q_1 + Q_2 = 669.24$ kVAR

(i) Therefore, to find overall power factor, $= \cos(\tan^{-1}(Q/P))$ = **0.917, lagging**

(ii) Line current, I_l: Well, Total apparent power, $S = \sqrt{(P^2 + Q^2)} = \sqrt{3} V_l I_l$

$\Rightarrow I_l = \sqrt{(1543750^2 + 669240^2)}/\sqrt{3} V_l$

= 88.3 A

(iii) Capacitors to correct power factor to 0.95 lagging.

For power factor 0.95, => Q/P must be 0.329. As P will not change (no additional resistive compenents), that means net Q must be 507.89 kVAR. Therefore, Capacitors must generate (669.24 – 507.89) kVAR = 161.35 kVAR.

Now, how many VARS generated?

Well, $= 3V_{ph}^2/X_c$

$= 3x(11000)^2 x\omega C$

=> **C = 1.42 μF**

4.

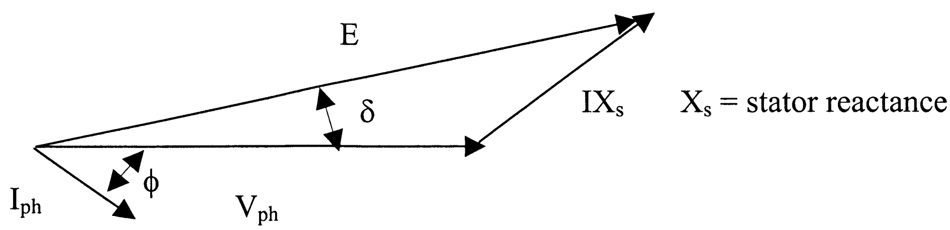
(a) Stator is stationary, whereas rotor is on bearings and is free to rotate. Dc current is fed into the rotor coils by slip-rings and brushes. This produces a magnetic field which interacts with the rotating magnetic field produced by the three-phase windings of the stator. When the prime-mover is rotated, the rotor magnetic field will also rotate, and an emf is induced in the stator windings. For steady torque to be produced, the basic principle is that the rotor and stator fields must rotate at the same speed, and there must be a phase lag (load angle not equal to zero) between these two fields. The two conditions which allow this are (i) The rotor and stator-driven fields must have the same number of poles, and (ii) the rotor speed must be equal to the stator-field speed.

(b) Now, Power, $P = 3V_{ph}I_{ph}\cos\phi$

Power = 300 MW => $I_{ph} = 300\times10^6/(3\times22000/(\sqrt{3}\cos\phi)$

= **8287 A**

To calculate load angle, δ and line-line excitation EMF, E, consider phasor diagram:



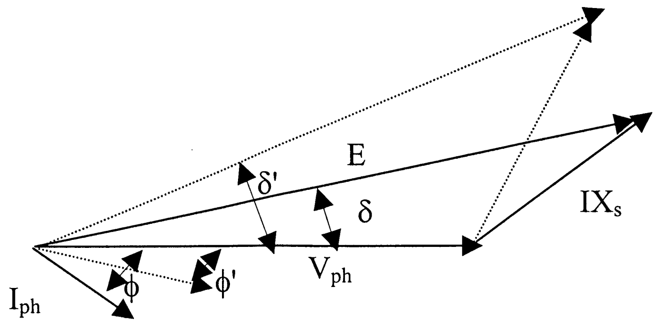
By cosine rule, $E = \sqrt{(V + IX_s\sin\phi)^2 + (IX_s\cos\phi)^2}$

=> **E = 20.3 kV => $E_{line} = \sqrt{3}E$ = 35.16 kV**

Load angle, by sine rule, $\sin\delta = IX_s\cos\phi/E$ => **$\delta = 35.5^o$**

(c) Well, excitation remains constant, as will terminal voltage and stator reactance. Therefore, E moves in an arc. Power is proportional to the vertical height of the triangle shown above.

i.e.:



Now, Power, $P = 3VI\cos\phi$

By sine rule, $P = 3VE\sin\delta/X_s$

All are constants apart from δ & δ'.

Therefore, (New Power)/(Old Power) = $\sin\delta'/\sin\delta$

=> $\sin\delta' = 4\sin\delta/3 = 0.774$

=> **$\delta' = 50.74^o$**

New stator current?

Well, by cosine rule, $(I'X_s)^2 = E^2 + V^2 - 2EV\cos\delta'$

=> **I' = 10488 A**

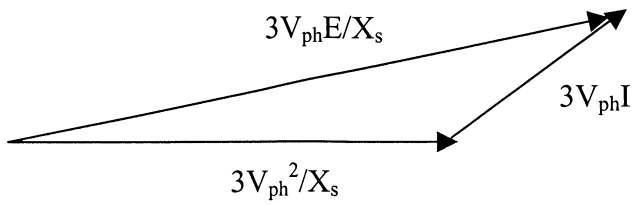
Power factor?

$P = 3VI'\cos\phi'$ => **$\cos\phi' \sim 1$**

(d)

Operating chart: scale sides of phasor diagrams above by factor $3V/X_s$

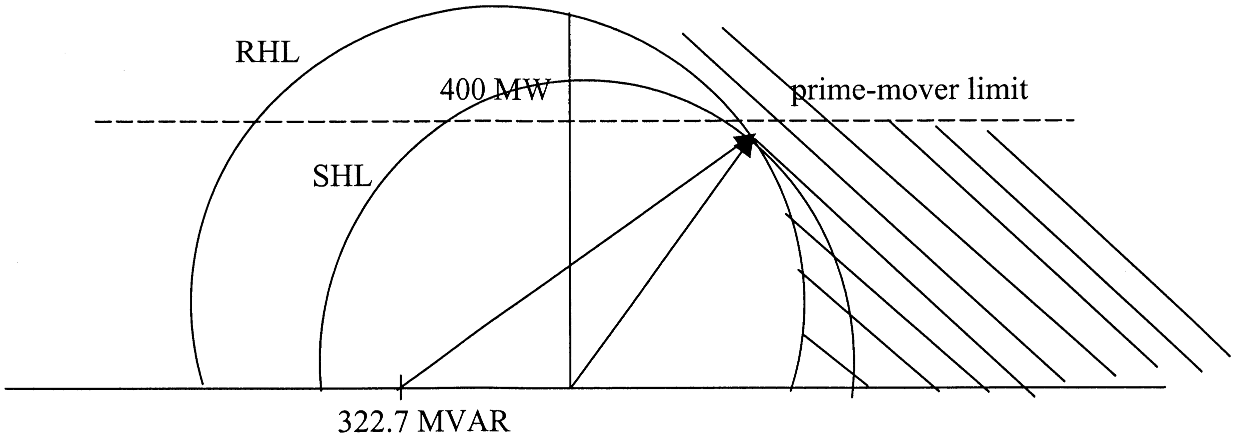
i.e.



Side lengths: $3V_{ph}^2/X_s$ = 322.7 MVAR

$3V_{ph}I$ = stator heating limit (SHL) = VA rated = 500 MVA

$3V_{ph}E/X_s$ = rotor heating limit (RHL) = 490 MVAR i.e.



5.

(a) (i) Armature EMF equation

Well, conductor of length l in a magnetic field of strength B, and moving with velocity v, has an induced EMF = Blv (from Faraday's law and Lorentz Force).

Average B field per pole is (flux per pole)/(pole area)

$= \phi/(2\pi rl/2p)$.........p = number of pole pairs

$\Rightarrow$ EMF = Blv = $p\phi v/\pi r$

But, velocity, $v = r\omega$ (spinning about an axis at angular velocity ω, at a distance r from the axis, where r is the coil's radius).

$\Rightarrow$ EMF = $p\phi\omega/\pi$

If there are n conductors, EMF = $np\phi\omega/\pi$ = $\mathbf{e_a = K\phi\omega}$ ....$K = np/\pi$, a constant

(ii)Torque equation

Force on a current, i_a in a wire of length l, in a magnetic field B is $F = Bi_a l$ (Lorentz force).

But, from before, average $B = p\phi/\pi rl$ $\Rightarrow$ Force = $p\phi i_a/\pi r$

Torque for n conductors is $Fnr = pni_a\phi/\pi$ = $\mathbf{T = K\phi i_a}$

(b) Power, $P = T\omega$, where T is torque and ω is angular velocity. If machine delivers 30 kW, and 1.5 kW are lost, then must generate 31.5 kW

Convert rpm to rads/s. 1000rpm = 2πx1000 rad/min = 2πx1000/60 rad/s = 104.72 rad.s-1.

$\Rightarrow T = P/\omega = 31500/104.72 = 300.8$ Nm

To find values of field current, use databook equations for dc motors.

i.e.

$V_a = e_a + i_a r_a$

$\Rightarrow V_a i_a = e_a i_a + i_a^2 r_a$

but, $T\omega = K\phi i_a e_a/K\phi = e_a i_a$

$\Rightarrow V_a i_a = T\omega + i_a^2 r_a$

$\Rightarrow 500 i_a = 31500 + 0.8 i_a^2$

$\Rightarrow i_a^2 - 625i_a + 39375 = 0$

$\Rightarrow i_a = 553.9$ A or 71.1 A

Now, open circuit test $\Rightarrow i_a = 0$, so V_a as measured is actually e_a. We need to find values of i_f corresponding to the above values for i_a.

$e_a = V_a - i_a r_a$

$\Rightarrow$ for $i_a = 553.9$ A, $e_a = 56.9$ V, and for $i_a = 71.1$ A, $e_a = 443.12$ V.

Now, 56.9 V @ 1000 rpm = 569x0.8 V @ 800 rpm (remember, e_a scales linearly with speed)

= 45.52 V

And, 443.12 V @ 1000 rpm = 354.5 V @ 800 rpm.

From data in table 1, graphical estimate is that at $V_{oc} = 354.5$ V, $i_f = 3.3$ A, and 45.52 V corresponds to $i_f = 0.4$ A.

Choose value of i_f corresponding to lowest value of i_a (71.1 A), i.e. $i_f = 3.3$ A. Other value of i_f means using i_a of 553.9 A, as compared to 71.1 A, so is 61 times less efficient!

(c) Remember, $T = K\phi i_a = K' i_f i_a$

$\Rightarrow$ if i_f is decreased from 3.3 A to 1.75 A while T remains constant, then i_a is increased by factor of 3.3/1.75 = 1.89.

$\Rightarrow e_{a2} = V_a - i_{a2} r_a$

= 500 – 142.2x0.8

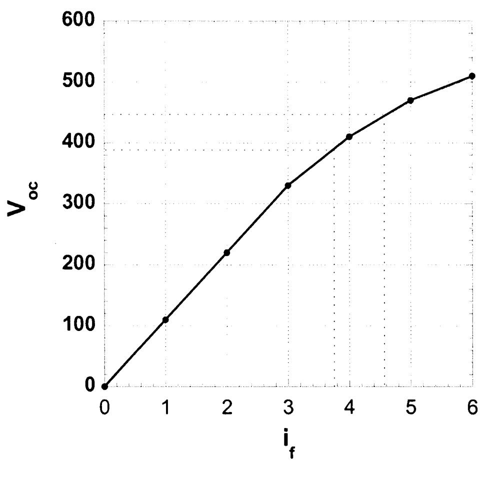
= 386.24 V

But, $e_{a2}/e_{a1} = (K_2' i_{f2}\omega_2)/(K_1' i_{f1}\omega_1) = (K_2'/K_1)x(\omega_2/1.89\omega_1)$

$\Rightarrow \omega_2 = 1.89\omega_1 x386.24/443.12x(K_1/K_2')$

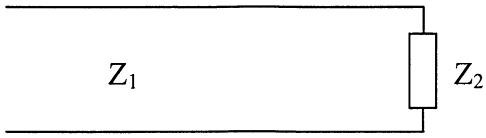
Now, graphically, $K_1/K_2' \sim 1.09$

= 1790 rpm



6.

(a)



(i) In the case of a matched transmission line where $Z_1 = Z_2$, then the reflection coefficient is zero at the end of the line. All the signal power is therefore transmitted to the load.

(ii) For $Z_1 \neq Z_2$, some power is reflected, and the voltage reflection coefficient is $\rho = (Z_2 - Z_1)/(Z_2 + Z_1)$.

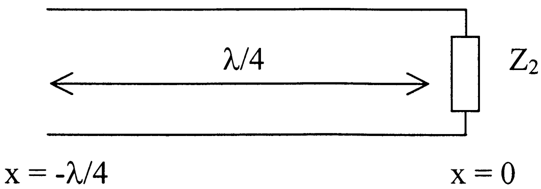
(b) Reflected voltage, $V_- = \rho V_+$

$$V_- = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right) V_+$$

$$= \left(\frac{j150 - 70}{j150 + 70}\right) V_+$$

$$= (0.642 + j0.766) V_+$$

(c)



Total voltage at $x = -\lambda/4$ is the sum of the forward and reflected voltages:

$$V\left(x = -\frac{\lambda}{4}\right) = V_+ e^{-j\frac{2\pi}{\lambda}\left(\frac{-\lambda}{4}\right)} + V_- e^{+j\frac{2\pi}{\lambda}\left(\frac{-\lambda}{4}\right)}$$

i.e.

$$= j(V_+ - V_-)$$

We can use a similar argument for the current to find $I(x = -\lambda/4) = j(I_+ - I_-)$

But, $I_+ = V_+/Z_1$, and $I_- = -V_-/Z_1 = -\rho V_+/Z_1$

The input impedance as observed at x = -λ/4 is

$$Z_{in}\left(\frac{\lambda}{4}\right)=\frac{V(x=\text{-}\lambda/4)}{I(x=\text{-}\lambda/4)}$$

$$=\frac{j(V_+-V_-)}{j(I_+-I_-)}$$

$$=\frac{(V_+-\rho V_+)}{\frac{V_+}{Z_1}+\rho\frac{V_+}{Z_1}}$$

$$=Z_1\left(\frac{1-\rho}{1+\rho}\right)$$

$$=0.023-j32.7 Ohm$$

~ j32.7 Ohm

(d) Case where x = λ/2, use a similar argument

$$V\left(x=-\frac{\lambda}{2}\right)=V_+e^{-j\frac{2\pi}{\lambda}\left(\frac{-\lambda}{2}\right)}+V_-e^{+j\frac{2\pi}{\lambda}\left(\frac{-\lambda}{2}\right)}$$

$$=-(V_++V_-)$$

$$Z_{in}\left(\frac{\lambda}{2}\right)=\frac{V(x=\text{-}\lambda/2)}{I(x=\text{-}\lambda/2)}$$

$$=\frac{-(V_++\rho V_+)}{-\frac{V_+}{Z_1}+\rho\frac{V_+}{Z_1}}$$

$$=Z_1\left(\frac{1+\rho}{1-\rho}\right)$$

$$=4.3+j148$$

$$\sim j150 Ohm$$

The input impedance is an oscillatory function of the transmission line length, and for l = 5λ/4, it is the same as for λ/4, i.e. it is –j32.7 Ohm

7.

(a) Wave equation

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}=\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$$

$$Test, E_y = E_0 e^{j(\omega t-\beta z)}$$

$i.e.$

$$\frac{\partial^2\left(E_0 e^{j(\omega t-\beta z)}\right)}{\partial z^2}=\varepsilon_0\mu_0\frac{\partial^2\left(E_0 e^{j(\omega t-\beta z)}\right)}{\partial t^2}$$

$$i.e. \beta^2=\varepsilon_0\mu_0\omega^2$$

This is a solution for a plane wave propagating in the Z direction provided that $\beta/\omega = \sqrt{\varepsilon_0\mu_0}$

(b) $E_x = E_0 e^{j(\omega t - \beta z)}$

also propagates in the positive Z direction, the only difference being the direction of polarisation.

(c) The signal is broadcast isotropically, hence power per unit area, P at a distance r from the source is

$P = 5000/(4\pi r^2)$ Watt.m^{-2}

But, the receiver area is only 0.02 m^{-2}, hence the power received at the antenna, P_{rec} is:

$P_{rec} = P \times 0.02$ Watt $= 100/(4\pi r^2)$ Watt

For the maximum distance r_{max}, the power received is only 2×10^{-9} Watt, hence

$2\times10^{-9} = 100/(4\pi r_{max}^2)$

=> $r_{max} = 63$ km

(d)

If the receiver is placed at 90% r_{max}, then the power received is 20% above the minimum required level. If the receiver is inclined at an angle θ to the optimum angle for picking up the signal, then the effective area is reduced by a factor cos θ.

Hence, the antenna can be mis-oriented by θ_{max} and still pick up the minimum required signal where

$\cos(\theta_{max}) = 0.8$

$\Rightarrow \theta_{max} = 37^{\circ}$