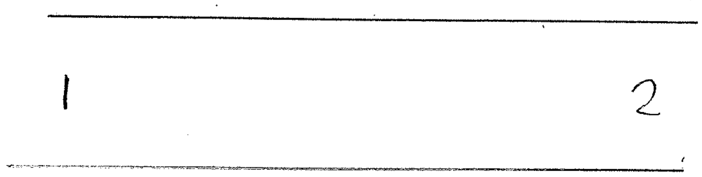
ENGINEERING TRIPOS, PART IB 2003

PAPER 4 - THERMOFLUID MECHANICS

CRIB

(A) SECTION. A PAPER 4



$V_1 = 150\,ms^{-1}$ $\qquad$ $V_2 = 175\,ms^{-1}$

$T_1 = 600K$ $\qquad$ $T_2 = 580K$

$P_1 = 130kPa$

CONTINUITY

$$\rho_1 V_1 = \rho_2 V_2$$

$$\frac{P_1}{\cancel{R}T_1} V_1 = \frac{P_2}{\cancel{R}T_2} V_2$$

$$P_2 = \underline{\underline{107.7kPa}}$$

SFEE $\quad q - W_x = h_2 + \frac{1}{2}V_2^2 - h_1 - \frac{1}{2}V_1^2$

$$q = C_p.(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)$$

$$q = \underline{\underline{-18.5\,kJ/kg}}$$

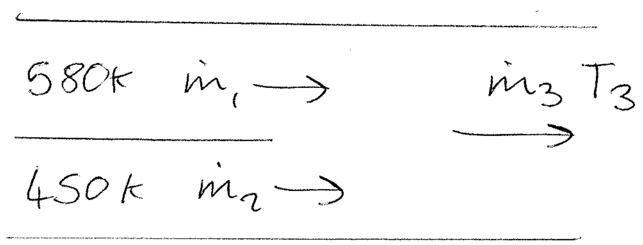
(B) i) $\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

$$\Delta S = 15.7 J/kg K$$

ii) FRICTION IN PIPE + HEAT TRANSFER

iii) FRICTION IN PIPE + HEAT TRANSFER ACROSS FINITE TEMP DIF

(c)



CONT $\quad \dot m_3 = \dot m_1 + \dot m_2 = \dot m_1 + 3\dot m_1$

ENERGY $\quad T_3 \dot m_3 = T_2 \dot m_2 + T_1 \dot m_1$

$$T_3 = \frac{3\times 450 + 580}{4} = \underline{\underline{482.5K}}$$

$$\Delta \dot S_{TOT} = \Delta \dot S_1 + \Delta \dot S_2$$

NB $P_1 = P_2$

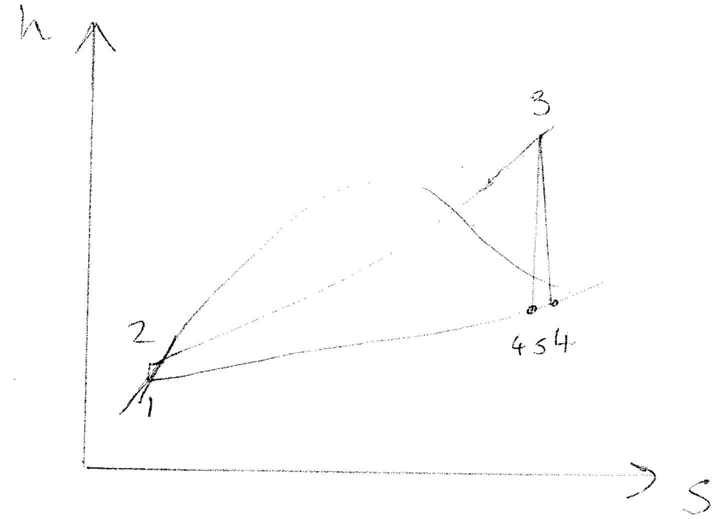
$$4\dot m_1 \, \Delta S_{TOT} = \dot m_1 C_p \ln\frac{T_3}{T_1} + 3\dot m_1 C_p \ln\frac{T_3}{T_2}$$

$$\Delta S_{TOT} = \frac{1.13e^3}{4} \ln\frac{482.5}{580} + \frac{3\times 1.13e^3}{4} \ln\frac{482.5}{450}$$

$$\Delta S_{TOT} = \frac{28.4}{4} = \underline{\underline{7.1 \text{ J/kgK}}}$$

HEAT-TRANSFER ACROSS FINITE TEMP DIF

2a)



$\Delta h_{SP} = V \Delta P$ $\qquad V_1 = 0.001004 m^3/kg$

$= \underline{2.003 kJ/kg}$

(B) $q_b = 3500 kJ/kg$

$\eta_t = 0.9$

$h_3 = h_2 + q_b = h_1 + \Delta h_{SP} + q_b$

$= 3623.4 kJ/kg$

CHART $\Rightarrow h_{4s} = 2300 kJ/kg$

$\eta_T = 0.9 = \dfrac{h_3 - h_4}{h_3 - h_{4S}}$

$h_4 = 2432.3 kJ/kg$

$\eta = \dfrac{W_{NET}}{Q_{IN}} = \dfrac{3623.4 - 2432.3 - 2}{3500} = 0.34$

CHART $x = 0.45$

ii) AT THE NEW CONDITION

$$h_4 = h_g = 2553.7 \text{ kJ/kg}$$

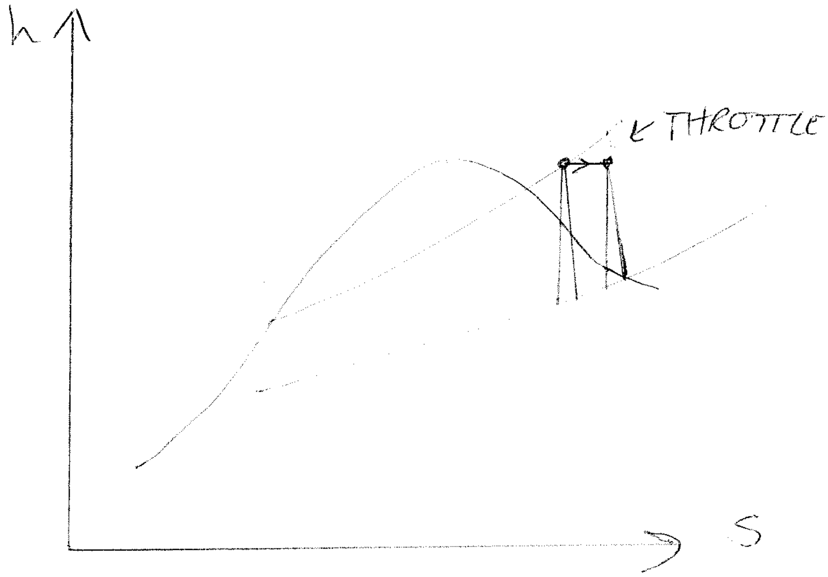
$$\therefore \eta = 0.9 = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\Rightarrow h_{4s} = \underline{2434.8 \text{ kJ/kg}}$$

NOW IN A THROTTLE h REMAINS UNCHANGED

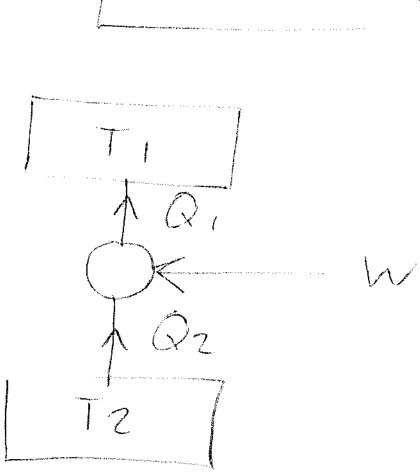
AND $S_{4s} = S_3$

$$\Rightarrow P = \underline{7 - 8 \text{ bar}}$$



3) (A) CLAUSIUS INEQUALITY

$$\boxed{0 \geq \sum \frac{Q}{T}}$$



$$COP = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

CLAUSIUS

$$0 \geq \frac{Q_2}{T_2} - \frac{Q_1}{T_1}$$

$$\frac{Q_2}{T_2} \leq \frac{Q_1}{T_1}$$

$$\frac{Q_2}{Q_1} \leq \frac{T_2}{T_1}$$

$$Q \geq Q_2 \frac{T_1}{T_2}$$

$$COP = \frac{Q_2}{Q_1 - Q_2} \leq \frac{Q_2}{Q_2\frac{T_1}{T_2} - Q_2} = \frac{T_2}{T_1 - T_2}$$

(3) (B) $T_2 = -223°C = 50K$

$Q_2 = 2kJ/hr = 0.5555 J/s$

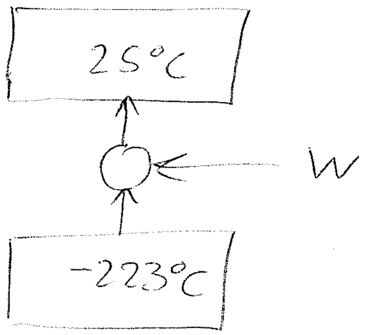
$T_1 = 25°C = 298K$

MIN POWER $\Rightarrow$ REVERSIBLE CYCLE

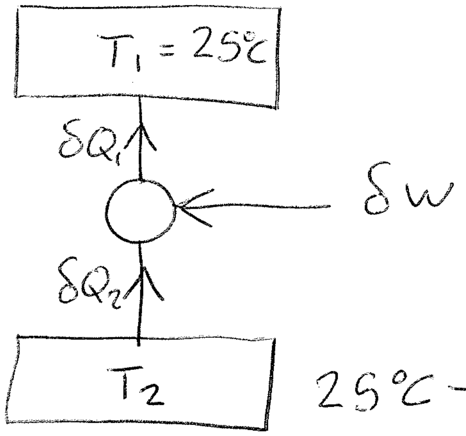
$$\therefore \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_1 = \underline{\underline{3.3078 W}}$$

$$W = Q_1 - Q_2 = \underline{\underline{2.75 W}}$$



3(e)



25°C → −223°C

CLAUSIUS

$$0 \geq \frac{\delta Q_2}{T_2} - \frac{\delta Q_1}{T_1}$$

HEATS FOR MIN WORK

$$\delta W = \delta Q_1 - \delta Q_2$$

$$\delta W = \left(\frac{T_1}{T_2} - 1\right)\delta Q_2$$

$$\boxed{\text{NB} \quad \delta Q_2 = -2\,\delta T_2} \quad \cancel{\frac{T_1}{T_2} - 1}$$

$$\delta W = \left(\frac{T_1}{T_2} - 1\right) \times \left(-2\,\delta T_2\right)$$

$$W = -2\int\left(\frac{T_1}{T_2} - 1\right)\delta T_2$$

$$= -2\left[T_1 \ln T_2 - T_2\right]_{298}^{50}$$

$$= -2\left(T_1 \ln\left(\frac{50}{298}\right) - (50 - 298)\right)$$

$$W = 567.9\,J$$

Q_2 IS FIXED AND SO FOR IDEAL WORK Q_1 IS MIN (HAS NO MAX)

4 (a) $$T\,ds = dh - v\,dp = dh - \frac{dp}{\rho} \quad \Rightarrow \quad \Delta h = \frac{\Delta p}{\rho} + \int T\,ds$$

$\rho \leftarrow$ const

SFEE $$\cancel{\dot{Q}}^{\,0} - \cancel{\dot{W}_x}^{\,0} = \dot{m}\Delta\left(h + \tfrac{1}{2}v^2 + gz\right)$$

$$\therefore \quad 0 = \dot{m}\left[\int T\,ds + \underbrace{\frac{\Delta p}{\rho} + \Delta\left(\tfrac{1}{2}v^2\right) + \Delta(gz)}_{\Delta p_0}\right]$$

$$\Rightarrow \quad \dot{m}\frac{\Delta p_0}{\rho} = \dot{m}\frac{p_{02} - p_{01}}{\rho} = -\dot{m}\int T\,ds \qquad \text{qed.}$$

Alternatively: $$\int T\,ds = \int dh - \int \underbrace{v}_{=\frac{1}{\rho} = \text{const}}\,dp = \int dh_0 - \int \frac{dp_0}{\rho} = \Delta h_0 - \frac{\Delta p_0}{\rho}$$

SFEE $\dot{Q} - \dot{W}_x = 0 = \dot{m}\,\Delta h_0$ hence result.

(b) (i) Continuity: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$$\rho A_1 u_1 + \rho A_2 u_2 = \rho A_3 u_3 \qquad \rho = 1000\ \text{kg/m}^3$$

$$\dot{m}_1 = \rho\pi\frac{D_1^2}{4}u_1 = 157.07\ \text{kg/s}$$

$$\dot{m}_2 = \rho\pi\frac{(D_2^2 - D_1^2)}{4}u_2 = 188.49\ \text{kg/s}$$

$$\therefore \quad \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 345.6\ \text{kg/s}$$

But $\dot{m}_3 = \rho\pi\frac{D_2^2}{4}u_3 \quad \Rightarrow \quad \underline{\underline{u_3 = 1.76\ \text{m/s}}}$

(ii) Streamlines are parallel $\therefore$ no pressure gradient perpendicular because $\frac{dp}{dr} = \frac{\rho v^2}{r}$ & $r = \infty$

$$\therefore \quad \frac{dp}{dr} = 0 \quad \Rightarrow \quad p_1 = p_2 = \text{const}$$

(iii) SFME

$$p_1 A_1 + p_2 A_2 - p_3 A_3 = \dot{m}_3 u_3 - (\dot{m}_1 u_1 + \dot{m}_2 u_2)$$

4 cont

but $p_1 = p_2$ & $A_1 + A_2 = A_3$ ∴

$$(p_1 - p_3)\,\pi \frac{D_3^2}{4} = \dot{m} u_3 - \dot{m} u_2 - \dot{m} u_1$$

$\Rightarrow \quad p_1 - p_3 = -13862 \qquad p_3 = 10^5\ \mathrm{Pa}$

$\Rightarrow \quad p_1 = p_2 = \underline{\underline{0.8614\ \text{bar}}}$

(iv) $p_{01} = p_1 + \frac{1}{2}\rho V_1^2 = 2.8613$ bar — neglect $\rho g z$ terms

$p_{02} = p_2 + \frac{1}{2}\rho V_2^2 = 0.86638$ bar

$p_{03} = p_3 + \frac{1}{2}\rho V_3^2 = 1.01548$ bar

Rate of loss of M.E. $= \dfrac{\dot{m}_3 P_{03}}{\rho} - \dfrac{(\dot{m}_1 P_{01} + \dot{m}_2 P_{02})}{\rho} = \underline{\underline{26.2\ \text{kW}}}$

(v) By conservation of energy, no $\dot{Q}$, no $\dot{W}_x$

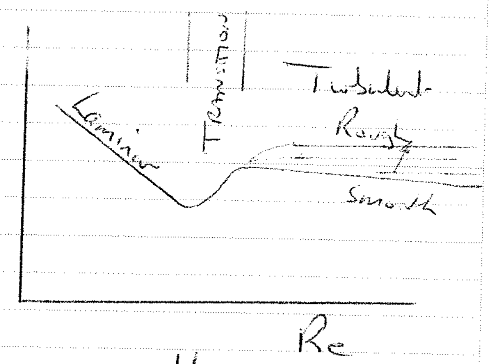
∴ rate of increase of int. E. $= \underline{\underline{26.2\ \text{kW}}}$

Alternatively; $T\,ds = du + p\,\cancel{dv}$ $\quad 0$ (incompressible)

$$\therefore \quad \Delta U = \int T\,ds = \frac{\dot{m}_1}{\rho}(P_{01} - P_{03}) + \frac{\dot{m}_2}{\rho}(P_{02} - P_{03})$$

$= $ as above.

Q5 (a) In the high Re region, there are only small variations in frictional effects (which dominate over pressure drag in "aerodynamic" devices such as pumps). The boundary layers are thin and surface roughness tends to dominate the frictional terms.



(b) Vol. flow rate $= Q$ $\quad L^3T^{-1}$

Rotational speed $= N$ $\quad T^{-1}$

Diameter $= D$ $\quad L$

Let $\phi = f_1(Q, N, D)$

$$\begin{matrix} - & & L^3T^{-1} & T^{-1} & L \end{matrix}$$

$$\phi = f_2\left(\frac{Q}{D^3}, N, \cancel{D}\right)$$

$$\begin{matrix} T^{-1} & T^{-1} & L \end{matrix}$$

$$\phi = f_3\left(\frac{Q}{ND^3}, \cancel{N}\right)$$

$$\begin{matrix} - & & - & T^{-1} \end{matrix}$$

∴ Let $\phi = \underline{\underline{\dfrac{Q}{ND^3}}}$ as is simplest expression

Pi theorem: 4 variables, 2 dimensions, 2 ndg ✓

5 (c) Extra dimensional variables:

Density ρ ML^{-3}

Pressure diff. ΔP $ML^{-1}T^{-2}$

By similar process to above, Pi theorem gives $5-3 = 2$ ndg's

Let

$$\Delta p = f_1(Q, N, D, \rho)$$
$$ML^{-1}T^{-2} \quad L^3T^{-1} \quad T^{-1} \quad L \quad ML^{-3}$$

$$\frac{\Delta P}{\rho} = f_2(Q, N, D, \cancel{\rho})$$
$$L^2T^{-2} \quad L^3T^{-1} \quad T^{-1} \quad L \quad ML^{-3}$$

$$\frac{\Delta P}{\rho \frac{N^2}{L^2}} = f_3\left(\frac{Q}{N}, \cancel{N}, D\right)$$
$$L^3 \quad T^{-1} \quad L$$

$$\frac{\Delta P}{\rho N^2 D^2} = f_4\left(\frac{Q}{ND^3}, \cancel{D}\right)$$
$$- \quad - \quad L$$

$\therefore$

Let $\frac{\Delta P}{\rho N^2 D^2} = \psi = f_5\left(\frac{Q}{ND^3}\right) = f_5(\phi)$

Strictly, as $\frac{\Delta P}{\rho N^2 D^2} = f\left(\frac{Q}{ND^3}\right)$ a ψ can be formed from any functional relationships between the two groups. However, the above is the simplest.

Also note that "by inspection" is sufficient to yield the required groups

5 d) $\Delta p = a - bQ^2$

$\Delta P = \psi \rho N^2 D^2 \qquad Q = \phi N D^3$

$\Rightarrow \psi \rho N^2 D^2 = a - b\phi^2 N^2 D^6$

$\Rightarrow \psi = \frac{a}{\rho N^2 D^2} - \frac{b\phi^2 \cancel{N^2} D^{\cancel{6}\,4}}{\rho \cancel{N^2} \cancel{D^2}}$

$\underline{\underline{\psi = 0.13 - 549.7\phi^2}}$

(e) $\Delta P_{pipe} = 4C_f \frac{L}{D_{pipe}} \frac{1}{2}\rho V^2 ; \quad \frac{\pi D^2}{4} V = Q$

$\Rightarrow \Delta P_{pipe} = \frac{32}{\pi^2} \frac{L}{D_{pipe}^5} C_f \rho Q^2$

$[\Rightarrow \Delta P_{pipe} = 1.1088 \times 10^7 Q^2]$

(f) The characteristic of the pump and of the pipe can be equated at the operating point in non-dimensional or in dimensional space.

In non-dimensional space $\psi_{Pump} = \psi_{pipe} ; \phi_{pump} = \phi_{pipe}$

$\frac{\Delta P_{pipe}}{\rho N^2 D^2_{pump}} = \frac{32}{\pi^2} \frac{L}{D^5_{pipe}} \rho \frac{(\phi^2 N^2 D^6_{pump})}{\rho N^2 D^2_{pump}} C_f$

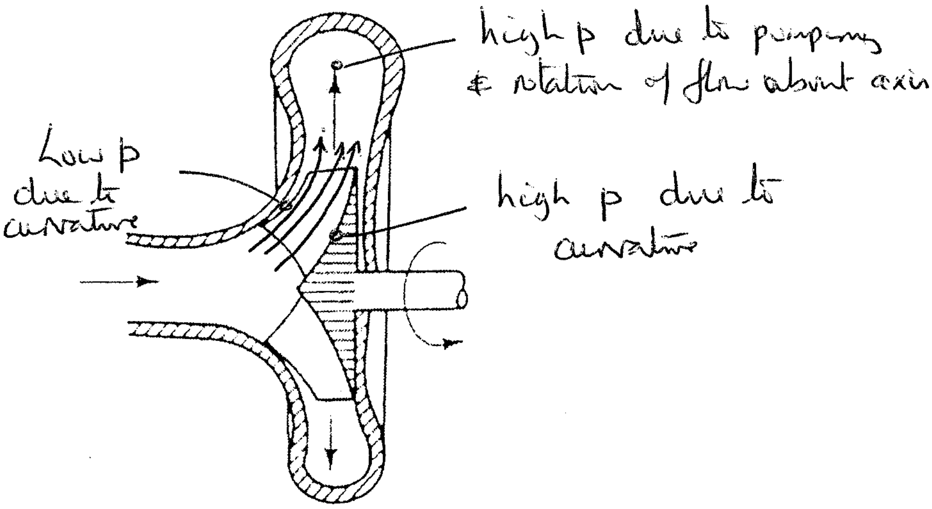
$\Rightarrow \psi = 1478.48\phi^2$

Equating terms: $\psi = 1478.4\phi^2 = 0.13 - 549.7\phi^2$

$\Rightarrow \phi = 0.008006 \qquad \psi = 0.09476$

$\Rightarrow \underline{\underline{Q = \phi N D^3 = 0.01 m^3/s}} \qquad \underline{\underline{\Delta P = \rho N^2 D^2 \psi = 112.2 kPa}}$

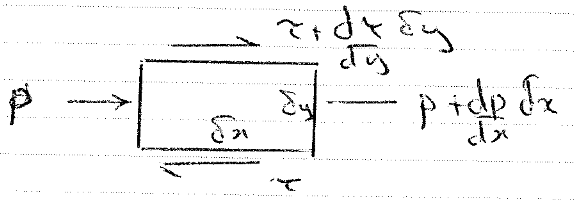
5 (g)


high p due to pumping & rotation of flow about axis
Low p due to curvature
high p due to curvature

6 (a) m is conserved

rate of displacement of mass $= \dot{m} = \rho A V = \rho \pi \frac{D^2}{4} V = \underline{\underline{\rho \pi R^2 V}}$

(b)



$p = p(x)$

$\tau = \tau(y)$

$u = u(y)$

SFME

$$-\frac{dp}{dx}\delta x \delta y + \frac{d\tau}{dy}\delta y \delta x = 0$$

$$\frac{dp}{dx} = \frac{d\tau}{dy}$$

Newton:

$$\tau = \mu \frac{du}{dy}$$

$$\therefore \quad \frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}$$

$$\Rightarrow \quad \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + By + C = u.$$

Now $\quad y = 0 \Rightarrow u = 0$

$\quad\quad y = h \Rightarrow u = V$

[Stationary frame of reference]

$$\therefore \quad C = 0$$

$$B = \frac{1}{h}\left[V - \frac{1}{2\mu}\frac{dp}{dx}h^2 \right]$$

Cont.

$$\therefore \quad u(y) = \frac{1}{2\mu}\frac{dp}{dx}\left[y^2 - yh\right] + \frac{Vy}{h}$$

(c) $$\dot{m} = \int \rho u \, dA = \int_0^h \rho u \, 2\pi R \, dy$$

$$= \rho 2\pi R \int_0^h u \, dy = \rho 2\pi R \left[\frac{-1}{12\mu}\frac{dp}{dx}h^3 + \frac{Vh}{2}\right]$$

$$= \rho \pi R h \left[V - \frac{h^3}{6\mu}\frac{dp}{dx}\right]$$

(d) $$\frac{dp}{dx} = \frac{p_2 - p_1}{L}$$

Equate mass flows from (a) & (c) gives

$$\cancel{\rho} 2\pi R \left[-\frac{p_2 - p_1}{L}\frac{h^3}{12\mu} + \frac{Vh}{2}\right] = \cancel{\rho}\pi R^2 V$$

$$\Rightarrow \quad p_1 - p_2 = \frac{6 V \mu L (R - h)}{h^3}$$