ENGINEERING TRIPOS PART IB

Thursday 6 June 2002 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer at least ***one*** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

*Answer at least **one** question from this section*

1 For the feedback system of Fig. 1

(a) Show that the closed-loop transfer function relating $\bar{y}(s)$ to $\bar{w}(s)$ is

$$\frac{9(s-2)}{(s+1)^2}$$

[6]

(b) Determine the open-loop and closed-loop poles for the system in Fig. 1 and indicate their position on an Argand diagram. Comment on the open-loop and closed-loop stability of the system. [4]

(c) By using the Initial and Final Value Theorems, or otherwise, find the initial and final values of the response $y(t)$ to a unit step on $w(t)$, and the initial value of the slope of the response.

Sketch the response $y(t)$. [10]

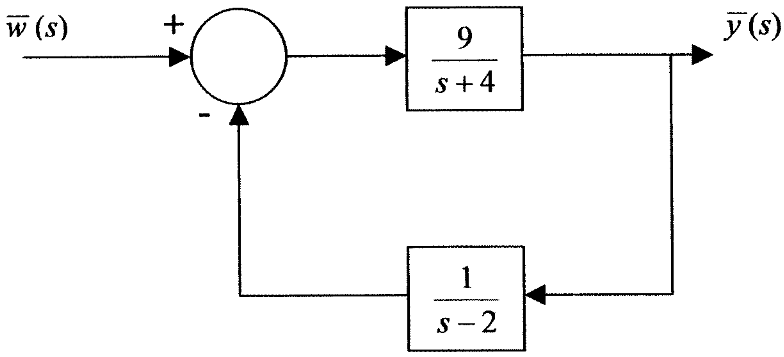

Fig.1

Note:

Final Value Theorem

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s\bar{x}(s)$$

Initial Value Theorem

$$\lim_{t \to 0^+} x(t) = \lim_{s \to \infty} s\bar{x}(s)$$

2 The linearised differential equation relating the instantaneous rate of change of the phase ϕ at the output of a voltage controlled oscillator (VCO) to the input voltage v is given by

$$\frac{d\phi}{dt} = \frac{1}{T_v} v$$

where $\frac{1}{T_v}$ is the gain of the VCO.

A controller of the form

$$v(t) = e(t) + \frac{1}{T_f} \int_0^t e(\tau)\, d\tau$$

is implemented where

$$e(t) = K[\theta(t) - \phi(t)]$$

and $\theta(t)$ is an input disturbance.

(a) Draw a block diagram of the closed-loop system and verify that the closed-loop transfer function relating $\bar{v}(s)$ to $\bar{\theta}(s)$ is given by

$$\bar{v}(s) = \frac{T_v s(1 + T_f s)}{\frac{T_f T_v}{K} s^2 + T_f s + 1} \bar{\theta}(s)$$

[9]

(b) If $K=1$, choose T_f and T_v such that the closed-loop poles lie at $s = -0.1 \pm j0.1$ [4]

(c) Sketch the variation of the voltage $v(t)$ in response to the disturbance

$$\theta(t) = \begin{cases} 0 & t < 0 \\ -0.1 & t \geq 0 \end{cases}$$

What is the initial rate of decay of $v(t)$ in this case? [7]

3 (a) Explain the meaning of the terms *gain margin* and *phase margin* of a feedback system. [4]

(b) Estimate the gain and phase margin of the closed-loop control system shown in Fig. 2 if $K(s) = 1$ and $G(s)$ is asymptotically stable with the Bode diagram given in Fig. 3. [6]

(c) The simple controller in (b) is replaced by a compensator having the transfer function

$$K(s) = \frac{6(5+s)}{(30+s)}$$

[8]

Using the semilog paper provided, sketch the Bode diagram for $K(s)$ and hence estimate the new gain and phase margins.

(d) Comment on the closed-loop behaviour in the two cases. [2]

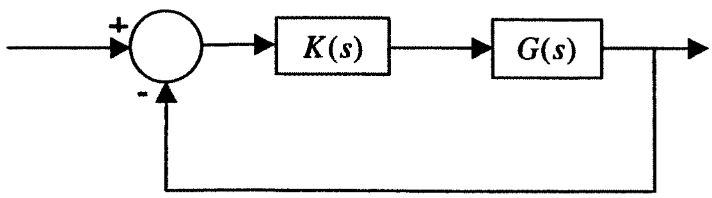

Fig.2

Note: the Bode Diagram of *G(s)* alone is given in Fig. 3. An extra copy is provided on a separate sheet and it should be handed in with your answer if constructions are made on it.

(cont.

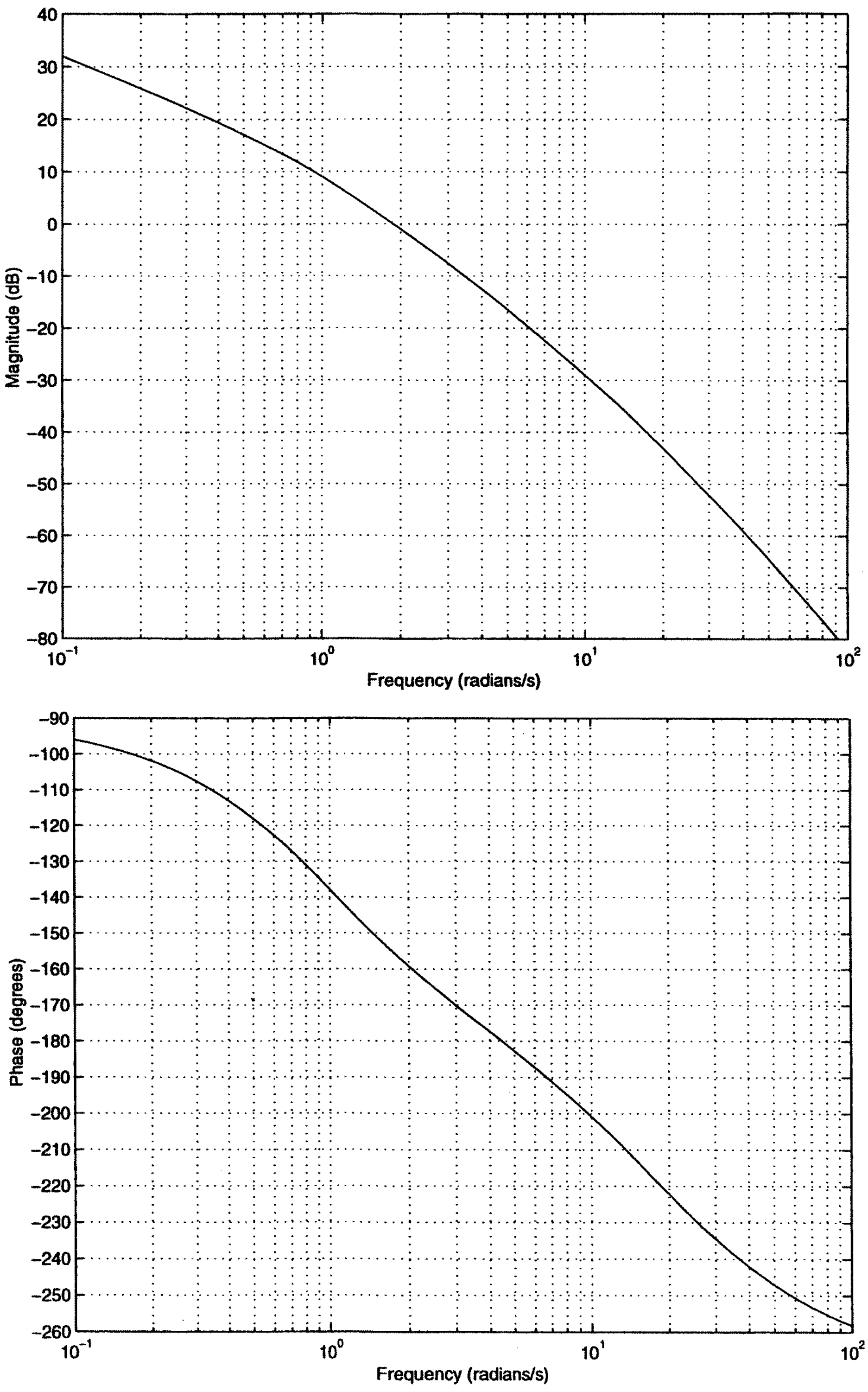

Fig. 3

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4 (a) State the Nyquist stability criterion as it applies to the system of Fig. 4 in the case when K=1 and $G(s)$ is asymptotically stable. [4]

(b) The stable system $G(s)$ is to be controlled as shown in Fig. 4. The Nyquist diagram for $G(s)$ is given in Fig. 5. For what range of K values is the transfer function

$$\frac{KG(s)}{1+KG(s)}$$

stable?

Give the location of two of the poles of this transfer function when K has the largest value allowed by the Nyquist stability criterion. What is the closed-loop frequency of oscillation in this case? [6]

(c) What is the Gain Margin when K=1 and what is the value of K if a Gain Margin of 1.5 is required? Sketch the variation of the magnitude of the closed-loop frequency response from r to y with frequency when the Gain Margin is 1.5. Pay particular attention to the response near ω=0 and also near to the frequency where the response attains its maximum modulus.
Briefly describe the step response of this system with reference to your plotted frequency response. [10]

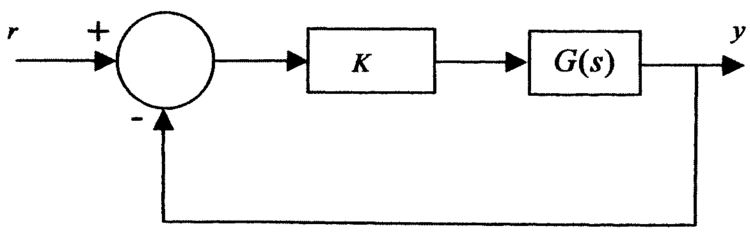

Fig. 4

Note: an extra copy of Fig. 5 is supplied on a separate sheet. This may be annotated with your constructions and handed in with your answer to Question 4.

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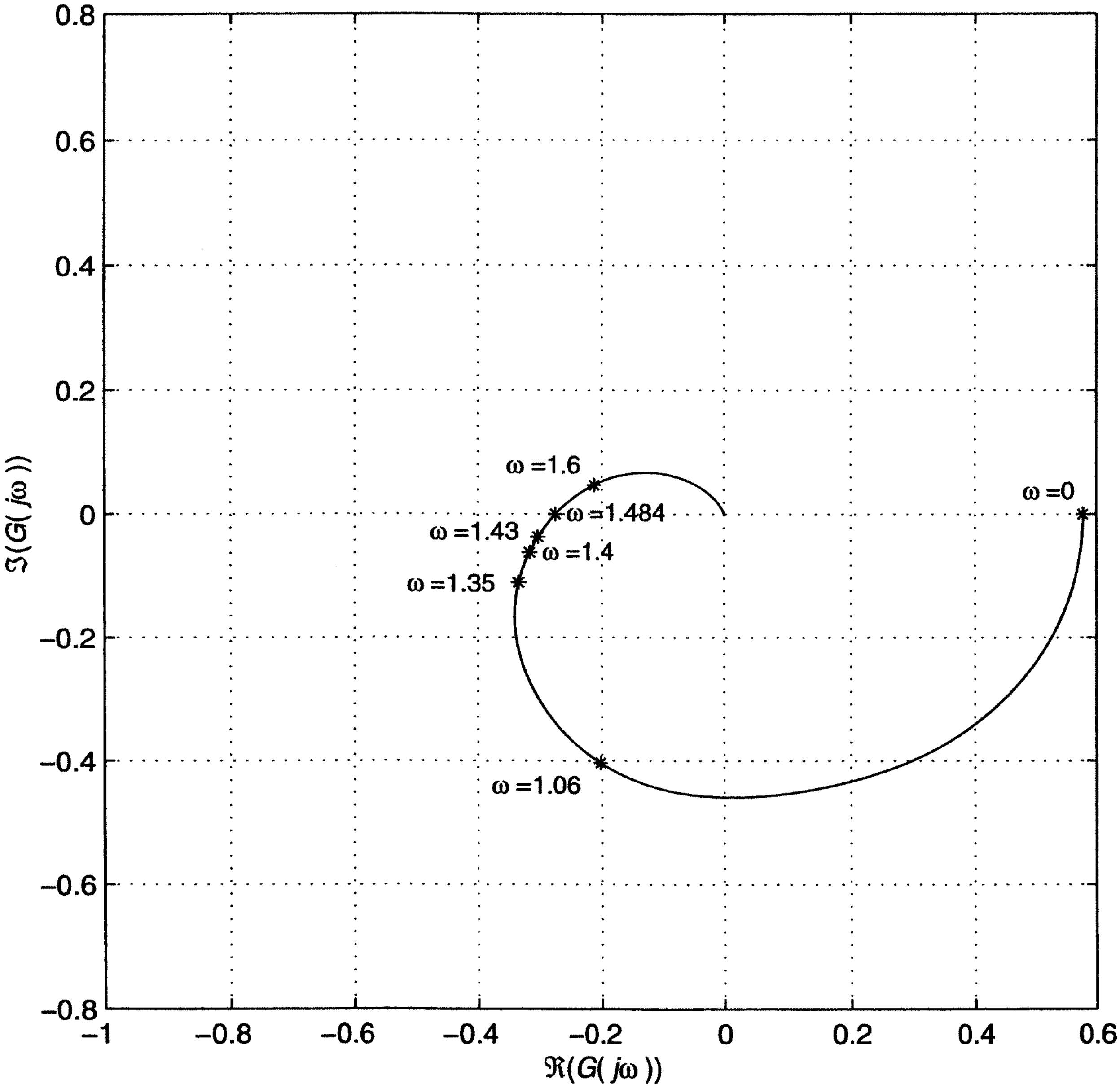

Fig. 5

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SECTION B

*Answer at least **one** question from this section*

5 A random binary data signal is clocked at 2400 bit/s.

(a) Sketch the power spectrum $P(\omega)$ of the data signal if rectangular pulses are used (equal in duration to the bit period) and if the two data levels are $+V$ and $-V$.

Assuming that frequencies up to the second zero in the power spectrum must be transmitted, what is the required system bandwidth? [4]

(b) To raise the maximum bit rate of the system the binary data signal of the format in (a) is passed through a filter with a half-sine impulse response $h(t)$,

$$h(t) = \left(\frac{\pi}{2t_s}\right)\sin\left(\frac{\pi t}{t_s}\right) \qquad 0 < t < t_s$$

$$h(t) = 0 \qquad \text{elsewhere}$$

Show, using the Fourier transform relations in the Electrical and Information Data Book, that this filter has a frequency response magnitude

$$|H(\omega)| = \frac{\pi}{4}\left|\text{sinc}\left(\frac{\omega t_s - \pi}{2}\right) + \text{sinc}\left(\frac{\omega t_s + \pi}{2}\right)\right|$$

$$\text{where } \text{sinc}(x) = \left(\frac{\sin(x)}{x}\right)$$

[6]

(c) Sketch $|H(\omega)|$ and determine the new maximum possible bit rate again assuming that frequencies up to the second zero in the resultant power spectrum $P(\omega)|H(\omega)|^2$ must be transmitted and that the system bandwidth remains the same as in part (a). [10]

6 Double Sideband Amplitude Modulation (DSB-AM) of a carrier with frequency f_c and amplitude a_0 by an information-bearing signal $x(t)$ is defined by the equation

$$s(t) = \left(a_0 + x(t)\right)\cos 2\pi f_c t \quad .$$

(a) If the information-bearing signal is a cosine wave, $x(t) = a_x \cos 2\pi f_m t$, show that the DSB-AM signal may be described as a sum of the central carrier-frequency component and two sidebands. [4]

(b) Determine an expression for the power efficiency of the DSB-AM in (a) in terms of the modulation index m_A, where $m_A = a_x / a_o$.
Hence, what is the maximum power efficiency if envelope demodulation is to be employed? [7]

(c) The envelope demodulator shown in Fig. 6 is used to demodulate $s(t)$. Using the Fourier series for a unity amplitude half-wave-rectified cosine wave given in the Electrical and Information Data Book, derive an expression for $u(t)$ and show that it contains a term proportional to the information-bearing signal $x(t)$. If $f_m \le 5\,\text{kHz}$, what is the maximum value of the time constant RC? [9]

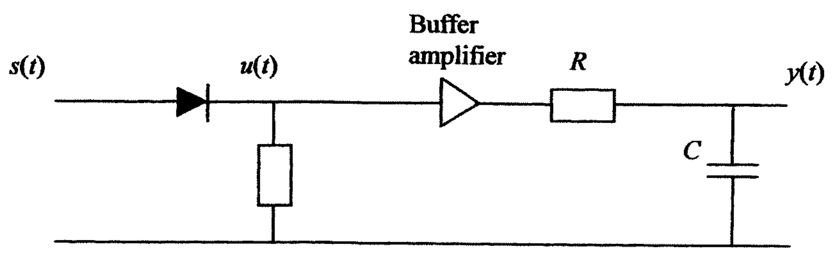

Fig. 6

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