ENGINEERING TRIPOS PART IB

Monday 3 June 2002 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 A rotor is assembled by bolting together on a single shaft four similar uniform discs A, B, C and D each of radius 0.05 m, thickness 0.02 m and mass 1.2 kg. The rotor is supported on two bearings 0.1 m apart as shown in Fig. 1 and is spinning at 10,000 rpm.

(a) The rotor assembly is mounted horizontally in a vehicle which is turning at a rate of 0.5 rad s^{-1}.

(i) Find the polar moment of inertia of the rotor assembly. [2]

(ii) Find the force acting on the bearings due to gyroscopic effects. [4]

(b) Discs A, B and C have unbalances of 0.0002, 0.0010 and 0.0011 kgm respectively while disc D is in perfect balance.

(i) Show clearly on a diagram how would you orientate the discs for static balance. [4]

(ii) The discs are now to be assembled in one of two orders: BCAD or BCDA. Which of these orders will generate the lower vibration? [6]

(iii) For BCDA find the magnitude of the dynamic out-of-balance force acting on each of the bearings. [4]

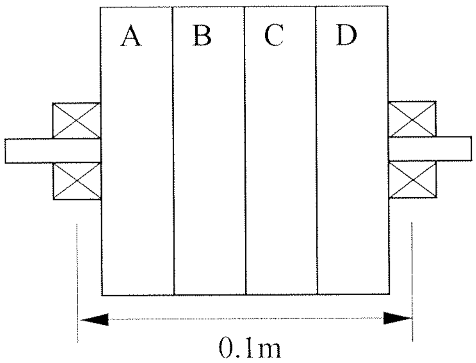

Fig. 1

2 A light crank AB of length $a\sqrt{2}$ is shown in Fig. 2. A light slider at B joins AB to a second light rigid bar EDCF which rotates about the point D. The angle at C is 90° and the point D is distance $2a$ from A. A mass m is fixed at the point E which is distance a from D.

At the instant shown DAB is 45°, DA is parallel to CF and the crank is driven at speed ω and with angular acceleration $\alpha = \omega^2$.

Note: If you wish to construct velocity and acceleration diagrams a separate sheet is provided which you should hand in with your script.

(a) Show that $\omega_{EDCF} = -\omega$ and that the speed of sliding of B on the bar is $2a\omega$. [4]

(b) Find the acceleration of the point B on the bar EDCF. [6]

(c) Show that the acceleration of E is $a\omega^2\mathbf{i} - 3a\omega^2\mathbf{j}$. [6]

(d) Find the torque at A needed to drive the mechanism at the instant shown (ignore both friction and the effect of gravity on the mass at E). [4]

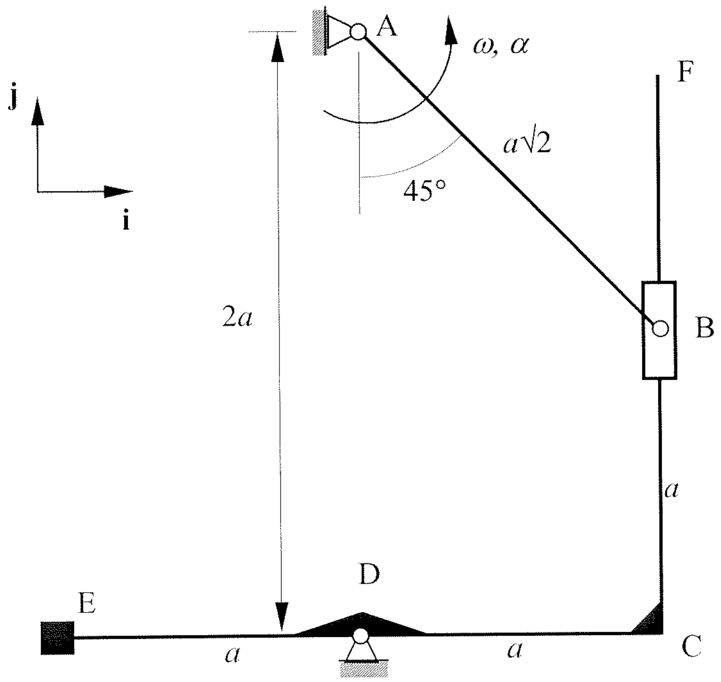

Fig. 2

3 An infinitely-variable gearbox is shown schematically in Fig. 3. Two cones A and B are fixed to input and output shafts respectively as shown. They are coupled by a light disc which is free to rotate on a layshaft. The layshaft itself can slide as shown in a direction parallel to the conical surfaces and to change gear the operator slides the layshaft. The total polar moments of inertia of the input and output shaft assemblies and flywheels are J and $2J$ respectively. The disc makes contact with cone A at a radius r_a and with cone B at radius r_b. The input speed is ω and the output speed is Ω.

(a) Find Ω in terms of ω, r_a and r_b. [2]

Initially the disc is in position 1 where $r_a/r_b = 2$ and the input and output speeds are ω_1 and Ω_1 respectively. The disc is then moved instantaneously to position 2 where $r_a/r_b = \frac{1}{2}$. While at position 2 some slippage occurs between the disc and the rotors until new steady speeds ω_2 and Ω_2 are reached.

(b) Find ω_2 and Ω_2 in terms of ω_1. [12]

(c) Compute the energy lost during the gear change for a gear change where ω_1=100 rad s^{-1} and ω_2=200 rad s^{-1}, taking $J = 4$ kg m^2. [6]

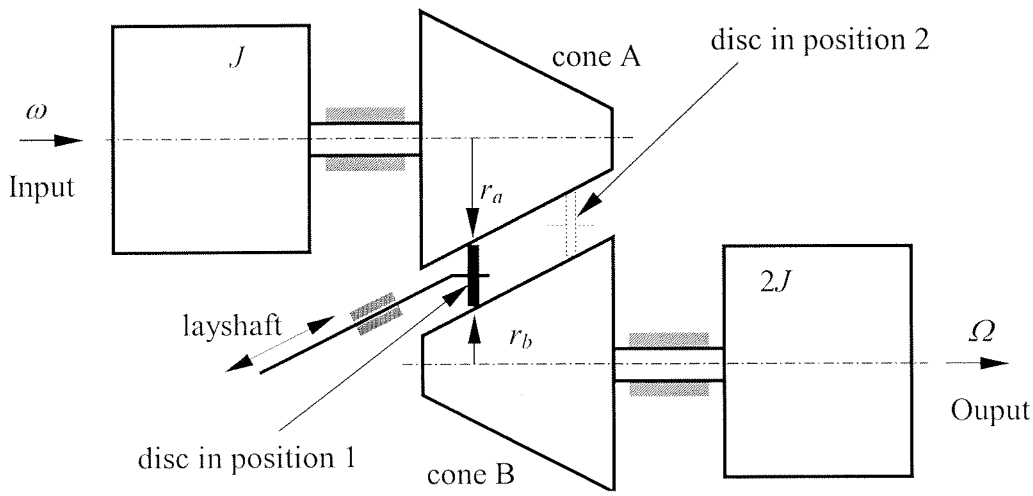

Fig. 3

SECTION B

4 A solid rubber ball of mass m and radius a rolls without slip on a horizontal plane with a speed u towards a vertical rigid wall as shown in Fig. 4(a). The ball strikes the wall and on its rebound it is observed that the ball jumps up at an angle of 45° and that the ball is not spinning. The moment of inertia of the ball is $\frac{2}{5}ma^2$.

(a) What do you understand by the term "ideal impulse" and how must weight forces be treated? [2]

(b) Show that the speed of the ball v immediately after rebound is $\frac{2\sqrt{2}}{5}u$ where u is the speed just before impact. [8]

(c) Estimate the coefficient of friction between the ball and the wall. [3]

(d) The ball bounces a few times before rolling back along the plane as indicated in Fig. 4(b). What is w, the eventual speed of the ball, in terms of u? [7]

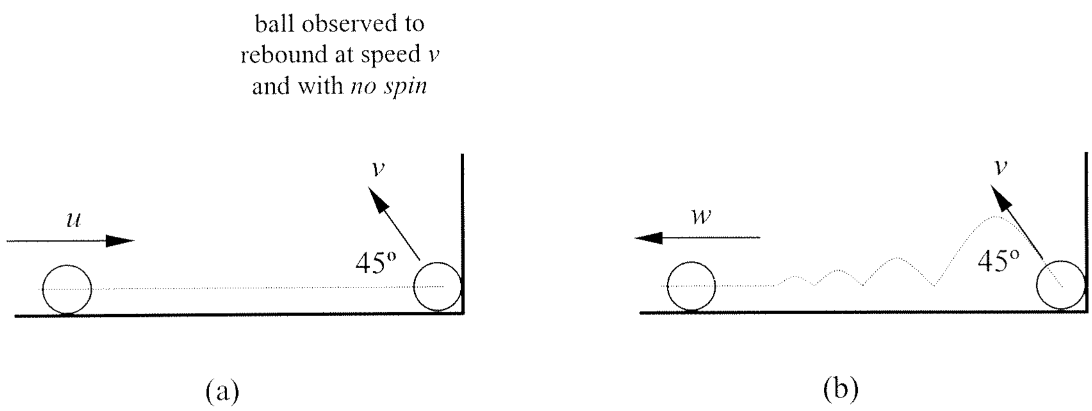

Fig. 4

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5 A light smooth rigid wire frame ABC is shown in Fig. 5. Distance AB is a and angle ABC is 90°. The frame is constrained to move only within the plane of ABC and rotates with steady angular velocity ω about A. A small mass m is glued to the frame at distance a from B . At time t=0 the glue fails and the mass begins to slide freely along BC. The distance from B at any time t>0 is denoted x. Unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ that rotate with the frame are defined in the figure.

The effects of gravity on the motion should be neglected.

(a) Find a vector expression for the velocity of the mass in terms of x and its derivatives. Use the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. [4]

(b) Show that the acceleration of the mass is $(-a\omega^2 + 2\dot{x}\omega)\mathbf{e}_1 + (x\omega^2 - \ddot{x})\mathbf{e}_2$. [4]

(c) Hence obtain a differential equation describing the variation of x with time. [6]

(d) Show that $x = a\cosh\omega t$ and find the value of x at which the force between the wire frame and the mass vanishes. [4]

(e) How are your answers affected if the direction of ω is reversed? Give a physical interpretation of your answer. [2]

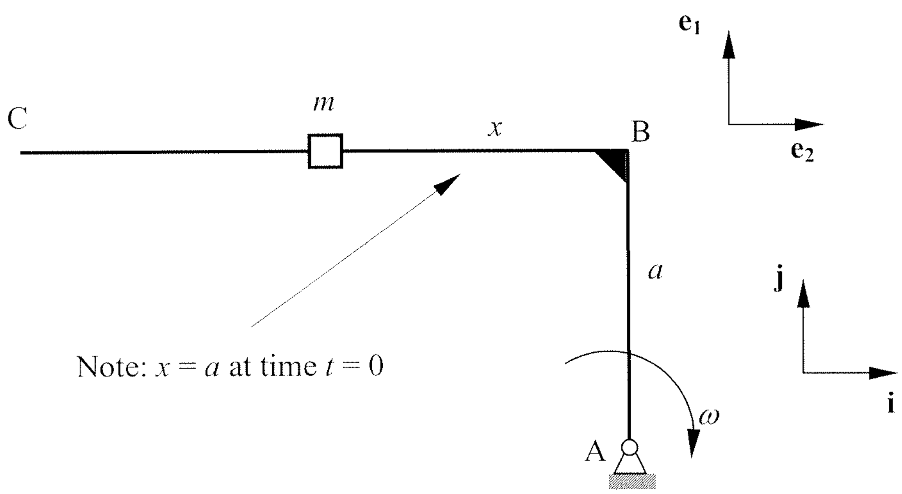

Fig. 5

6 A uniform bar AB of mass m and length $2a$ is supported from a single point O by two light threads OA and OB each of length $a\sqrt{2}$, as shown in Fig. 6. The bar is initially at rest. The thread OB is cut and the bar is free to move under the action of gravity. Its motion is described by the rotations θ of the thread OA and ϕ of the bar as indicated in the figure.

Immediately after the thread OB is cut:

(a) find the **i** and **j** components of the acceleration of G, the centre of the bar, in terms of a, θ, ϕ and their time derivatives; [6]

(b) show that the bar AB has angular acceleration $\ddot{\phi} = \dfrac{3g}{5a}$ and find also the tension in thread OA; [6]

(c) Show that the acceleration of point C (midway between points G and B) is $a\ddot{\theta}\,\mathbf{i} - a(\ddot{\theta} + \tfrac{3}{2}\ddot{\phi})\,\mathbf{j}$ and hence find the bending moment in the bar at G. [8]

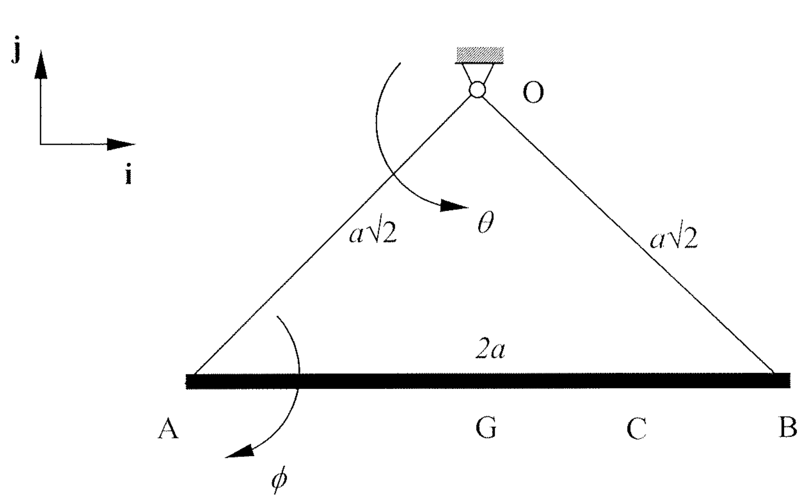

Fig. 6

END OF PAPER

EXTRA SHEET

DO NOT PRINT.

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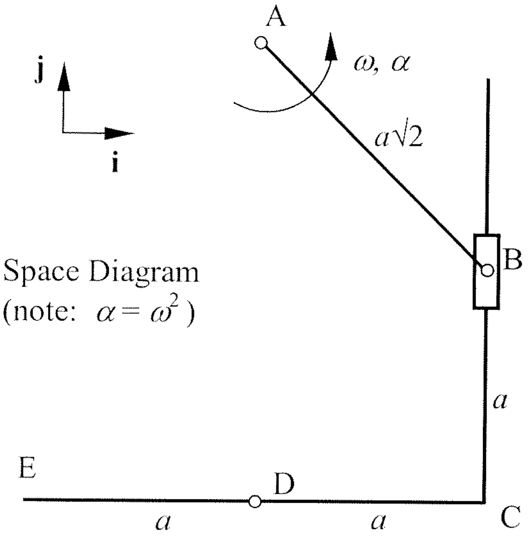
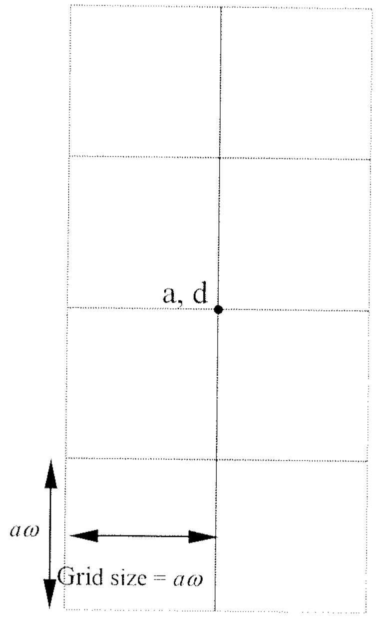
MECHANICS

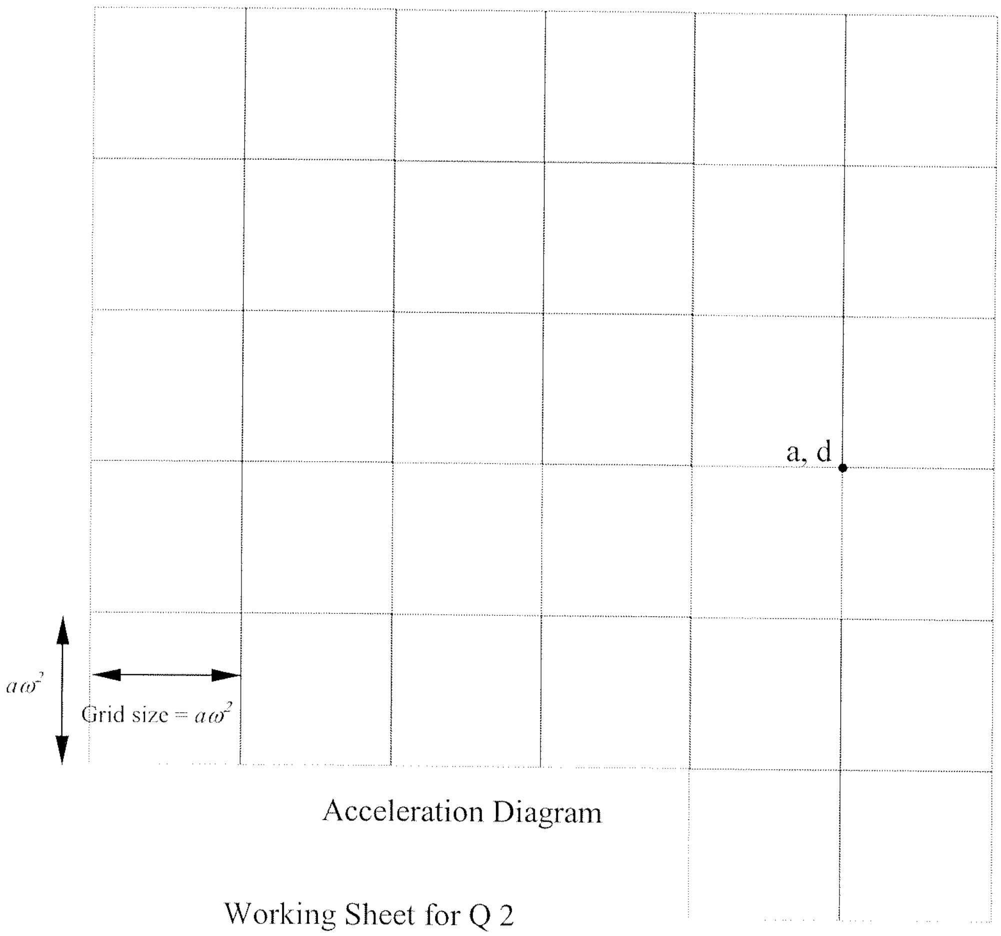
CANDIDATE NUMBER:



Space Diagram
(note: $\alpha = \omega^2$)



Velocity Diagram



Acceleration Diagram

Working Sheet for Q 2
(may be handed in with your script)