ENGINEERING TRIPOS PART IB

Monday 3 June 2002 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1(a) The pin-jointed truss in Fig. 1(a) has three weightless members each of length L. The horizontal and vertical members BQ and DQ each have cross-sectional area A and the diagonal member CQ has cross-sectional area $2A$. It is initially unstressed. All bars have Young's Modulus E and all behaviour is linear elastic.

When a horizontal force H is applied to node Q as shown in Fig. 1(a), show that:

(i) the forces in the three members BQ, CQ and DQ are $\frac{2H}{3}$, $\frac{\sqrt{2}H}{3}$ and $-\frac{H}{3}$ respectively. [5]

(ii) the vertical and horizontal displacements of point Q are $\frac{HL}{3EA}$ upwards and $\frac{2HL}{3EA}$ to the right respectively. [5]

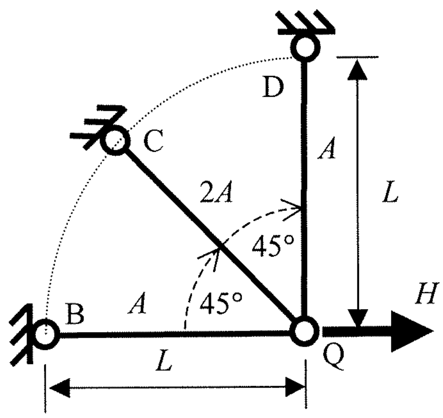

Fig. 1(a)

(b) The pin-jointed truss in Fig. 1(b) has eight members each of length 1.2 m spaced uniformly round a circle so that each is at 45° to its adjacent members.

(i) How many redundancies does this truss have? [2]

(ii) A horizontal force of 10 kN is now applied at node Q acting in the direction shown in Fig. 1(b). Using the results of part (a) or otherwise, determine the bar forces and the vertical and horizontal displacement of Q. [8]

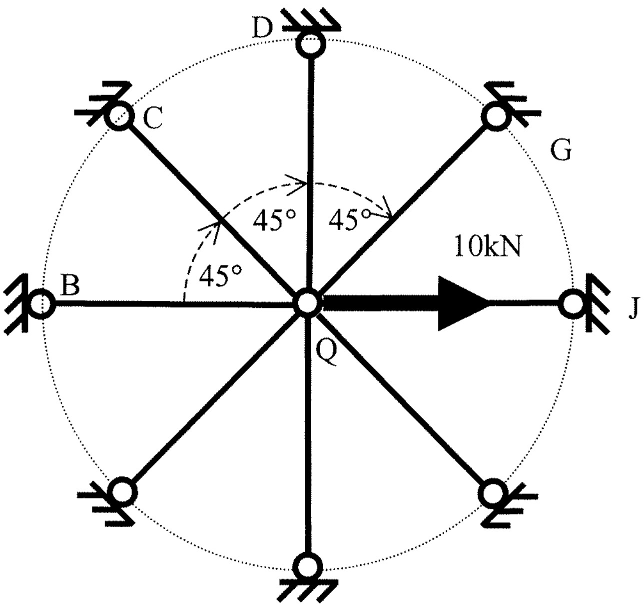

Fig. 1(b)

2 The experimental apparatus shown in Fig. 2(a) consists of a 2 m long aluminium alloy circular cylindrical tube with internal diameter 800 mm and wall thickness 5 mm. It is capped at each end by hemispherical aluminium alloy domes welded to the main cylindrical tube. The wall thickness of the domes is also 5 mm.

(a) The apparatus is internally pressurised to a gauge pressure of 2500 kPa. Using the usual assumptions for thin-walled structures:

(i) calculate the three principal stresses at a typical point C on the cylindrical tube (well away from the ends) and draw the Mohr's circles of stress; [5]

(ii) calculate the corresponding strains in the principal directions and draw the Mohr's circles of strain. [5]

(b) With the gauge pressure maintained at 2500 kPa, equal and opposite axial torques of magnitude T about the longitudinal axis are applied to each end of the cylindrical tube at sections A-A and B-B as shown. Assuming that the aluminium alloy obeys Tresca's criterion and has a uniaxial yield stress of 280 MPa, calculate the maximum torque that can be sustained before inelastic behaviour occurs at C. Draw the corresponding Mohr's circles of stress just before failure and mark on the graph the values of the three principal stresses. [6]

(c) The principal strains at C, corresponding to the principal stresses determined in part (b) above, are –1414, –1034 and 3906 microstrain. Two strain gauges, located on the outer wall of the cylinder at C, are aligned perpendicular to each other and at 45° to the hoop and longitudinal directions as shown in Fig. 2(b). Just before yield in the pressure-plus-torque test of part (b), these gauges give readings of 3716 and 507 microstrain. One of the strain gauges is known to have been damaged and is suspected of giving erroneous results. Determine which of these two readings is incorrect. [4]

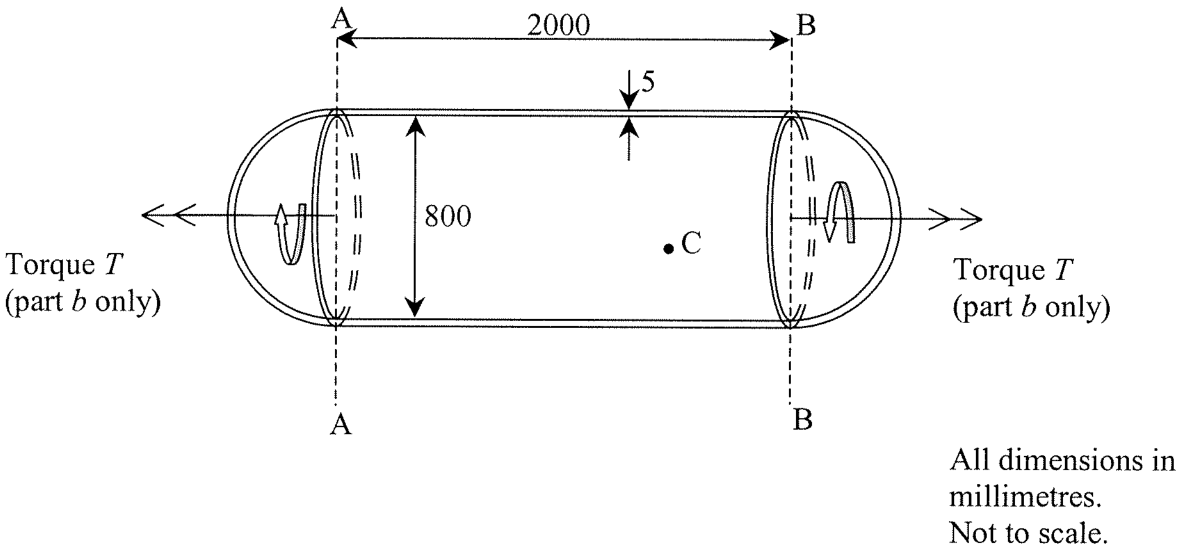

Fig. 2(a)

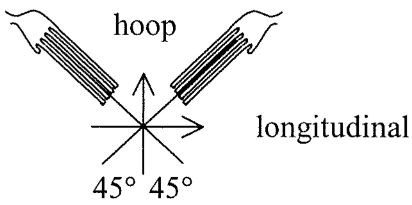

Fig. 2(b)

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3 The weightless frame shown in Fig. 3 consists of continuous members BCD and CF connected to each other by a pin joint at C. Both members are rigidly connected to supports at B and F. All members have flexural rigidity EI and coefficient of thermal expansion α. The frame is initially unstressed, and linear elastic behaviour is to be assumed throughout.

(a) Determine the number of redundancies in the structure. [2]

(b) All members undergo a uniform temperature rise T and the supports B and F do not move. Assuming that the members are axially incompressible:

(i) determine the bending moments everywhere caused by the temperature rise illustrating your answer with a carefully drawn and labelled bending moment diagram; [6]

(ii) determine the corresponding components of displacement and rotation of point D. [6]

(c) If all members have axial rigidity EA and axial compressibility is now taken into account, show that the bending moments calculated in part (a) reduce by a factor

$$\frac{1}{\left(1+\frac{3I}{AL^2}\right)}$$

[6]

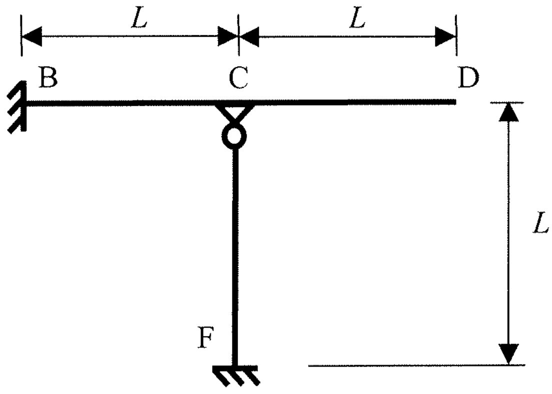

Fig. 3

SECTION B

4 The two-span structure shown in Fig. 4 is to be constructed using a single steel beam over the entire length from A to C. The design live load, with factors of safety already included, comprises a vertical point load $P = 120$ kN applied midway between A and B and a uniformly distributed load $w = 60$ kN/m applied between B and C. The yield stress of steel may be taken as $\sigma_y = 300$ N/mm^2. Self-weight of the beam may be ignored.

(a) Postulate two simple collapse mechanisms, one in each span of the structure, and hence, using the *upper bound theorem,* select a single UB (Universal Beam) that would be suitable to use over the entire span from A to C. (For the mechanism in span BC assume a hinge forms at mid-span). [6]

(b) As an alternative design approach, use the *lower bound theorem* to find the UB with smallest cross-sectional area from the Structures Data Book that would be suitable to use over the entire span from A to C. [10]

(c) What would be the effect on the collapse load of this structure if the support at B settled vertically by 50 mm relative to the supports at A and C? (Note that the beam is constrained to remain in contact with the support at B when this settlement occurs). [2]

(d) Would it be valid to perform the same *upper bound* calculation as in part (a) above if cast iron with the same strength as steel (i.e. 300 N/mm^2) had been specified as an alternative material? Explain your answer. [2]

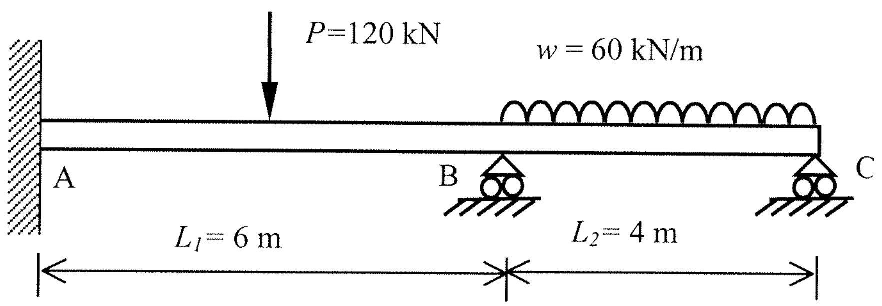

Fig. 4

5 Two collapse mechanisms have been postulated, as shown in the typical cross-sections in Figs. 5(a) and 5(b) respectively, for a long strip foundation that can be assumed to be a rigid block supported by soil. The soil is assumed to be a rigid-plastic continuum of uniform isotropic material with yield strength k in shear. A uniform line load F_i per unit length is applied vertically along the centreline of the block. A second uniform line load of magnitude αF_i per unit length, which is directly proportional to the vertical load, is applied horizontally along the length of the block. The subscript i identifies the collapse mechanism considered in each of the Figures. The interface between foundation block and the soil is rough so assume there is no slippage.

(a) Estimate the failure load F_1 in terms of the shear strength k, block dimension b and load parameter α, using collapse mechanism no.1, shown by dashed lines in Fig. 5(a). Assume the three rigid regions of soil in this mechanism are all equilateral triangles with the same fixed dimensions. [7]

(b) Find an upper bound estimate of the load F_2 in terms of the shear strength k, block dimension b and load parameter α, which will cause collapse using the alternative slip-circle collapse mechanism no.2, shown in Fig. 5(b). [7]

(c) For what range of values of α would the failure load F_1 in the mechanism in Fig. 5(a) be *less than* the failure load F_2 in the mechanism shown in Fig. 5(b)? [3]

(d) For each of the two mechanism topologies postulated in Fig. 5, what geometric parameter(s) might you vary in order to search for the optimised failure mode geometry? [3]

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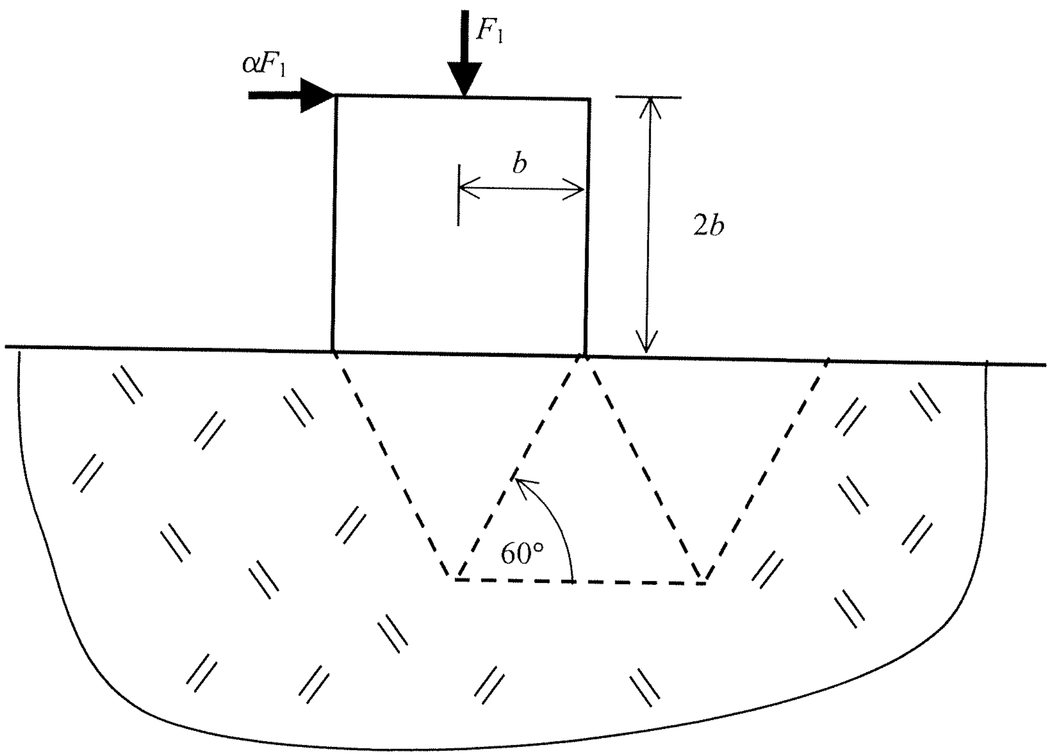

Fig. 5(a) Collapse Mechanism No.1

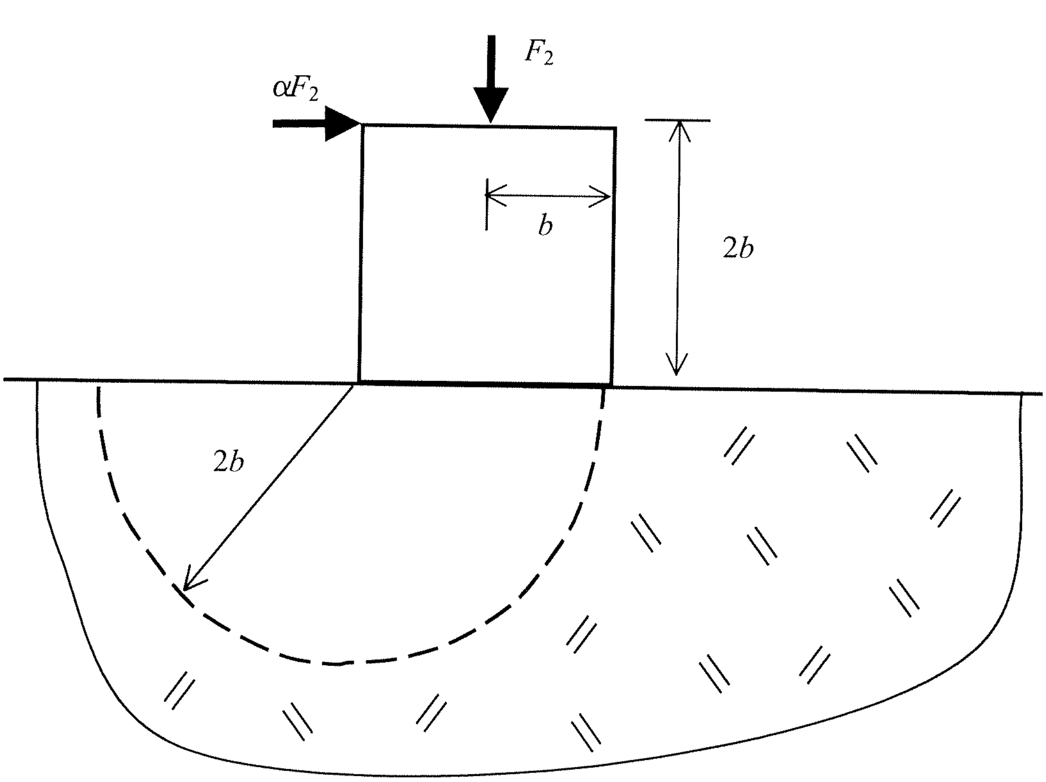

Fig. 5(b) Collapse Mechanism No.2

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6 A steel pole is fabricated by joining two thin-walled tubes with different lengths, outside diameters (O.D.) and wall thicknesses (t), as shown in Fig. 6(a). The base of the larger tube is fixed rigidly to a vertical support at A. Rigid diaphragms are located at the ends of each tube so that local effects can be ignored. At the free end of the pole, D, three force resultants are applied: (i) a couple T = 50 kNm about its longitudinal axis (acting clockwise when viewed from the tip D towards the base A) (ii) a horizontal force P = 500 kN at the centre of the diaphragm and (iii) a vertical force F = 30 kN acting through the centre of the pole. The self-weight of the pole can be ignored.

(a) Calculate the value of bending moment, shear force and torque at A, B, C and D and sketch the corresponding diagram of each over the full length of the pole AD. [3]

(b) Show that for a thin-walled tube the polar second moment of area J is equal to $2\pi r^3 t$ where r = radius and t = wall thickness. Hence calculate the numerical value of torsional rigidity GJ kNm^2 for each tube. [4]

(c) Calculate the rotation in degrees at the end of the pole at D about its longitudinal axis relative to the fixed support at A. [5]

(d) Compute the longitudinal stress and the shear stress on the face of cross-section AA' at the base of the pole due to the applied torsion, T, axial load, P and vertical load, F:

(i) at location A, at the top of the cross-section. [4]

(ii) at location G, on the horizontal centreline of the cross-section. [4]

Note: Refer to Fig. 6(b) for position of the centroid, C, of a thin-walled semi-circular section.

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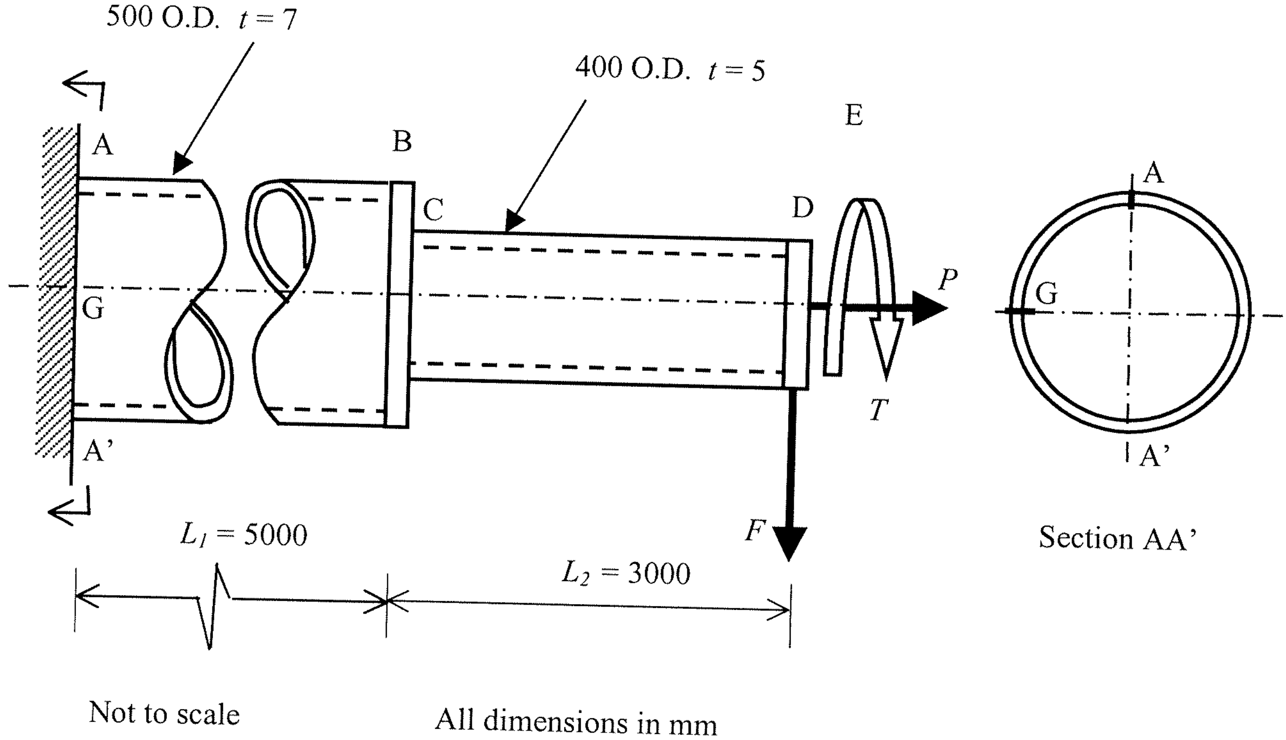
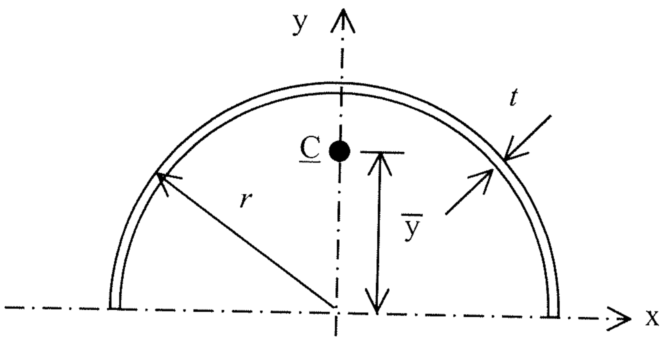


Fig. 6(a)

Distance from x-axis to centroid of area of a thin-walled semi-circular section of radius r is:



$$\bar{y} = \frac{2r}{\pi}$$

Fig. 6(b)

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