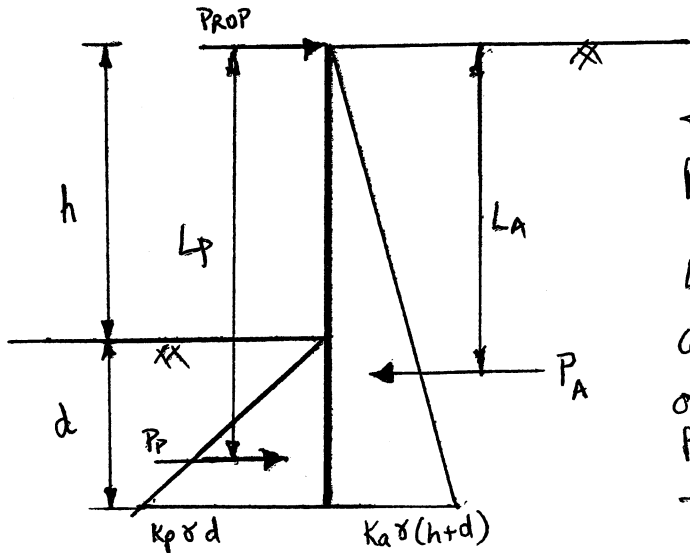


Answers to Question 1:



At the instant of rotational failure, full active and passive pressures act on the wall, as shown.

Let unit weight of dry sand be γ . Consider the wall just on the verge of outward rotational failure about the prop.

Triangular active and passive pressure distributions develop as shown in the figure, with resultant forces P_A and P_p respectively with lever arms L_A and L_p respectively (measured from top).

Consider the active zone:

$$P_A = \frac{1}{2} K_a \gamma (h+d) \times (h+d) = \frac{1}{2} K_a \gamma (h+d)^2$$

$$L_A = \frac{2}{3} (h+d)$$

K_a = Co-efficient of active earth pressure

Consider the passive zone:

$$P_p = \frac{1}{2} K_p \gamma d^2$$

$$L_p = h + \frac{2}{3} d$$

K_p = Co-efficient of passive earth pressure

Taking moments about the prop

$$P_A \cdot L_A = P_p \cdot L_p$$

$$\therefore \frac{1}{2} K_a \gamma (h+d)^2 \cdot \frac{2}{3} (h+d) = \frac{1}{2} K_p \gamma d^2 (h + \frac{2}{3} d)$$

$$\therefore \frac{(h+d)^3}{d^2 (h + \frac{2}{3} d)} = \frac{3 K_p}{2 K_a} \quad \dots \text{Eq 1.} \quad [5]$$

1 b). $L = 8 \text{ m}$

$\phi' = 30^\circ$

(see Data sheet).

$$\therefore K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.33$$

$$K_p = \frac{1}{K_a} = \frac{1}{0.33} = 3$$

$$\therefore \frac{3K_p}{2K_a} = \frac{3 \times 3}{2 \times \frac{1}{3}} = 13.5$$

$$\therefore \frac{(h+d)^3}{d^2(h+\frac{2}{3}d)} = 13.5$$

$$\therefore \frac{(8+d)^3}{d^2(8+\frac{2}{3}d)} = 13.5$$

Solve by Trial & Error :

i) Try $d = 2 \text{ m}$
LHS = 26.79

ii) Try $d = 4 \text{ m}$
LHS = 10.125

iii) Try $d = 3 \text{ m}$
LHS = 14.79 ✓ ok.

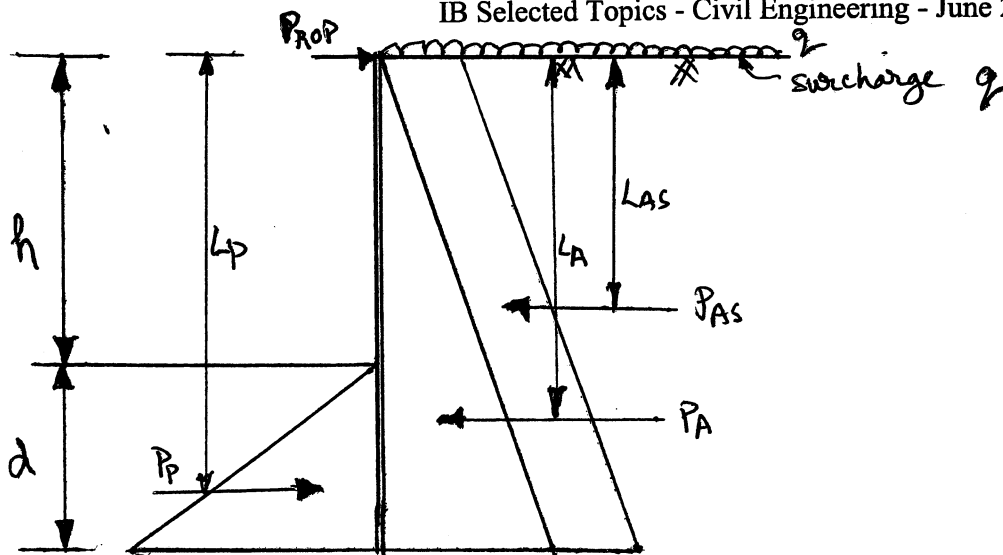
iv) Try $d = 3.5 \text{ m}$
LHS = 12.01 ✓ ok.

v) Try $d = 3.25 \text{ m}$
LHS = 13.25 ✓ ok. This is very close to RHS

Hence $d = 3.25 \text{ m}$

[5]

1c)



Surcharge q causes additional pressure $K_a q$ constant with depth to act on the back of wall, as shown in Figure.

Resultant force $P_{AS} = K_a q (h+d)$

P_A, P_P & L_A, L_p as before.

Factor of safety against rotation failure $F = \frac{P_P \cdot L_p}{P_A \cdot L_A + P_{AS} \cdot L_{AS}}$

$L_{AS} = \frac{(h+d)}{2}$

$\therefore F = \frac{\frac{1}{2} K_p \gamma d^2 (h + \frac{2}{3}d)}{\frac{1}{2} K_a \gamma \frac{2}{3} (h+d)^3 + K_a \frac{q}{2} (h+d)^2}$

Substituting $h = 8m, d = 6m, K_a = \frac{1}{3}; K_p = 3$
 $q = 20 \text{ kN/m}^2$ & $\gamma = 20 \text{ kN/m}^3$

$F = \frac{\frac{1}{2} \times 3 \times 20 \times 6^2 (8 + \frac{2}{3} \times 6)}{\frac{1}{2} \times \frac{1}{3} \times 20 \times \frac{2}{3} (14)^3 + \frac{1}{3} \times \frac{20}{2} (14)^2} = \frac{12960}{6097 + 653.3} = 1.9199$
 $= \underline{\underline{1.92}}$

[10]

Question 2)

a) i) Tunnel centreline at depth of 12.5 m

Consider stability ratio $N = \frac{\sigma_v - \sigma_T}{S_u}$

where σ_v = total vertical stress at tunnel axis level

σ_T = temporary support pressure

S_u = Undrained shear strength of clay

$$\sigma_v = 5 \times 18 + 7.5 \times 17 = 90 + 127.5 = 217.5 \text{ kPa}$$

With no support pressure $\sigma_T = 0$ i.e. open face mode.

$S_u = 25 \text{ kPa}$ for soft clay.

$$\therefore N = \frac{217.5 - 0}{25} = \underline{\underline{8.70}} > 5.$$

This is very high. If N exceeds about 5, face support is needed to prevent collapse.

$$\text{For } N = 5 \quad \frac{217.5 - \sigma_T}{25} = 5 \Rightarrow \sigma_T = 92.5 \text{ kPa}$$

This means that the tunnel must be constructed in 'closed face' mode with a face support pressure of at least 92.5 kPa (and preferably higher).

A pressurised face tunnelling machine must therefore be used.

ii) Tunnel centreline at depth of 35 m

$$\begin{aligned} \sigma_v &= 5 \times 18 + 15 \times 17 + 5 \times 20 + 10 \times 20 \\ &= 90 + 255 + 100 + 200 = 645 \text{ kPa} \end{aligned}$$

Without support pressure $\sigma_T = 0$ i.e. in open face mode.

$S_u = 175 \text{ kPa}$ for stiff clay

$$\therefore N = \frac{\sigma_v - \sigma_T}{S_u} = \frac{645 - 0}{175} = 3.686 \approx \underline{\underline{3.7}} < 5$$

This shows that the tunnel could be safely constructed in open face mode, and this could be done with mechanised tunnelling shield (i.e. a mechanical digger excavating the unsupported face of the stiff clay).

[8]

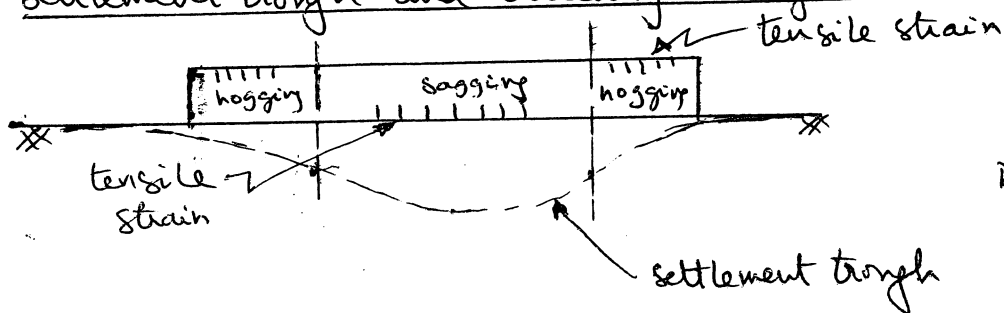
2b) Tunnel Centre-line at 25 m.

If the centre-line of the tunnel is at a depth of 25 m, the Crown of the tunnel would be 2 m above the base of the highly permeable sandy gravel, i.e., the top half of the tunnel would be in highly permeable sandy gravel and the bottom half would be in stiff clay.

Water inflow in the sandy gravel would be a major problem. This would cause collapse of the face. This could be controlled by the following alternative techniques:

- i) tunnelling in open face mode but using compressed air to balance the water pressure (but note that the air pressure would need to be high - about $24 \times 10 = 240$ kpa)
- ii) dewatering by pumping - install deep wells to pump water from sandy gravel layer.
- iii) grouting of sandy gravel layer (using cement grout or chemical grout) to reduce its permeability and thereby reduce the amount of water.
- iv) use closed face slurry or earth pressure balance tunnelling machines [6]

2c) Settlement trough and building damage



Differential settlement causes curvature and tensile strain

○ tunnel

Masonry buildings are vulnerable to tensile strain and are more vulnerable in hogging mode than in sagging mode, because in hogging mode tensile strains are induced in the top of the building where there is no restraint. In the sagging mode, the tensile strains are induced at the bottom of the building and the foundations and the ground tend to resist this. Buildings positioned to one side of the

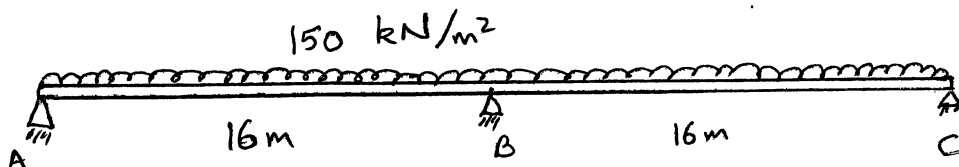
tunnel are therefore more prone to damage than those immediately above the tunnel.

Possible protective measures:

- i) strengthen the fill (mainly gravel) by grouting to form a stiff 'raft' of strengthened ground.
- ii) strengthen the building using tie rods. An effective technique but tends to be disruptive to building owner (and often politically unacceptable).
- iii) Strutless Jacking to control differential settlement. This involves cutting the foundations and inserting hydraulic jacks.
- iv) Compensation grouting involves installation of grouting tubes in the ground between the tunnel and the building foundation, usually from a shaft. Grout is injected during tunnelling to control settlement of the building. Instrumentation is needed for monitoring.

[6]

3 a)

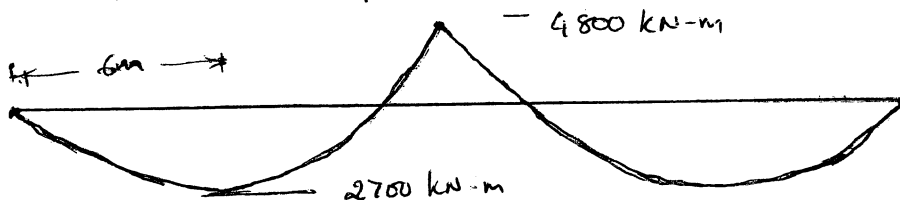


$$\text{Total load} = 150 \times 32 \times 1 = 4800 \text{ kN}$$

$$\text{Central reaction } R_B = \frac{5}{8} \times 4800 = 3000 \text{ kN}$$

$$\therefore \text{End reactions } R_A = R_C = 900 \text{ kN}$$

\therefore Points of zero shear force will be at $\frac{900}{150} = 6 \text{ m}$ from each end.



$$\text{Maximum Sagging moment} = 900 \times 6 - 150 \times 6 \times \frac{6}{2} = \underline{\underline{2700 \text{ kN-m}}}$$

$$\text{Maximum hogging moment} = \frac{150 \times 16^2}{2} - 900 \times 16 = \underline{\underline{4800 \text{ kN-m}}}$$

[4]

$$\left[\begin{array}{l} \text{Point of contraflexure} \quad 900x - \frac{150}{2}x^2 = 0 \\ x = \frac{900 \times 2}{150} = \underline{12 \text{ m}} \\ \text{Not asked for but useful for detailing} \end{array} \right]$$

3 b) If depth is governed by sagging bending

$$2700 \times 10^6 = 0.15 f_{cu} b d^2 \quad [\text{from Data sheet}]$$

$$f_{cu} = 40 \text{ N/mm}^2 \quad b = 1000 \text{ mm}$$

$$\Rightarrow d = \sqrt{\frac{2700 \times 10^6}{0.15 \times 40 \times 1000}} = 670.82 \text{ mm}$$

$$\text{Say } d = \underline{700 \text{ mm}}$$

$$\text{Guess } x = 0.5$$

$$2700 \times 10^6 = 0.87 \times 460 \times 700 \left(1 - \frac{0.5}{2}\right) A_s \quad [\text{from Data sheet}]$$

$$A_s = 12850.7 \text{ mm}^2 \approx 12850 \text{ mm}^2$$

$$\Rightarrow x = 2.175 \left(\frac{460}{40}\right) \cdot \frac{12850.7}{(1000 \times 700)} = 0.459$$

Repeat with this x

$$2700 \times 10^6 = 0.87 \times 460 \times 700 \left(1 - \frac{0.459}{2}\right) A_s$$

$$A_s = 12508.8 \approx \underline{12509 \text{ mm}^2}$$

This is only a small change, so repeating with changed x is not really needed.

$$40 \text{ mm bar @ } 100 \text{ mm centres} = \underline{12560 \text{ mm}^2}$$

[6]

3 c) To carry hogging moment of ~~4800~~ $4800 \text{ kN}\cdot\text{m}$

$$0.15 f_{cu} b d^2 = 2940 \text{ kN}\cdot\text{m} \quad \text{Carried by concrete}$$

$$\therefore \text{Steel must carry the rest} = 4800 - 2940 = 1860 \text{ kN}\cdot\text{m}$$

$$\therefore 0.75 f_y A_s' (700 - 60) = 1860 \text{ kN}\cdot\text{m}$$

or choose suitable f_y value

$$0.75 \times 460 \times A_s' (700 - 60) = 1860 \text{ kN}\cdot\text{m}$$

$$\Rightarrow \therefore A_s' = 8424 \text{ mm}^2$$

\Rightarrow 40 mm bars @ 150 mm centres.

Probably choose 125 centres.

Tension steel then becomes

$$0.87 f_y A_s = 0.75 f_y A_s' + 0.2 f_{cu} b d$$

$$= 0.75 \times 460 \times 8424 + 0.2 \times 40 \times 1000 \times 700$$

$$= \text{~~8291160~~} = 8.50628 \times 10^6$$

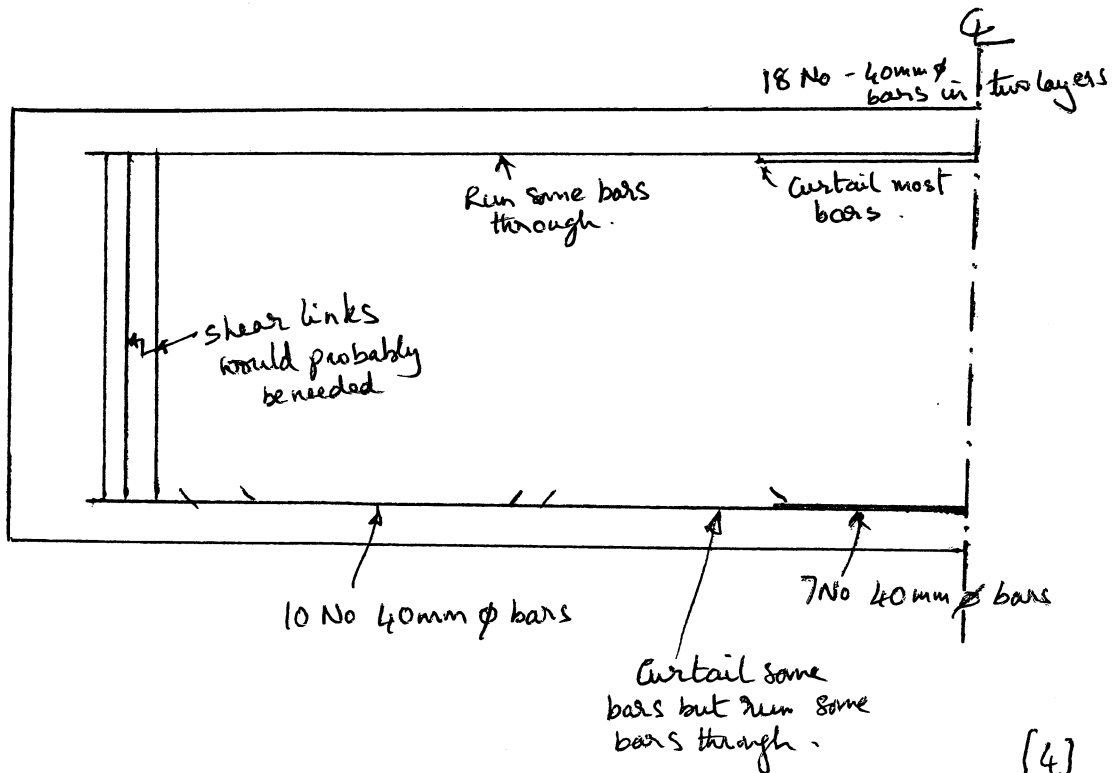
$$\therefore A_s = 21255 \text{ mm}^2$$

= 40 mm bars @ 60 mm cs.

\therefore Probably need 2 layers.

[6]

3 d)

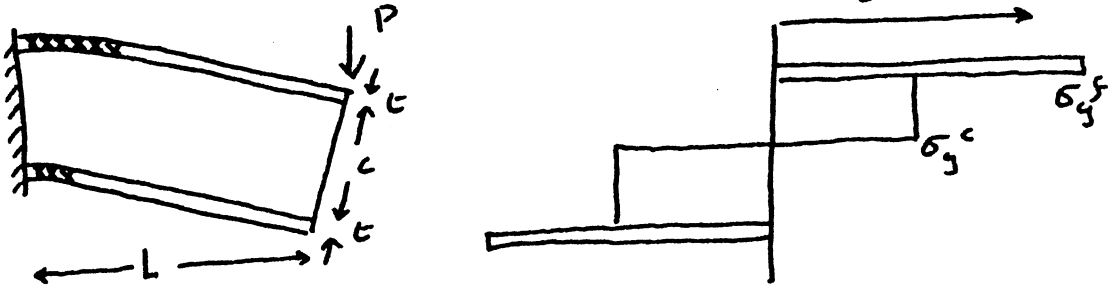


[4]

4. a) Sandwich panels consist of two stiff, strong skins separated by a lightweight core. The separation of the skins by the core increases the first and second Moments of Area of the panel, with little increase in weight. This produces an efficient lightweight structure for resisting bending and shearing loads.

Hexagonal honeycombs, lattice materials, and metal or polymer foams are used as core materials in sandwich panels for lightweight structural applications. [5]

- b) Face yield:



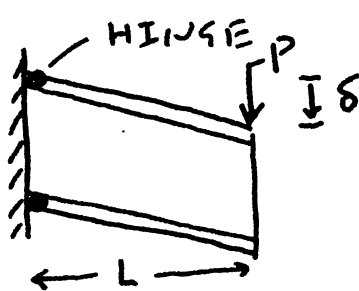
$$PL = \frac{bc^2}{4} \sigma_y^c + bt(c+t) \sigma_y^f$$

$$\text{i.e. } P = \frac{bc^2}{4L} \sigma_y^c + \frac{bt(c+t)}{L} \sigma_y^f$$

$$\text{Since } t \ll c, \quad P = \frac{bc^2}{4L} \sigma_y^c + \frac{bct}{L} \sigma_y^f$$

[4]

Core shear:



$$M_P \rightarrow \quad P\delta = 2 \left(\frac{\sigma_y^f b t^2}{4} \right) \frac{\delta}{L} + \tau_y^c L b c \frac{\delta}{L}$$

$$\therefore P = \frac{\sigma_y^f b t^2}{2L} + \tau_y^c b c$$

(Note that the shear stress across the core is approximately uniform, because of the skins.) [4]

- c) Neglecting core contribution to face yield:

$$P^* = \frac{bct \sigma_y^f}{L}$$

$$I_{max} m = Lb(2tP_f + cP_c)$$

For a given P^* and c ,

$$t = \frac{P^* L}{bc \sigma_y^f}$$

9

$$\text{So } m = Lb \left(\frac{2P^*LP_f}{b\sigma_y^f c} + P_c c \right)$$

$$\frac{dm}{dc} \propto \frac{-2P^*LP_f}{b\sigma_y^f c^2} + P_c = 0 \text{ for min mass}$$

$$\therefore c_{opt} = \sqrt{\frac{2P^*LP_f}{b\sigma_y^f P_c}}$$

$$m_{min} = Lb \left(\frac{2P^*LP_f}{b\sigma_y^f c_{opt}} + P_c c_{opt} \right)$$

$$= Lb \sqrt{\frac{2LP^*P_f P_c}{b\sigma_y^f}} + \frac{2LP^*P_f P_c}{b\sigma_y^f}$$

$$\therefore m_{min} = 2L \sqrt{\frac{P^*P_f P_c Lb}{\sigma_y^f}}$$

[7]

5. a) i) Casting using a polymer foam precursor:

An open-cell polymer foam of the desired cell size and relative density is selected. This is then coated with a mould casting slurry (ceramic powder) which is then dried and embedded into a casting sand. The mould is then baked, both to harden the casting material and to decompose and evaporate the polymer template, leaving behind a 'negative image' of the foam. The mould is then filled with a metal alloy and cooled. The mould materials are then removed, leaving the metal equivalent of the original polymer foam.

(The ERG DUOCEL foam is made by this method, which typically produces open-cell foams with pore sizes of 1-5 mm, and relative densities as low as 5%.)

ii) Metal deposition on cellular preforms:

Open-cell polymer foams can serve as templates upon which metals are deposited by Chemical Vapour Decomposition (CVD), by evaporation, or by electrodeposition. After several tens of micrometres of the metal have been deposited, the polymer is burned out by heating in air. This results in a cellular metal structure with hollow ligaments: a subsequent sintering step is then used to densify the ligaments.

(The INCO nickel foam is produced in this manner, by CVD. Typically, foams with pore sizes 100-300 μm & relative densities in the range 2-5% can be made by this technique.)

[6]

- b) i) For simplicity, open-celled metal foams can be modelled as shown in Figure 5, with dimensions chosen such that the relative density and surface area density of the model match those of the real foam (which can be measured).

The cylinders shown in Fig. 5 are each shared by 4 cells, so only $\frac{1}{4}$ of them belong exclusively to the unit cell shown. Assuming $d \ll h$, the relative foam density is:

$$\rho = \frac{\rho^*}{\rho_s} = \frac{12 \times (\pi d^2/4) h \times \frac{1}{4}}{h^3}$$

$$\rightarrow \rho = \frac{3\pi}{4} \left(\frac{d}{h}\right)^2$$

Similarly, the surface area density of the foam is:

$$\alpha = \frac{\text{Total surface area of solid in unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{12 \times \pi d h \times \frac{1}{4}}{h^3} = \frac{3\pi d}{h^2} \quad [5]$$

- ii) For a 20 ppi foam,

$$h = \frac{25.4}{20} = 1.27 \text{ mm}$$

And since $d/h = \frac{2}{\sqrt{3\pi}} \rho^{1/2}$, $\alpha = \frac{2\sqrt{3\pi}}{h} \rho^{1/2}$

$$\therefore \alpha = \frac{2\sqrt{3\pi}}{1.27 \times 10^{-3}} \times 0.1^{1/2} = \underline{1529 \text{ m}^2/\text{m}^3}$$

To qualify as a compact heat exchanger, α must be larger than $700 \text{ m}^2/\text{m}^3$.

Therefore, a 20 ppi foam with relative density 0.1 does qualify. [4]

- iii) In general, there are 3 principles guiding the design:

- 1) The foam needs to have a reasonably high relative density and be made of a highly conductive material, so that heat can be rapidly transported from the heat source to the bulk of the foam. Aluminium and copper are good candidates.
- 2) A high surface density, and turbulent flow of the coolant within the foam, is then required to facilitate heat transfer between solid and fluid.
- 3) A low pressure drop in the coolant is desirable, to minimise pumping power.

The *thermal* efficiency of this type of heat exchanger is defined as

$$I = \frac{\text{Rate of heat removal}}{\text{pumping power}}$$

Requirements 1 and 2 call for high relative density, surface area density, and coolant velocities; but these tend to compromise requirement 3. Simple models based on the cubic unit cell have shown that, for a given pore size, I is maximised when the relative density is in the range $0.1 \sim 0.2$. (Increasing the pore size for a given relative density can raise the value of I , but at the expense of making the heat exchanger larger.)

[5]

6. a) i) The main contributions to the cost of manufacturing a component are:
- The capital cost of the equipment used to make the component (generally a non-dedicated cost), C_c
 - The cost of dies, jigs and other special tooling (a dedicated cost), C_d
 - Time-dependent costs (labour, overheads, administration, &c.), C_t
 - The cost of energy per unit of production, C_e
 - The cost of information (research, development costs, royalties, &c.) per unit of production, C_i
 - The cost of the material, C_m

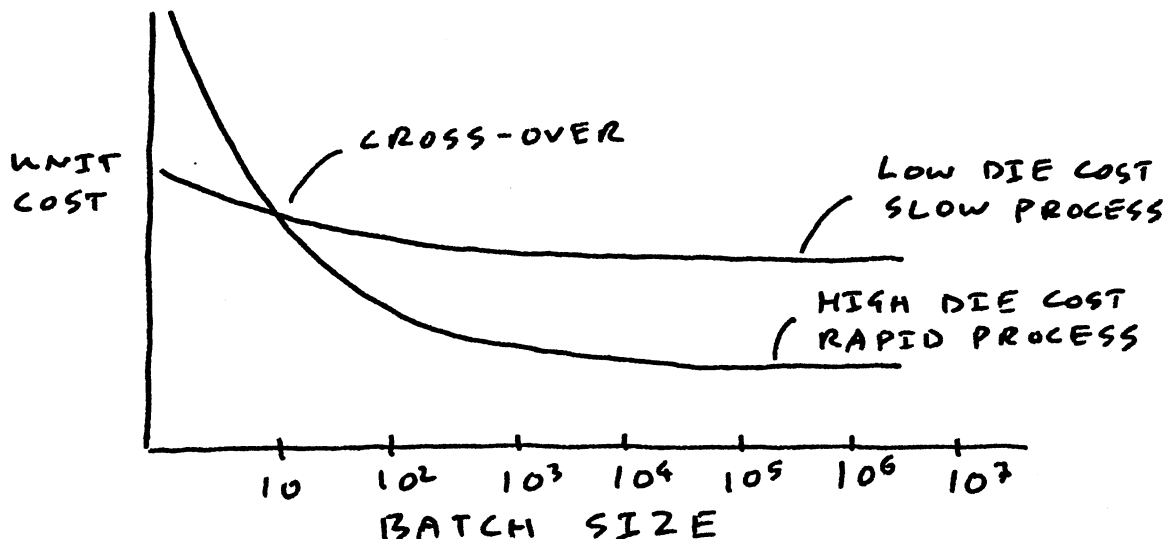
These are combined as follows:

$$C = C_m + \frac{C_d}{n} + \frac{1}{\dot{n}} \left(\frac{C_c}{t_{wo}} + C_t \right) + C_e + C_i x$$

where n is the batch size, \dot{n} is the rate of production (units per hour) and t_{wo} is the capital write-off time.

[5]

- ii) Cost varies with batch size as shown in the sketch:



The curves differ because of the differing capital, die, and time-dependent costs, and the rate of production. This leads to cross-overs, as shown, making one process economic at low batch size and another at high. Die costs often dominate at low batch sizes: the development of low-cost dies that can survive a small batch run, avoiding the need for costly long-life dies, can extend the economic range of a given process.

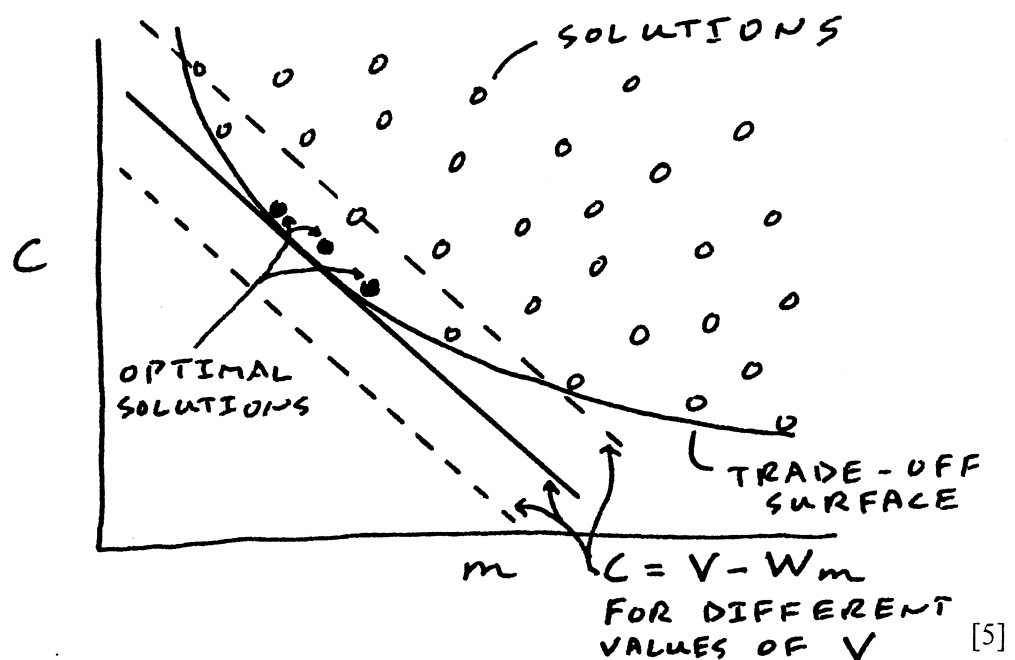
[5]

- b) i) When a problem has two objectives – minimising both mass m and cost C , for instance – a conflict arises: the cheapest solution is not the lightest, and vice versa. The best combination is sought by constructing a trade-off plot (see sketch below) using mass as one axis, and cost as the other. The lower envelope of the points on this plot defines the trade-off surface, and the solutions that offer the best compromise lie on this surface.

To get further, we need a value function. Define the function

$$V = C + Wm$$

where W is a constant (a weight factor) that defines the acceptable additional cost to reduce the mass by 1 unit. The best solutions are found where the line defined by this equation is tangential to the trade-off surface. (Note that a low value of V is more desirable than a high one.)



ii) For problems involving two or more competing objectives, the relative weighting of one objective with respect to each of the others must be established. These weight factor are found in two principal ways:

- 1) By engineering analysis – for instance, by calculating the cost of the fuel saved by reducing the weight of a vehicle by 1 unit;
- 2) By asking experts (or potential customers) for their best estimates, using a structured interviewing technique.

[5]

SELECTED TOPICS

Part Ib- Paper 8 - Section C - Aerothermal Technology

Q7 (a) $P_{02} = 46 \text{ kPa}$ $P_R = 1.65 \therefore \underline{\underline{P_{023} = 75.9 \text{ kPa.}}}$

$T_{02} = 260 \text{ K}$ $\eta = 0.88 = \frac{T_{0235} - T_{02}}{T_{025} - T_{02}}$

$T_{0235} = T_{02} \times 1.65^{\gamma-1/\gamma} = 299.99$

$\therefore T_{023} = \underline{\underline{305.45 \text{ K}}}$

(b) Nozzle is isentropic $\therefore T_{11} = \left(\frac{P_{11}}{P_{023}}\right)^{\gamma-1/\gamma} T_{023}$

$\dot{Q} = 0$ $\dot{W}_s = 0$ $= \left(\frac{29}{75.9}\right)^{\frac{0.4}{1.4}} \cdot 305.45 = 232.03$

SFEE: $V_{11} = \sqrt{2c_p(T_{023} - T_{11})} = \underline{\underline{385.1 \text{ m/s}}}$

(c) $P_{03} = 25 P_{023} = 25 \times 75.9 = 1897.5 \text{ kPa} = P_{04}$

$T_{03} = 818 \text{ K}$

$T_{04} = 1600 \text{ K}$

Work balance $W_T = W_C$ for HPT

$(T_{03} - T_{023}) = (T_{04} - T_{045})$

$\rightarrow T_{045} = -(818 - 305.45) + 1600 = \underline{\underline{1087.45 \text{ K}}}$

$\eta_T = 0.89 \Rightarrow T_{0455} = T_{04} - \eta_T^{-1}(T_{04} - T_{045}) = 1017.55$

$P_{0455} = P_{04} \left(\frac{T_{0455}}{T_{04}}\right)^{\gamma/\gamma-1} = \underline{\underline{389.24 \text{ kPa}}}$

SELECTED TOPICS

Part Ib- Paper 8 - Section C - Aerothermal Technology

7 (d) $BPR = 9$ $\eta_{LPT} = 0.9$

Work Balance for $fan + LPT$:

$$(1 + BPR)(T_{02s} - T_{02}) = 1.(T_{04s} - T_{05})$$

$$\therefore T_{05} = \underline{632.95 \text{ K}}$$

$$\therefore T_{05s} = T_{04s} - \eta_{LPT}^{-1}(T_{04s} - T_{05}) = 582.45$$

$$\therefore P_{05} = P_{04s} \left(\frac{T_{05s}}{T_{04s}} \right)^{\frac{\gamma}{\gamma-1}} = 389.24 \left(\frac{582.45}{1087.45} \right)^{3.5} = \underline{43.77 \text{ kPa}}$$

Isentropic nozzle

$$\therefore T_a = T_{05} \left(\frac{P_a}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} = 632.95 \left(\frac{29}{43.77} \right)^{0.2857} = 562.71$$

$$\therefore \text{SPPE} \Rightarrow V_a = \sqrt{2c_p(T_{05} - T_a)} = \underline{376.67 \text{ m/s}}$$

(e)

$$P_1 = 29 \text{ kPa} \quad \therefore T_1 = T_{02} \left(\frac{P_1}{P_{02}} \right)^{\frac{\gamma-1}{\gamma}} = 227.89 \text{ K}$$

$$\Rightarrow V_1 = \sqrt{2c_p(T_{02} - T_1)} = 254.67 \text{ m/s}$$

$$\text{Gross Thrust} = \dot{m}_c (BPR V_a + V_a) = X_c$$

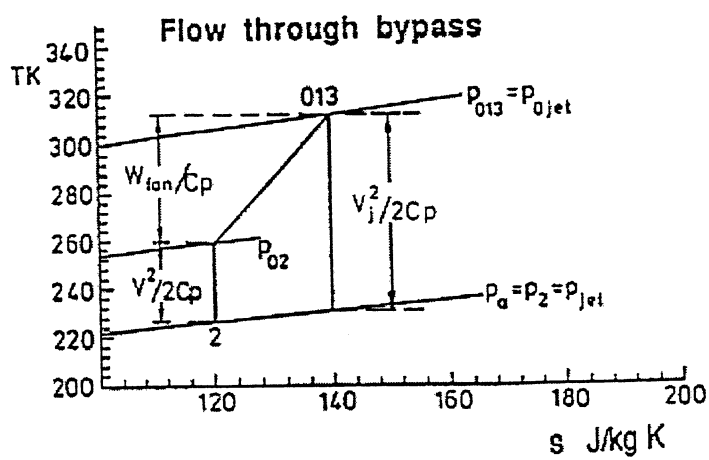
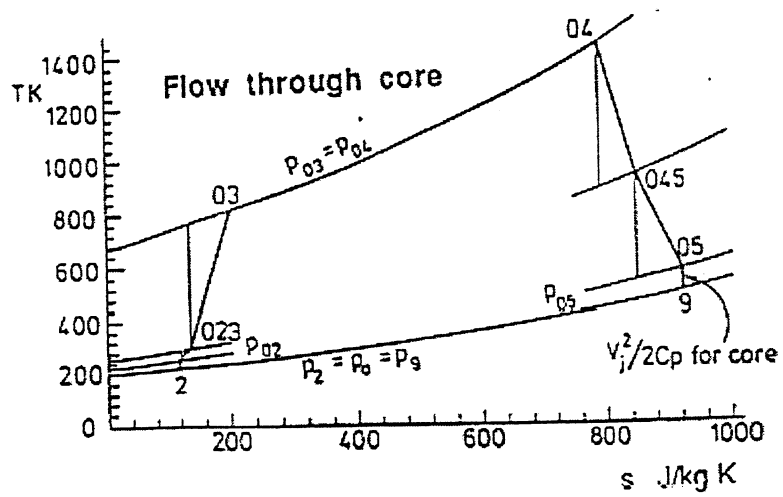
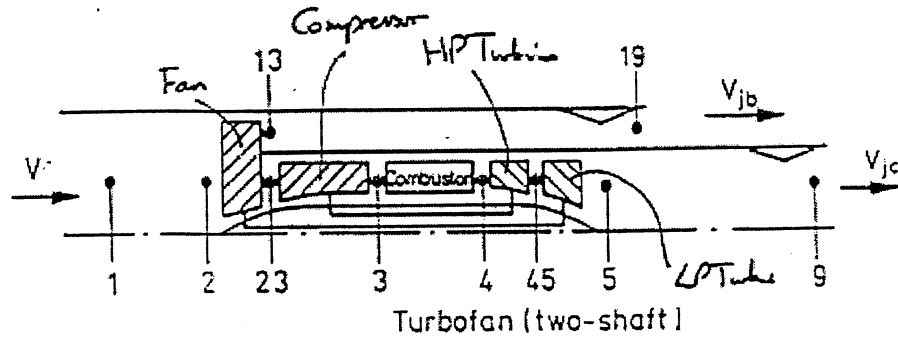
$$X_c = \dot{m}_c (9 \times 385.1 + 376.67) = \dot{m}_c 3842.6$$

$$\text{Net Thrust} = X_c - \dot{m}_c (1 + BPR) V_1$$

$$\Rightarrow 400 \times 10^3 = \dot{m}_c (3842.6 - 10 \times 254.67)$$

$$\dot{m}_c = \underline{308.67 \text{ kg/s}}$$

8e) The turbofan will have two or three shafts because of the need for the fan, due to its larger diameter, to rotate at a lower rotational speed. This is needed because the pressure rise and blade stresses (both proportional to blade speed squared) and Mach number (blade speed) must not exceed certain limits. The pressure rise/Mach number limits exist due to the risk of boundary layer separation, shock wave strength and noise.



SELECTED TOPICS

Part Ib- Paper 8 - Section C - Aerothermal Technology

8(d).

$$\eta_p = \frac{\text{Power to plane}}{\text{Rate of } \Delta KE \text{ to gas}}$$

High $\eta_p \rightarrow$ less wasted KE

$$\eta_p = \frac{\text{flight speed} \times \text{Thrust}}{\frac{1}{2}(\dot{m}_a + \dot{m}_p) v_j^2 - \frac{1}{2} \dot{m}_a v^2} = \frac{v[(\dot{m}_a + \dot{m}_p) v_j - \dot{m}_a v]}{\frac{1}{2}(\dot{m}_a + \dot{m}_p) v_j^2 - \frac{1}{2} \dot{m}_a v^2}$$

Above assumes bypass velocity = core jet velocity. Neglect mix

gives
$$\eta_p = \frac{\dot{m}_a v (v_j - v)}{\frac{1}{2} \dot{m}_a (v_j^2 - v^2)} = \frac{2v}{v + v_j}$$

Now,

$$\eta_{TH} = \frac{\Delta KE \text{ to gas}}{\dot{m} LCV} \quad \eta_o = \frac{\text{Power to Plane}}{\dot{m}_f LCV} \Rightarrow \eta_o = \eta_{TH} \eta_p$$

$$sfc = \frac{\dot{m}_f}{\text{Thrust}} \Rightarrow \eta_o \propto \frac{1}{sfc} \left(\propto \eta_p \text{ for given } \eta_{TH} \right)$$

Civil engines use high BPR because

require high $\eta_o \therefore$ high η_p

$\Rightarrow v_j \approx v$ but net Thrust $\rightarrow 0 \therefore$ high \dot{m}_a needed.

Typical BPR $\sim 6-9$

BPR limited by

- (i) fan tip speed (see (c)) which fixes RPM for given dia (increases with BPR) therefore requiring disproportionately more LPT stages (or gear box) \Rightarrow increased wt & cost
- (ii) large size leading to increased nacelle drag, increased cost, installation problems

(e) Breguet's range eqⁿ requires ML/D (or VL/D) maximised for best range. This corresponds to $M \sim 0.8-0.85$ and $C_{Lopt} \sim 0.5$ for best LD etc. As a result, cruise speed \gg take off speed.

Flight speed (~ 260 m/s) much greater than at takeoff (~ 90 m/s). Lift @ cruise similar to take off \therefore since $L \propto \rho V^2 C_L$, ρC_L must reduce as V rises. In fact C_L at take off much higher than at cruise ($C_{Lopt} \sim 0.5$) but difference not enough due to much increased $\frac{1}{2} \rho V^2 \therefore \rho$ must be reduced by flying at high altitude. As fuel is burned, W falls $\therefore L$ falls $\therefore \rho$ must fall \therefore altitude should increase to maintain optimum condition

SELECTED TOPICS

Part Ib- Paper 8 - Section C - Aerothermal Technology

9 (i) Net thrust is force transmitted to airframe by the engine

$$X_{NET} = \underbrace{(\dot{m}_{air} + \dot{m}_{fuel}) V_{jet}}_{\text{Exit Momentum Flux}} - \underbrace{\dot{m}_{air} V_{aircraft}}_{\text{Ram Drag = Inlet Momentum}}$$

$$\text{Gross Thrust} = X_G = \text{Exit Momentum Flux} \quad \text{Ram Drag} = \text{Inlet Momentum}$$

Gross Thrust is the thrust produced by the engine in the absence of ram drag (see above)

When stationary, $X_G = X_{NET}$

Neglecting air; $X_{NET} = 0$ when $V_{jet} = V_{aircraft}$

(b) At Mach 0.9:

$$T_{01} = T_1 \left[1 + \frac{\gamma-1}{2} M_1^2 \right] = 252.15 \text{ K} = T_{02}$$

$$P_{01} = P_1 \left[\frac{T_{01}}{T_1} \right]^{\frac{\gamma}{\gamma-1}} = 32.64 \text{ kPa} = P_{02}$$

$$V_1 = M_1 \sqrt{\gamma R T_1} = 0.9 \times 295.28 = \underline{\underline{265.75 \text{ m/s}}}$$

$$\begin{aligned} X_G &= X_{NET} + X_{ram} = 50 \times 10^3 + 66.5 \times 265.75 \\ &= \underline{\underline{67673 \text{ N}}} \end{aligned}$$

(c) Because the nozzle is choked, the exit conditions cannot influence the operation of the engine (no information can travel upstream through the nozzle). Therefore, the pilot has only the fuel flow rate with which to control the engine at given P_{02} & T_{02} . The use of inlet stagnation conditions removes the dependency on flight speed.

The fuel flow rate determines the rate of energy release. Therefore, the appropriate N-D group is

SELECTED TOPICS

Part Ib- Paper 8 - Section C - Aerothermal Technology

$$\frac{\dot{m} \text{ LCV}}{\sqrt{\rho_{023}} D^2 P_{023}}$$
 - 'energy release rate'
 velocity force

Alternatively, the mass flow (non-dimensional) depends on the operating point of the engine. Then we may also use

$$\frac{\dot{m}_{air} \sqrt{\rho_{02}}}{D^2 P_{02}}$$
 - "momentum flux"
 - "force"

(d) For the flight & test in ground, $\dot{m}_{air} \sqrt{T_0} = \text{same value}$

$$\therefore \dot{m}_{air \text{ TEST}} = \dot{m}_{air \text{ cruise}} \sqrt{\frac{T_{02 \text{ cruise}}}{T_{02 \text{ TEST}}}} \frac{P_{02 \text{ TEST}}}{P_{02 \text{ cruise}}}$$
 sec (b)

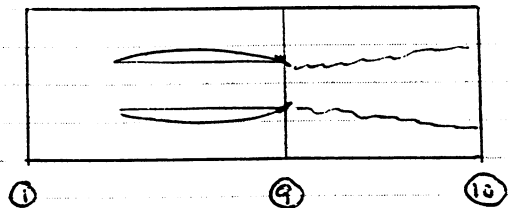
$$= 66.5 \sqrt{\frac{252.15}{288}} \frac{101}{32.64} = \underline{\underline{192.5 \text{ kg/s}}}$$

(e)

$X_G = \dot{m} V_{10}$; $P_{10} = P_{amb}$

But (conservation of mom)

$\dot{m} V_{10} + P_{10} A_{10} = \dot{m} V_9 + P_9 A_9$



$\therefore X_G = (\dot{m} V_9 + P_9 A_9) - P_{10} A_{10}$

$= (\dot{m} V_9 + P_9 A_9) - P_{amb} A_9$

where $\dot{m} V_9 + P_9 A_9 = \text{Impulse Func. } \dot{e}_9$

$$\therefore X_{G \text{ TEST}} = (X_{G \text{ cruise}} + P_{amb} A_9) * \frac{P_{01 \text{ TEST}}}{P_{01 \text{ cruise}}} - P_{amb \text{ TEST}} A_9$$

$$= \underline{\underline{197 \text{ kN}}}$$

Cribs

Q. 10

a] mobility = velocity/field $\mu = v/E$

considering a tube of current of velocity v that passes a point in 1 second,

current density = charge density x velocity = $eN.v$

$$v = \mu E$$

$$\sigma = J/E = eNv/E = eN\mu E/E = eN\mu$$

b] $N = 4 \times (4.09 \times 10^{10})^{-3} = 5.85 \times 10^{28} \text{ m}^{-3}$?

$\sigma = Ne\mu$, so $\mu = 6.3 \times 10^7 / [5.85 \times 10^{28} \times 1.6 \times 10^{-19}] = 6.7 \times 10^{-3} \text{ m}^2/\text{V.s}$ ✓

~~for real Si.~~

c] $\sigma = 2 \times 10^{20} \times 1.6 \times 10^{-19} \times 6.7 \times 10^{-3} = 0.21 \text{ ohm}^{-1} \text{ m}^{-1}$ ✓

d] equations needed are

$$P = V.I$$

$$V = E.L$$

$$t = L/[\mu E]$$

$$W/L = c$$

$$I = eNWd\mu E$$

where W, L, d are usual dimensions of channel.

So, substitute for W and E ,

$$P = V.I = V.eNcLd\mu V/L = eNcd\mu V^2$$

$$V^2 = 2 / (1.6 \times 10^{-19} \times 10^{22} \times 50 \times 10^{-5} \times 0.1) = 25$$
 ✓

$$V \leq 5 \text{ V}$$
 ✓ ↑

Combine 2 and 3

$$L^2 = \mu Vt = 0.1 \times 20 \times 10^{-12} \times 5 = 10^{-11}$$
 ✓

$$L = 3.1 \times 10^{-6} \text{ m}$$

Q. 11

Photoelectric effect. Minimum frequency for any electrons. Electron KE proportional to light frequency above this threshold, rather than to light intensity., as expected from power density arguments. Current density proportional to light intensity ✓

b] $E = hv = hc/\lambda = 6.6 \cdot 10^{-34} \times 3 \cdot 10^8 / 400 \cdot 10^{-9} = 4.95 \cdot 10^{-19} = 3.09 \text{ eV}$

c] $\phi = A \exp(ik.x) + B \cdot \exp(-i k.x)$

$$E = \hbar^2 k^2 / 2m$$

Boundary conditions give

$$\phi = A' \sin(k.x)$$

$$k = n\pi/L$$

$$E = \hbar^2 (n\pi/L)^2 / 2m$$

d) Take the case of $n=1$.

$L = 200 \text{ nm}$. Photon wavelength = 400 nm .

$$E = (1.05 \times 10^{-34} \times 3.14 \times 1/200 \times 10^{-9})^2 / [2 \times 0.9 \times 10^{-30}] = 1.5 \times 10^{-24} \text{ J}$$
$$= 9.4 \times 10^{-6} \text{ eV}$$

small compared to photon.

Q.12

a) The various etch steps used include removal of resist, insulator, polysilicon, silicon and metals.

Wet etching:

This involves liquid chemicals to dissolve unwanted material to produce the required patterns. Examples are given below. The exact mixture of the chemicals needed, their temperature, degree of agitation etc all impact on the process. Generally wet etching is faster than dry etching but less controllable.

Examples:

Si etch, KOH + DI water 1:7 at 60°C – etch rate is around 0.5 μ m per minute

SiO₂ etch – P-etch, a mixture of 15 parts HF, 10 parts HNO₃ and 300 parts DI water at 25°C. etch rate 2nm/sec.

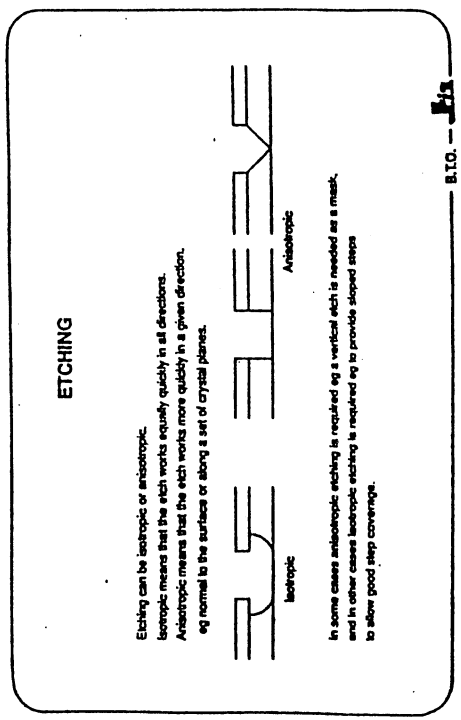
Al etch – 1 part HNO₃, 4 parts acetic acid, 4 parts orthophosphoric acid and 1 part DI water – etch rate around 35nm/min.

2

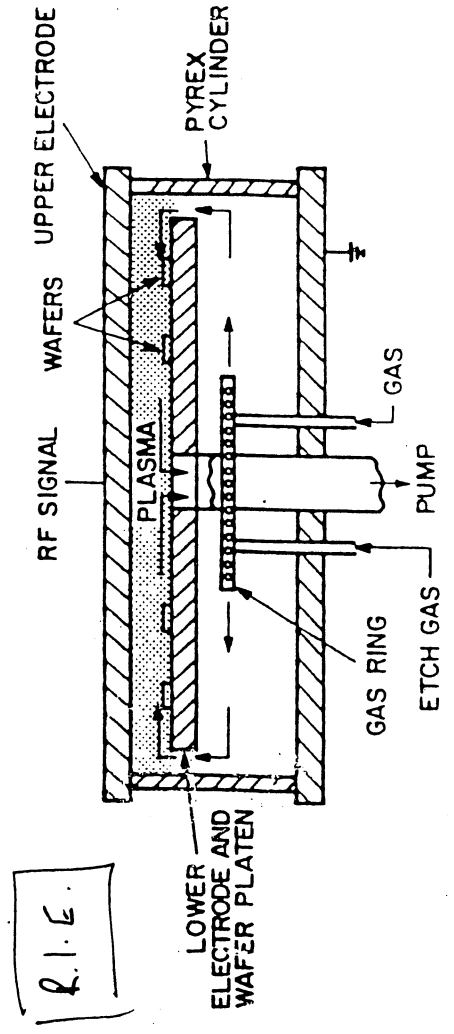
Dry etching includes two basic methods: plasma and sputter etching. Sputter etching is a mechanical process where the surface to be etched is bombarded with high energy ions or atoms which physically knock atoms off the surface.

Plasma etching is more closely related to wet etching where a chemical reaction takes place to remove material. The reactive species are produced by interaction in an RF plasma produced between two parallel plates.

Etches can be either isotropic or anisotropic.

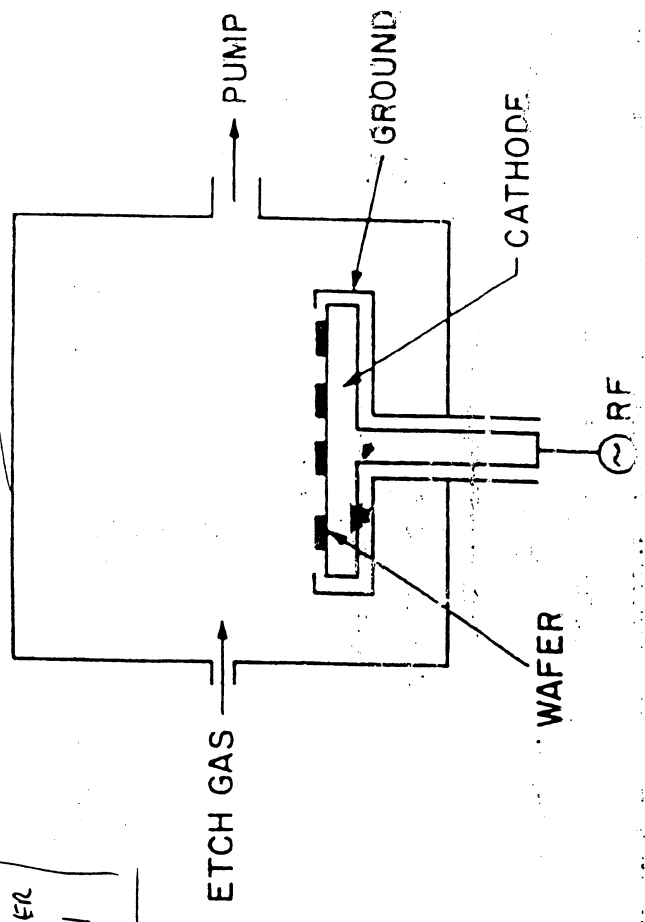


In general wet etches are isotropic and sputter etching can be either. Sputtering is generally anisotropic and is used principally to clean up surfaces, e.g. the removal of the native oxide from aluminium. This is difficult to etch chemically but can easily be sputtered off.



Sputter etch

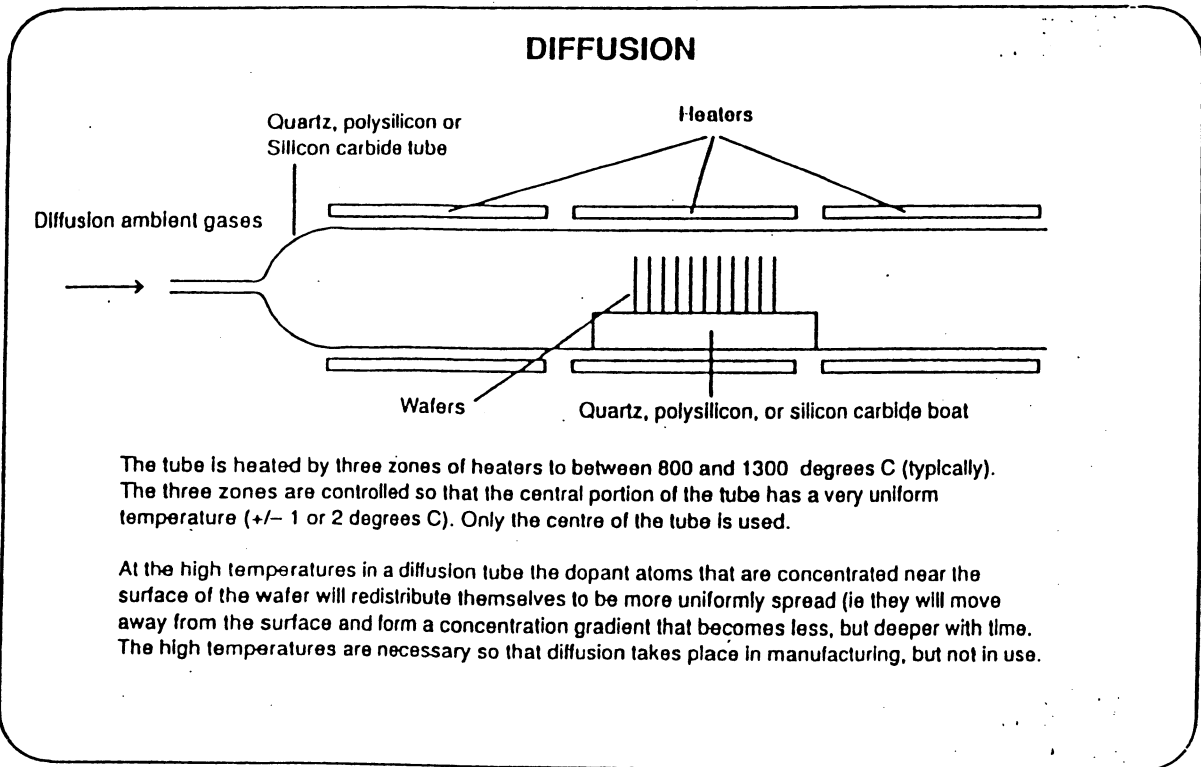
SPUTTER ETCH



b) Phosphorus or Arsenic are possible n-type dopants. There are two main techniques for doping either Diffusion or Ion implantation.

Diffusion – simple and cheap but time consuming. Difficult to control the dopant from spreading sideways.

Ion implantation – more expensive but faster and lower annealing temperature which leads to less sideways spread and better control.



DIFFUSION/ION IMPLANTATION

The above processes show how to produce the basic structure. Now we have to consider how to modify the semiconductors themselves to produce the required properties.

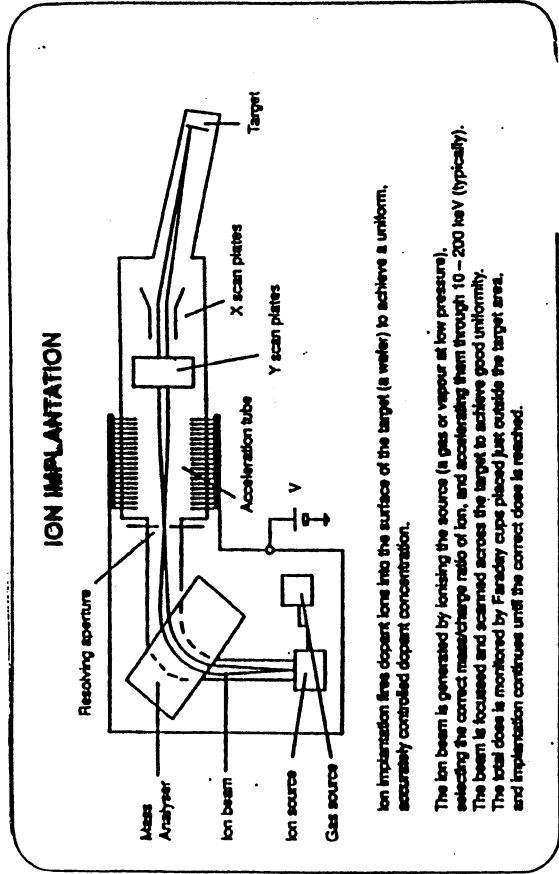
In order to produce most devices, it is necessary to use both N-type and P-type semiconductors. The most common method employed to alter the type of semiconductor is to use a technique known as diffusion. For example in the transistor structure described by Dr Robertson it may be used to define the source and drain regions.

Two types of diffusive processes are relevant to device formation.

Diffusion from an infinitive source where a constant surface concentration is maintained and diffusion from a constant total dopant.

ION IMPLANTATION

This is the introduction of ionized projectile atoms into samples. The energies of the atoms are sufficient to cause penetration, beyond the surface region. It has the advantage over diffusion in that it has the capability of precisely controlling the number of implanted dopant atoms and it takes place at much lower temperatures. A Schematic of such a system is shown in the next diagram.



The use of 10-500keV energies for boron, phosphorus or arsenic ions is sufficient to implant ions from about 100-10,000 Å below the silicon surface. The depth and type of implantation can be selected to meet a particular application.

As it is a directional process it also reduces the amount of sideways spread through apertures that is associated with the standard diffusion process.

However the ions cause a certain amount of damage to the crystal surface as they impact upon it in some instances the energies being sufficient to turn the surface into an amorphous layer.

In addition the atoms as implanted will be mostly inactive because on average they will tend to occupy interstitial rather than substitutional positions in the lattice.

Because of both of these problems a post implantation heat treatment is needed. If a shallow diffusion is required the heat treatment is short and at very low temperatures but if a deeper diffusion is required then a drive in stage is needed.

To summarise ion implantation has the advantage that it produces very uniform profiles and is very controllable. However, it is time consuming and is not practical for large doses. The damage to the lattice also increases as the dose increases and this also limits the highest practical dose (exact amount depends on the atom used as atom size is important factor).

$$c) \quad C(x, t) = C_s \operatorname{erfc} \left[\frac{x}{2(Dt)^{\frac{1}{2}}} \right]$$

$$C_s \text{ for P Silicon from Graph (a) at } 1050^\circ\text{C} \\ = 10^{27} / \text{m}^3$$

$$D \text{ for P Silicon at } 1050^\circ\text{C} = 1323 \text{ K} \\ 1000/\text{K} = 0.76$$

From Graph

$$D = 1.5 \times 10^{-3} \Rightarrow D = 1.5 \times 10^{-17} \text{ cm}^2/\text{s} \quad (1 \text{ cm} = 1 \times 10^{-2} \text{ m}) \\ = ~~1.5 \times 10^{-21} \text{ m}^2/\text{s}~~$$

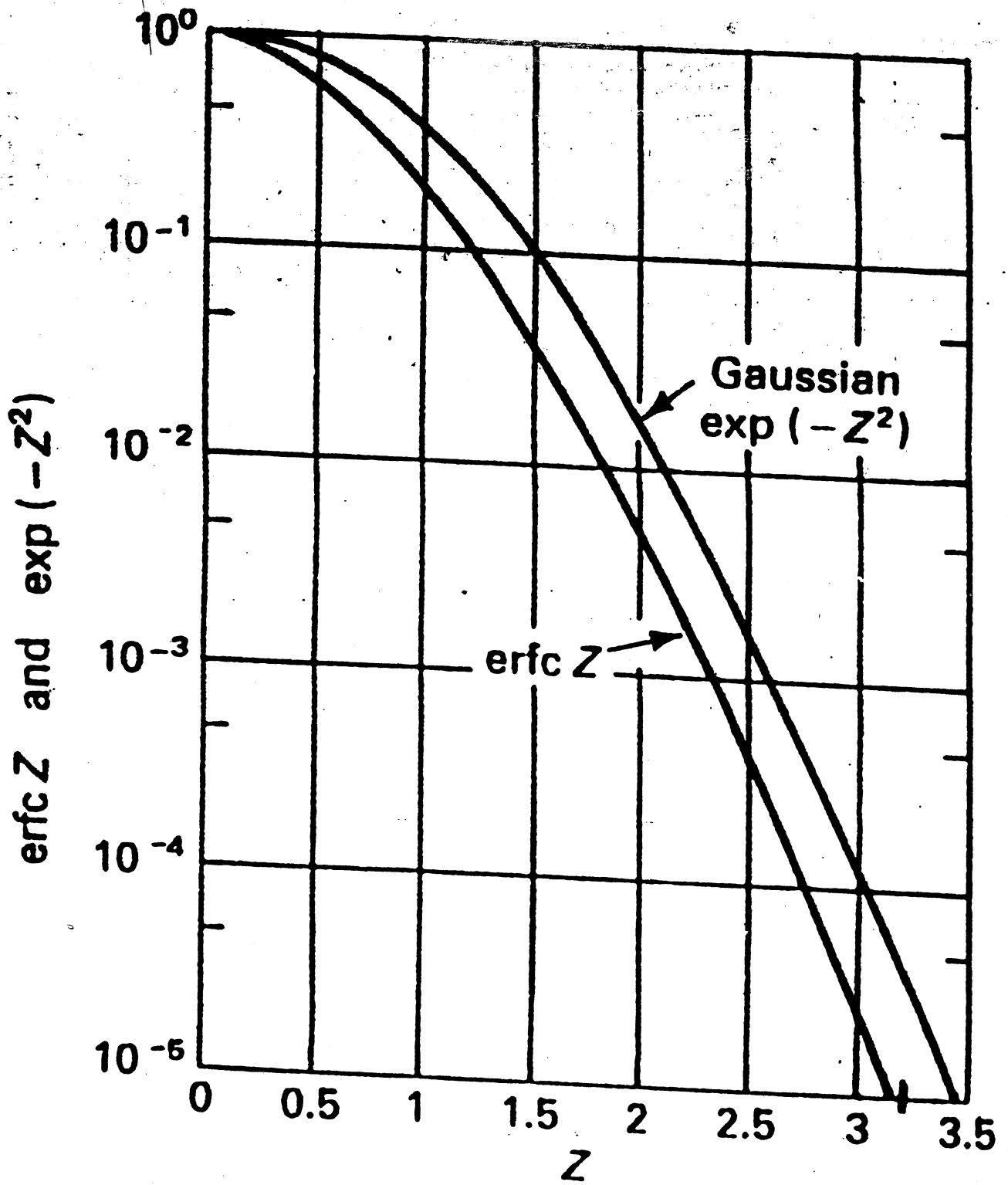
Junction formed when concentration of P atoms
= concentration of substrate i.e. at $5 \times 10^{22} / \text{m}^3$

$$1. \quad \frac{C(x, t)}{C_s} = \frac{5 \times 10^{22}}{10^{27}} = 5 \times 10^{-5}$$

$$\Rightarrow \operatorname{erfc} \left(\frac{x}{2(Dt)^{\frac{1}{2}}} \right) = 5 \times 10^{-5}$$

$$\frac{x}{2(Dt)^{\frac{1}{2}}} = 2.8 \quad \frac{x^2}{4Dt} = 7.84$$

$$\Rightarrow x = \sqrt{7.84 \times 4 \times 1.5 \times 10^{-17} \times 1.5 \times 60 \times 60} \\ = \sqrt{2.54 \times 10^{-12}} = ~~1.16 \mu\text{m}~~ \\ = 1.6 \times 10^{-6} \text{ m} = 1.6 \mu\text{m}$$



Part Ib, Paper 8, Section E “An Architecture for Wearable Computing”

Q 13

- (a) -A stateless client is simple
 - State is not lost if the client crashes
 - Creating new clients is easy
 - Management of the server side is potentially easier because it is centralized
 - Mobility comes “for free” because the client can be re-started at another place with no difficulty

- (b) -Basic primitive is “put a rectangle of pixel data at a given x,y position”
 - Initial interaction is
 - Authenticate in some way if required
 - Set up connection; server sends width and height in its “natural” format
 - Client chooses pixel format and encoding
 - Then
 - Client requests screen update
 - Server sends screen update
 - Client processes screen update
 - Client sends keyboard and pointer events as they happen

- (c) -Raw encoding is where pixel data is sent in left to right scanline order
 - Run length (RRE) encoding describes rectangle differences from a single background colour rectangle
 - Copy rectangle encoding is used where the client already has the pixel data somewhere in the frame buffer; this is good when a user moves a window across the screen
 - JPEG
 - MPEG

- (d) -The intrinsic design is that of a “suck” protocol hence data is only sent when a client requests it. This means the amount of network traffic is minimized for example when a fast server sends to a slow client
 - The pixel data encodings reduce the amount of data sent and allow choice of the most appropriate encoding for any stream
 - Pixel data caching can be used at the client end
 - Servers do not have to be workstation or computers

Q 14

(a) Benefits of multiprocessing:

- Better hardware utilization (using multiple peripherals in parallel)
- Can perform computation in “parallel” with peripheral processing
- It appears many processes are running in parallel (multiple applications/background processes)
- Can switch to a new process when waiting for data/user input

The definition of each of the given process states is:

Running: Process is using the CPU and is being executed. There can only be one running process per CPU in the system.

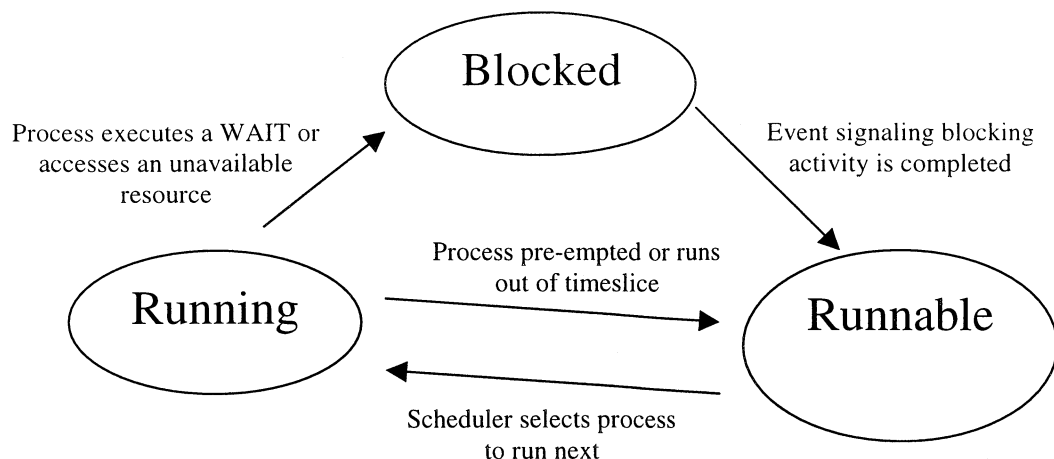
Runnable: Process could be run once the CPU is made available.

Blocked: Process is waiting for a resource to be made available.

A running process becomes runnable when it is pre-empted after consuming its time slice. A running process may also become blocked if it tries to access a resource that is not available, or while the data from the resource is being loaded. A runnable process becomes running when it is scheduled to have access to the CPU.

A blocked process becomes runnable once the resource it is waiting for becomes available.

This is summarized in the following state-transition diagram:



The actions performed by the different scheduling policies are:

- i) **Round robin:** A queue of runnable processes is maintained. When one process finishes (either by yielding or being preempted), then the next

one on the queue is scheduled and the old process is moved to the end of the queue. When a blocked process becomes free, it is added to the end of the queue. Therefore, each process is executed in turn, in a circular fashion.

- ii) **Priority based:** When the processor becomes available, the next process to be run is the one with the highest priority. This priority may be based on the amount of CPU the process used in the last time slice (to provide some level of fairness), or may be arranged so that I/O bound processes are given higher priority.

A preemptive scheduling mechanism is one in which a process can interrupt a running one in order to be processed in a more timely fashion. It can also mean that a process will be interrupted once it has used up its current time slice (to avoid letting a process have permanent access to the CPU).

In a non-preemptive scheme, a process must yield the CPU to another process, usually by an explicit system call. Also known as cooperative scheduling this is harder to guarantee fairness between processes and can result in slower response to events.

(b)

- i) In a time-slicing system, a compute bound process will usually be preempted from the processor, while an I/O bound one will usually yield the CPU as it waits for I/O to be completed.
- ii) Round robin will favor CPU bound processes, as each process will be given equivalent access to the processor. I/O bound processes will usually yield; so will have to wait a long time between accesses. Therefore, want to give higher priority to I/O bound processes as this will allow them to handle I/O events in a more timely manner and they will block quickly, giving the CPU back to compute bound processes.

(c)

i) A process is a heavyweight object consisting of a block of (protected) memory, session details (such as allocated devices) and at least one thread of execution.

A thread of execution is one of the many paths that can be taken through the process. It is a snapshot of the current location and the current contents of all the processor registers. Multiple threads can operate within the same process space, sharing the same memory region.

When switching between processes, the memory management unit will need to be reloaded with the new memory information. This will also usually result in the cache needing to be invalidated. As memory is shared between threads, switching between

them only requires the processor registers to be reloaded. It also does not invalidate the cache.

As there is much less state allocated to a thread, it is more efficient to switch between threads than. Hence they are sometimes known as lightweight processes.

ii) A WWW server potentially has many hundreds of concurrent connections operating in parallel. It is an I/O bound process (disk and network I/O), so there is frequent switching between these connections. It may also choose to cache frequently accessed pages in memory to avoid slow disk accesses.

Due to the smaller descriptor required for each thread, a more memory efficient implementation would be possible than when using processes. Similarly, being an I/O bound application it would be more efficient to switch between threads when they rapidly block on I/O. Finally, sharing a single memory image between multiple threads allows them to share any cached information, rather than require multiple copies of the data, saving memory and avoiding disk I/O to load multiple copies.

Q 15

- (a) In the fat model, the user carries all of their equipment around on their body. A large number of applications would run directly on the hardware, communicating via a bodynet. This avoids reliance on networked infrastructure and can be used to augment human senses. However, the weight and discomfort are an inconvenience, and the appearance can be off-putting.

In the thin model, the user carries the minimum amount of hardware, perhaps only a token to identify them to the system. Because the system knows who they are, existing computing resources can be tailored to that user without them belonging to the user – ubiquitous computing. However, rely on existing infrastructure and usually need wireless networking for ease of communication.

In the fat model, the user carries around their device, so therefore carries all of their information with themselves. This makes it very secure. If they have an augmented display, then only they can see the information. The device does not need a network connection, so the only security threat is from theft of the device (which loses all of the information).

In the thin model, minimal information is contained in the tag. However, this means all information must be available to all devices so they can be personalized. Sharing a device also allows other people to see the information. Stealing the tag can allow someone to pretend to be someone else.

- (b) Wireless networks provide only an intermittent connection to the main network. The quality of connection (bandwidth, bit error rate etc) is generally lower than a wired connection. Range is limited, so there may need to be handover between cells, requiring a complex protocol. May also need to handover between different networks (802.11 for in-building and GSM for wider area); these may provide different qualities of service, so applications may need to adapt. Peer-to-peer networking between devices is more complicated (for example, Bluetooth needs a rendezvous mechanism to synchronize devices before they can communicate).

Discovery of devices/services is problematic (but may be helped with a distributed object layer offering brokers, etc).

TCP/IP may not be the best network protocol because it presents a complex implementation and may not fit in the limited resources available in the mobile device.

An application-specific protocol can be optimized to perform specific tasks; can be made more efficient.

The networking overheads (additional information in the packet) may also not be appropriate given the limited available bandwidth.

Many benefits of TCP may also be redundant depending on the physical layer – for example GSM packets are already reliable, so an additional layer is unnecessary.

Addressing can be problematic – based around fixed IP address rather than any concept of a service. Need to add these layers on top of the basic protocol (e.g. distributed object broker).

When not transmitting or receiving, the wireless network can switch to a “sniff” mode, where it periodically checks for a network connection. This gives slower response, but reduces duty cycle and hence power. RSSI information can be used to detect approximate range of destination device and the transmit power could be reduced. In favorable signal conditions, the network can also transmit at a higher bit rate for a shorter time, allowing the transmitter to overall use up less power.

The operating system can maintain a list of which processes are using the wireless network and once no applications are using it, it can be turned off to conserve battery power.

To conserve power in other parts of the system, devices may be turned off when they are no being used. The processor can be halted when waiting for I/O to complete or when it is idle. The backlight on an LCD display can be reduced, or even turned off when not being interacted with. The voltage of the device can be reduced to lower power consumption – this can either be stepped or proportional to the work queue.

(c)

Reliability can be enhanced using memory protection – preventing one process from gaining access to another processes memory area or to privileged areas of memory such as used by the OS or devices.

Garbage collection and automatic resource recovery can help badly written software from crashing a system by avoiding the system running out of resources over time (especially memory; see JAVA).

Watchdog timers can be set before starting critical or dangerous tasks that may lockup the system. If a timer is not reset after a particular time period, or if the system is deadlocked or live locked, the timer interrupts will occur and can reset the system back into a known valid state.

User mode and privileged mode can prevent badly written software from accessing dangerous areas of memory, such as reconfiguring a device or the MMU. A switch from the 'safe' user mode is made during an operating system call and allows the trusted OS to have full access to the system whilst maintaining safety and security.

To support memory protection, the system needs to support some sort of memory management unit. This does not need to be a full paging system, but can just support a base and limit register.

A watchdog timer requires a timer with an interrupt.

User and privileged modes require support by the CPU and MMU, so that only certain operations can be carried out in certain modes.

Prof A Hopper