IB
ANSWERS
2003

Paper 1 MECHANICS – ANSWERS

1 (b) $\theta = \sqrt[3]{\dfrac{3ga}{8V^2}}$ (c) $-\dfrac{9}{8} mg \sin\theta \cos\theta$

2 (b) $\omega_n = \sqrt{\dfrac{(R-h)g}{h^2+k^2}}$

3 (a) 38.7 N and 86.6 N

(b) Disc A: 1.12 g at 153.4°; Disc C: 0.5 g at 90°

4 $\omega / 4$; $15/16$; $\omega = \dfrac{2\sqrt[4]{2}}{\sqrt{5}} \sqrt{\dfrac{g}{a}}$

5 $I_O = \dfrac{5}{3} ma^2$; $x\dot{\omega} - a\omega^2$

$$SF = \frac{m\dot{\omega}}{2a} x^2 - m\omega^2 x - \frac{m\dot{\omega}a}{2} + m\omega^2 a$$

$$BM = \frac{m\dot{\omega}}{6a} x^3 - \frac{m\omega^2}{2} x^2 - \frac{m\dot{\omega}a}{2} x + m\omega^2 ax - \frac{m\omega^2 a^2}{2} + \frac{m\dot{\omega}a^2}{3}$$

6 (a) $\dot{\phi} = -1.36\omega$; $\ddot{\phi} = 5.09\omega^2$

(b) torque $= -18.5 ma^3\omega^2$

2003 Part IB Paper 2

Answers

1 a) 2

b) (ii) $H_F = -3wL/92,\ V_F = -15wL/184,\ M_F = 3wL^2/92$
$H_G = 3wL/92,\ V_G = 107wL/184,\ M_G = wL^2/92$

2 a) $\gamma_{xy} = \dfrac{2(\varepsilon_{60} - \varepsilon_{120})}{\sqrt{3}}$

(i) $\varepsilon_{xx} = -200 \times 10^{-6}$ $\varepsilon_{yy} = 283 \times 10^{-6}$ $\gamma_{xy} = 202 \times 10^{-6}$.

(ii) $\sigma_{xx} = -26.6$ MPa $\sigma_{yy} = 51.5$ MPa $\tau_{xy} = 16.3$ MPa

(iii) $\sigma_1 = 54.8$ MPa $\sigma_2 = -29.9$ MPa $\theta = 11.3$ degrees clockwise

(iv) Factor of safety = 3.1

3 (a) 1

(b) (i) $\underline{t} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{5} \\ -2 \\ 0 \end{bmatrix} W$

(ii) $S_1 = \begin{bmatrix} \sqrt{5} \\ -2 \\ -\sqrt{5} \\ 2 \\ 1 \end{bmatrix}$

(iii).. $\underline{t} = \begin{bmatrix} 1.09 \\ -0.97 \\ 1.15 \\ -1.03 \\ 0.49 \end{bmatrix} W$

(iv) $\delta_{P(\,horizontal\,)} = -1.95\dfrac{WL}{EA}$

4 (a) 37.6 kN
(b) 437 mm, 109 mm
(c) 0.39 degs
(d) 774 MPa at bottom left corner as shown.
(e) 2.4 MPa

5 (a) Ductility
(b) (ii) $P_1 = 8(m' + m)$
(c) $P_2 = 6.93(m' + m)$
(d) $P_3 = 6.28(m' + m)$
(e) $P_3 = 6.28(m' + m)$

6 (a) (i) Equilibrium + Material Law
(ii) Compatibility + Material Law

(b) (iii) $H = W = 19.7$ kN

Answers Paper 3 , Materials.

1) 5.3 mm.
2) a) Ratio of times 852, b) Deflection 0.21 mm.

Paper 4

Q1 (a) (i) 107.7 kPa
(ii) -18.5 kJ/kg

(b) (i) 15.7 J/kgK
(ii) Friction in the pipe and heat transfer
(iii) Friction in pipe and heat transfer across a finite temperature difference.

(c) (i) 482.5 K
(ii) 7.1 J/KgK
(iii) Heat transfer across a finite temperature differnce

Q2 (a) 2.003 kJ/kg

(b) (i) 0.34
(ii) 0.95

(c) 7-8 bar

Q3 (a) $COP \leq \dfrac{T_C}{T_H - T_C}$

(b) 2.75 W

(c) (i) 567.9 J
(ii) Q_C is fixed so for ideal work Q_H is minimum. No maximum exists.

Q4 (b) (i) 1.76 m/s
(ii) Parallel streamlines have no perpendicular pressure gradient
(iii) 0.8614 bar
(iv) 26.2 kW dissipated
(v) 26.2 kJ/s increase

Q5 (b) $\phi = \dfrac{Q}{ND^3}$

(c) $\psi = \dfrac{\Delta p}{\rho N^2 D^2}$

(d) $\psi = 0.13 - 550\phi^2$

(e) $\Delta p_{pipe} = \dfrac{32LC_f \rho Q^2}{\pi D_{pipe}^5}$ or $\psi = 1478\phi^2$

(f) 0.01 m^3/s 112 kPa

Q6 (a) $\dot{m} = \rho \pi R^2 V$

(b) $u = \dfrac{1}{2\mu}\dfrac{dp}{dx} y(y-h) + V\dfrac{y}{h}$ where $\dfrac{dp}{dx} = \dfrac{p_2 - p_1}{L}$

(c) $\dot{m} = \rho \pi R h \left(V - \frac{h^3}{6\mu} \frac{dp}{dx} \right)$

(d) $p_1 - p_2 = \frac{6VL\mu(R-h)}{h^3}$

ENGINEERING TRIPOS, Part 1B 2003

Paper 5 – ELECTRICAL ENGINEERING

Answers

1. (a) $R_E = 99.75\ \Omega$, $R_2 = 170\ k\ \Omega$

(b) Yes, it is accurate at mid-band frequencies. At higher frequencies, a capacitor between the base and collector should be added.

(c) Gain fir h_{fe} of 400 = -1.46. With h_{fe} of 800, Gain = -1.48. Important for circuit design due to inherent variation in h_{fe} between transistors.

(d) The capacitors are there to remove dc offsets from the inout and output.

2. (a) This is a class A unity-gain buffer, used to couple a source of high output impedance with a load of low inout impedance.

(b) Gain = 0.998

(c) Efficiency = 25%

(d) Complementary follower is class B, so draws much less power and is more efficient.

3. (b) $I_{line} = 273.4$ A, $V_{line} = 138.6$ kV

(c) Power loss in lines = 1.77 MW, $V_{line} = 137.6$ kV

(d) Add capacitors in parallel with load. If star connected, C = 2.42 μF, if delta-connected, C = 809 nF

(e) I_{line} will be constant, because power factor and voltage remain the same.

4. (b) Curent = 3401 A, E = 11.006 kV

(c) Fault current = 11494 A, circuit breaker rating = 438 MVA

(d) Add reactance of 1.74 Ω/phase

5. (b) $R_0 = 202.6\ \Omega$, $X_0 = 80.6\ \Omega$.

(c) $R_2' = 0.69\ \Omega$, $X_1 = 0.53\ \Omega$, $X_2' = 0.8\ \Omega$.

(d) Torque = 43.95 Nm, efficiency = 97%

6. (a) $L = (\mu_0/2\pi)\ln(a/b)$

(b) $Z0 = \dfrac{\ln\left(a/b\right)}{2\pi}\sqrt{\dfrac{\mu_0}{\varepsilon_0\varepsilon_r}}$, 29 Ω

(c) 8%, 0.61 mm

7. (a) $n1/n2 = \sin\theta_t/\sin\theta_i$

(b) $(E_i^2/n_1)\cos\theta_i$

(c) minimum value = 0

Engineering Triops Part 1B
Paper 6. Information Engineering, June 2003

Answers

1. This was the most popular question and was in general well answered. Relatively few realised that the steady state frequency response could be used to solve part (d).
a) See notes
b) (i) $-1/K_p$
(ii) $-1/K_p$
c) Let $K(s)=K_p+K_I/s$
d)

2. Most candidates determined the transfer function required in part (a). The major source of difficulty was finding the required time domain response in part (c). Rather than using the Laplace approach to find the solution, a sizable number of candidates used the Mechanical Data Book incorrectly to find the time domain response.
a)
b) $\tau = 0.354$, $K_o=141$
c) Looks like a second order system response to a step input. Final value is 0.5

3. This question concerned Bode diagrams. The students were given a transfer function and had to estimate a number of parameters from its Bode diagram. Few students manage to solve this question. There were also a couple of bookwork questions that were reasonably well answered.
a) $T_1=1$, $T_2=1/300$, $a=1$ and $\omega_n=20$
b) See notes
c) GM=6.39dB, PM=10°

4. The bookwork element of this question was well answered. A sizable number of candidates could not determine the Nyquist diagram when a time delay was introduced. Consequently they could not perform the comparison required in the final part of the question.
a) See notes
b) $K_{max}=3.5$, $\omega=1.225$rad/s (0.195Hz)
c) $K_{max}=1.25$
d) Response with delay will be more oscillatory and will converge more quickly than original system

5. This question concerned sampling and quantization of analogue signals. In part (a) the majority of candidates were able to satisfactorily define the terms aliasing and quantization. Subsequently, one had to compute a filter transition bandwidth in the context of speech signals and establish the number of bits required to quantize a signal to satisfy a specified signal to noise ratio. In general the candidates knew the methodology to solve this part of the question but there were quite a lot of small mistakes due to a lack of attention.
a) See notes
b) 17.7%
c) See notes
d) N=6. Use a non-linear (i.e., logarithmic quantiser)

6. This question concerned analogue TV signals. In the first part of this question a numerical formula given in the paper had to be established. Numerous students adopted a reverse engineering approach to come up with the result using incorrect arguments. In the generally well answered second part of this question one had to compute a number of system bandwidths and answer some bookwork questions.
a)
b)
c) (i) 11.56MHz, (ii) 46.25MHz
d) Vestigial sideband (VSB) modulation

Engineering Tripos Part Ib Mathematical Methods Paper 7 2003
Answers

1. (a) $(\pi/d)V_0 \exp(-\pi x/d)\,[\cos(\pi y/d)\mathbf{i} + \sin(\pi y/d)\mathbf{j}]$
(b) V_0
(c) 0
(d) $V_0 \exp(-\pi x/d)\sin(\pi y/d)$ + constant

2. (a) $[-2xz, -2yz, 1]\exp(-x^2-y^2)$
(b) 0
(c) 0

3. (a) $A\cos(n\pi x/2\ell)\cos(\lambda t+\phi)$ where $\lambda=n\pi c/2\ell$ and n is odd.
(b) $\Sigma A_n \cos(n\pi x/2\ell)\cos(\lambda_n t+\phi_n)$, n odd

4. (a) 1, 1, 2
(b)
$$\begin{bmatrix}1&0&0\\1&1&0\\2&2&1\end{bmatrix}\begin{bmatrix}1&0&2&1&0\\0&1&-3&1&0\\0&0&2&-3&-1\end{bmatrix}$$
(c)
$$\mathbf{x} = \begin{bmatrix}5\\-9/2\\-5/2\\0\\0\end{bmatrix}+\lambda\begin{bmatrix}-4\\7/2\\3/2\\1\\0\end{bmatrix}+\mu\begin{bmatrix}-1\\3/2\\1\\0\\1\end{bmatrix}$$
(d) (i) $(1,1,2)^T$, $(0,1,2)^T$, $(2,-1,0)^T$, (ii) $(1,0,2,1,0)^T$, $(0,1,-3,1,0)^T$, $(0,0,2,-3,-1)^T$,
(iii) $(-4, 7/2, 3/2, 1,0)^T$, $(-1, 3/2, 1/2, 0,1)^T$, (iv) $(0,0,0)^T$

5. (a)
$$\begin{bmatrix}2&2&2\\2&-4&-2\\2&-1&3\end{bmatrix}\mathbf{x} = \begin{bmatrix}0\\5\\6\end{bmatrix}$$
(b)
$$\mathbf{A} = 1/6\begin{bmatrix}\text{sqrt}(2)&\text{sqrt}(3)&1\\\text{sqrt}(2)&-\text{sqrt}(3)&1\\\text{sqrt}(2)&0&-2\end{bmatrix}\begin{bmatrix}6\text{sqrt}(2)&-3\text{sqrt}(2)&3\text{sqrt}(2)\\0&6\text{sqrt}(3)&4\text{sqrt}(3)\\0&0&-6\end{bmatrix}$$
(c) $\mathbf{x} = (1/18)\,[8, -29, 21]^T$

6. (a) (N-1)/NT
(b) $|F_k| = |\Sigma\sinh(nT)\exp(-jnk2\pi/N)|$
(c) $0.5\,[\sinh T+\sinh((N-1)T) - (\cos(k2\pi/N)+j\sin(k2\pi/N))\sinh(NT)]/[\cos(k2\pi/N)-\cosh T]$

7. (a) (i) $(1/(1-t^2))\text{sinc}(\pi t)$
(ii) $2(1+\cos(2\omega))e\text{-}j\omega\tau$ where $-\pi/2 < \omega < \pi/2$, and 0 otherwise.
(b) (i) 0.107
(ii) 0.464

8. (a) 0.0062, (b) Poisson if n large and p small, (c) 0.9707, (d) 10 and 10, (e) 20 and 20.

Numerical answers to Paper 8, Part IB, 2002-3

Q3. (b) stability ratio =2.0 for stiff clay and =8.5 for soft clay

Q4. (a) (ii) force = 4.4 kN/m (b) (ii) force = 544 kN/m

Q5. (a) Bending moment = 124.8 kNm Shear force = 144 kN
(b) t=200mm As=2810 mm^2
(c) As=922 mm^2
(d) option 1 cost = £159/m option 2 cost = £143/m
(e) shear force capacity = 164.5 kN, so the design is OK

Q6. (c) $E_1 = \frac{1}{2}\bar{\rho}E_S$ $\sigma_{Y1} = \frac{1}{2}\bar{\rho}\sigma_{YS}$

(d) $\sigma_{Y2} = \frac{1}{4}\bar{\rho}^2\sigma_{YS}$

Q9. (a) V=241.1 ms-1 P_{02}=44.2 kPa T_{02}=254.9 K
gross thrust = 126 kN net thrust=39.24 kN

(b) $\frac{\dot{m}_a\sqrt{c_p T_{02}}}{AP_{02}}$ and $\frac{\dot{m}_f LCV}{AP_{02}\sqrt{c_p T_{02}}}$

(c) new total mass flow rate of air = 249.5 kg s^{-1}
net thrust = 26.54kN

Q10. (c) 14 stages are needed; ψ =0.396
(d) absolute flow angle at inlet = 38.4°
at inlet, angle = -63.4° at exit, angle = -50.38°

Q11. (b) wing area = 792.3 m^2 (c) ideal altitude = 39,680 ft.
(d) distance travelled = 10,850 km

Q12. (a) $\tau = \frac{L}{\mu E_d}$ (b) $E_{\max} = \frac{eN_D D}{\varepsilon_o \varepsilon_r}$ (c) D=3.6 μm, τ = 1.4 ns

(d) 10^{22} m^{-3} (e) transit time τ=1.0 ps, drain voltage = 1 V

Q13. (c) (i) d=1.3μm; (ii) C=10^{20} m^{-3}

Q14. (b) $\lambda = \frac{h}{\sqrt{2Em^*}}$ (d) $d_2 = \frac{d_1\varepsilon_2}{\varepsilon_{r1}}$ (e) Figure Of Merit = $\varepsilon_{r2}\sqrt{m_2^* V_2}$