ENGINEERING TRIPOS, Part 1B 2003

Paper 5 – ELECTRICAL ENGINEERING

Solutions

1.

(a) Summing up voltage drops from Vcc to ground: $Vcc = I_E R_E + V_{CE} + I_c x150\Omega$

Now, $I_c = 40$ mA, $I_B = 100\ \mu A$, and $I_E = I_c + I_B = 40.1$ mA

$=> 20V = 40.1x10^{-3}R_E + 10V + 40x10^{-3}x150$

$=> 40.1x10^{-3}\ R_E = 20 - 10 - 6 = 4V$

$=>$ **$R_E = 99.75\Omega \sim 100\Omega$**

Now, to find R_2, we know that (i) $V_{BE} = V_B - V_E = +0.7V$, and (ii) the transistor is drawing some current ($I_B = 100\ \mu A$), so ***we cannot just use the potential divider equation to find R_2***. Instead, we just sum currents at the base as follows: We name the current flowing through resistor R_2 as I_2, and then the current through the 120kΩ resistor (which we call R_1) is $I_2 + I_B$.

That leaves us with two equations:

(1) $V_B = 4.7V = I_2R_2$

(2) $I_2R_2 + (I_B + I_2)R_1 = Vcc$

$=> I_B + R_2 = 15.3/R_1$

$=> I_2 = 2.75x10^{-5}A$

$=>$ **$R_2 = 4.7/I_2 = 170\ k\Omega$**

(b) From the DC characteristics, we can say that $Ic = Ic(I_B, V_{CE})$, and $V_{BE} = V_{BE}(I_B, V_{CE})$

Therefore, the differentials of Ic and V_{BE} are:

$$\partial Ic = \frac{\partial Ic}{\partial I_B}\partial I_B + \frac{\partial Ic}{\partial V_{CE}}\partial V_{CE} \quad \text{and} \quad \partial V_{BE} = \frac{\partial V_{BE}}{\partial I_B}\partial I_B + \frac{\partial V_{BE}}{\partial V_{CE}}\partial V_{CE}$$

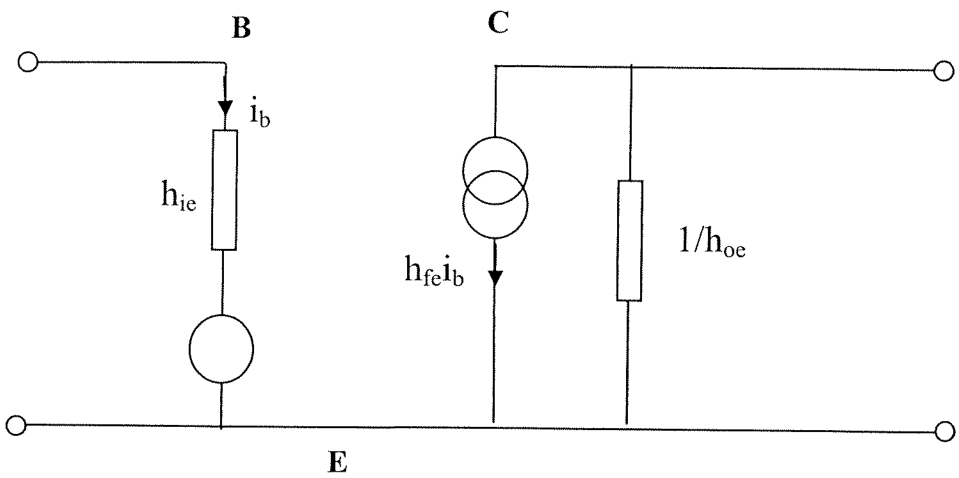
Using the h-parameters, that leaves us with:

$i_c = h_{fe}i_b + h_{oe}v_{ce}$..........this describes the output of the transistor

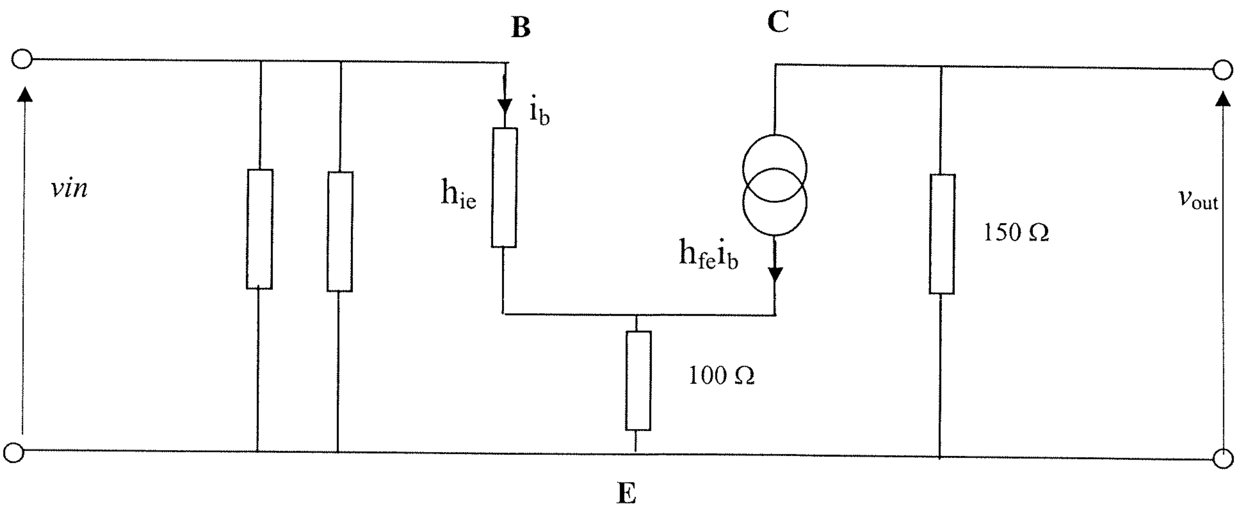
$v_{be} = h_{ie}i_b + h_{re}v_{ce}$..........this describes the input to the transistor

Yes, this is quite accurate, but only at mid-band frequencies. At higher frequencies, parasitic capacitances within the transistor mean the equivalent circuit must be modified. This is done by the addition of a capacitor between the base and the collector.

Small-signal equivalent circuit of a transistor:



(c) The small-signal equivalent of the circuit in Fig. 1 is:

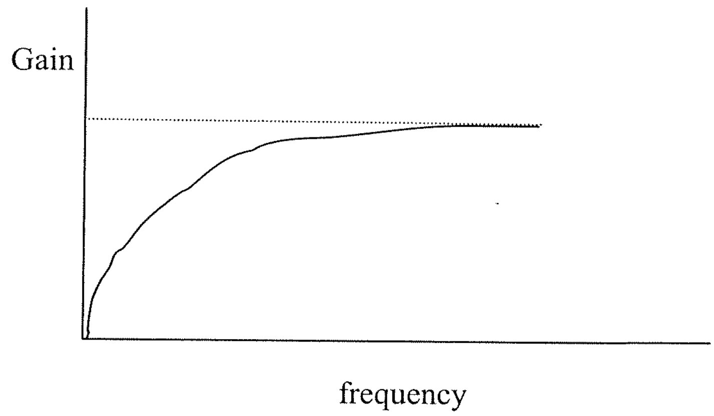


$$\text{Gain} = v_{out}/v_{in} = \frac{-100h_{fe}i_b}{h_{ie}i_b + (i_b + h_{fe}i_b)} = -\mathbf{1.46}$$

If we increase h_{fe} to 800 now, the gain increases to – **1.48**. Therefore, this circuit is relatively insensitive to variations in h_{fe}. This is an important consideration, because during fabrication of

ICs, there will always be significant variations in hfe between transistors due to statistical fluctuations. Thus, this type of circuit is ideal if you wish to have a predictable gain (which is almost always the case!).

(d) C_1 & C_2 are there to ensure that only ac signals are present at the input and output – by removing dc offsets.

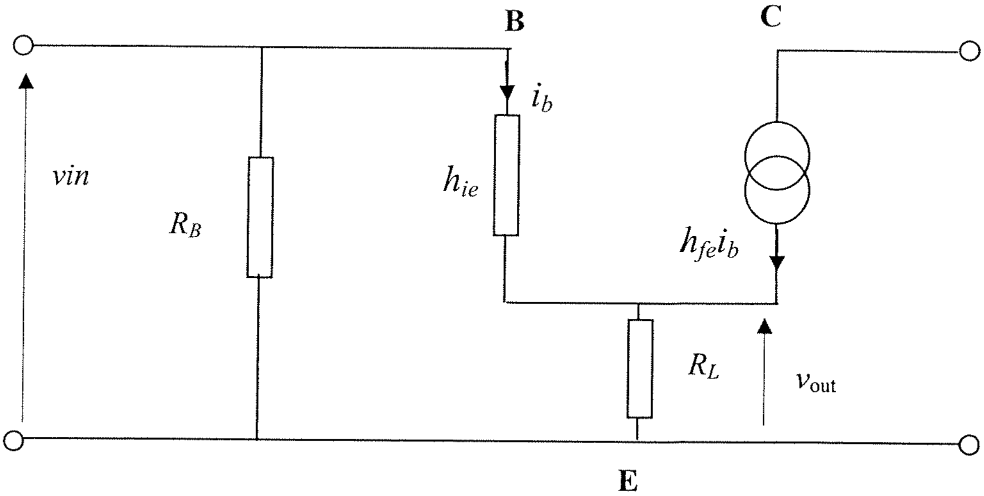


These capacitors cause the gain to roll off at low frequencies. If they were omitted, the gain would be almost independent of frequency (although stray capacitances in the circuit would probably give rise to a lower -3dB point at a very low frequency), and the circuit would be prone to problems with dc offsets, and would have a lower dynamic range.
Most students thought that C_1 and C_2 set the lower and upper -3dB frequencies, respectively. The upper 3dB frequency of this circuit will only be affected by parasitic capacitances, most of which are within the transistor itself.

2.

(a) This is a class A amplifier, as the transistor will be on all the time. The emitter follower is used in applications where you want to couple a source with a high output impedance with a load of low inputimpedance. It does this by having a high input impedance and low output impedance. It has a gain of almost 1, and is usually called a *buffer* amplifier.

(b) The small-signal equivalent circuit of this amplifier is:



At the input, $i_b = (v_{in} - v_{out})/h_{ie}$

At the emitter, $(1 + h_{fe})i_b = v_{out}/R_L$

=> putting both expressions for i_b equal to each other, we obtain:

$v_{in}R_L(1 + h_{fe}) = v_{out}h_{ie} + v_{out}R_L(1 + h_{fe})$

=> $v_{in}R_L(1 + h_{fe}) = v_{out}(R_L + h_{ie} + h_{fe}R_L)$

=> $\dfrac{v_{out}}{v_{in}} = \dfrac{R_L(1+h_{fe})}{R_L(1+h_{fe})+h_{ie}}$ = 2000x(251)/(2000x251 + 1000) = 0.998

i.e. **Gain = 0.998**

(c) Efficiency is the ac output power over the dc input power to the load.

Maximum possible output Voltage amplitude, Vo = Vcc/2 (i.e. max. possible peak-peak output voltage swing is from 0V to Vcc. As we are dealing with ac signals, that means we can get that when the ac signal is on a dc offset of Vcc/2). Therefore, the maximum possible ac current is $Vcc/(2R_E)$.

Thus, RMS output power = $\frac{1}{2}I_{max}V_{max} = Vo^2/(2R_E) = Vcc^2/(8R_E)$

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Now, the dc input power is the average current in the circuit times Vcc. The current through RE is Vcc/2R_E. (remember, maximum power when output is on a dc offset of Vcc/2 so output can swing from 0->Vcc).

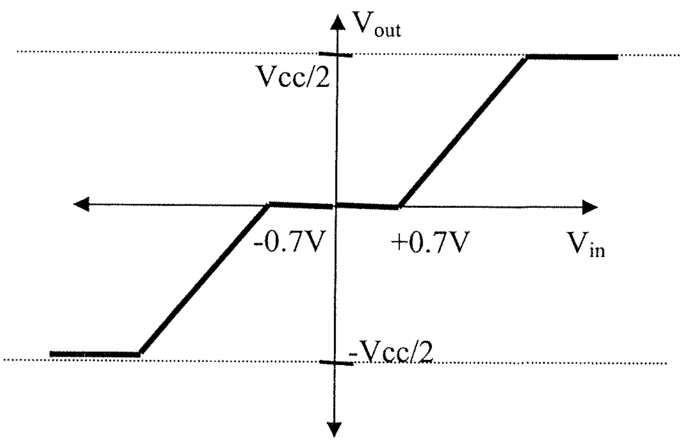
$$=> \text{efficiency, } \eta = \frac{V_o^2 / 2R_L}{V_{cc}^2 / 2R_L} \quad = - \quad \frac{V_o^2}{V_{cc}^2}$$

For maximum output swing, i.e. Vo = $V_{cc}/2$

η = 25 %

(d) The simple emitter follower is class A, so draws a lot of power even with no input signal, and there is also a flow of current through the load. The complementary follower is class B, so only one transistor is operating at a time, and there is only power dissipated when there is an input. The complementary follower also has much higher efficiency, approaching 75%.

(e)



There is no output for -0.7V < V_{in} < +0.7V *(0.7V is typical value for V_{BE})*
This is called *crossover distortion*, and can be reduced by biasing the transistors to the edge of conduction, by for example placing Zener diodes on the inputs.

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3.

(a) Three-phase is the most efficient due to the power per line that can be achieved. Adding more phases only improves power output by a relatively small amount, and requires extra cabling. Single-phase power only generates 2/3 as much power output. AC is used due to the ease of both generation and transmission. Ac motors are the cheapest and easiest to manufacture. Also, transformers can be used to step the voltage up for transmission (low I^2R losses) and down again for distribution. Finally, ac is useful because the voltage can be arbitrarily changed using transformers, which is useful as not all appliances use the same voltage.

Most students gave very sketchy answers here which were clearly not well-thought out.

(b) The total power loss, P is:

Power = Power in load + power in lines.

Let the current in the lines be I_{lines} (this is also the current in the load), and the resistance of the lines is 10Ω.

Then,

$P_{total} = P_{load} + 3x10I_{lines}^2$

And, $P_{load} = 3V_{ph}I_{ph}\cos\phi = \sqrt{3}V_{load}I_{lines}\cos\phi$

$\Rightarrow 50x10^6 = \sqrt{3}x132x10^3xI_{lines}x0.8$

$\Rightarrow \mathbf{I_{lines} = 273.4\ A}$

To find Sending-line voltage, we first need to find the apparent power S, and then we can use

$S = 3V_{ph}I_{ph} = \sqrt{(P^2 + Q^2)}$

Total real power then is:

$P_{total} = P_{load} + 3x10I_{lines}^2 \quad = \quad 50x10^6 + 3x(273.4)^2x10$

$= 52.24$ MW

Similarly,

$Q_{total} = Q_{load} + 3x31.8x10^{-3}x2\pi x50xI_{lines}^2$

$= \quad 50x10^6 x\tan(\cos^{-1}(0.8)) + 3\ x31.8x10^{-3}x2\pi x50x273.4^2$

$= 39.74$ MVAR

$\Rightarrow S = 65.6$ MVA

$\Rightarrow V_{ph} = S/3I_{ph} = 80.03$ kV

$\Rightarrow \mathbf{V_{line} = \sqrt{3}V_{ph} = 138.6\ kV}$

(c) Theload p.f. is now 0.9, and the voltage at the load is still 132 kV

$\Rightarrow 50\ mW = \sqrt{3}V_{load}I'_{lines}\cos\phi$

$=> I'_{lines} = 243$ A

The new power loss in the lines is $3I'_{lines}2R$ = **1.77 MW**

The new sending-line voltage is worked out as before:

$P_{total} = P_{load} + 3x10I_{lines}^2 \quad = 51.77$ MW

$Q_{total} = Q_{load} + 3x31.8x10^{-3}x2\pi x50xI_{lines}^2$

$= \quad 50x10^6xtan(cos^{-1}(0.9)) + 3\ x31.8x10^{-3}x2\pi x50x243^2$

= 25.93 MVAR

$=> 3V_{ph}I_{ph} = S = 57.9$ MVA

$=>$ **$V_{line} = \sqrt{3}V_{ph} = 137.6$ kV**

(d) To now have $\cos\phi = 0.9$, remember that it was 0.8.

$Q = P\tan\phi$, and P is 50 MW

$=> Q_{new} = 50xtan(cos^{-1}(0.9)) = 24.216$ MVAR

Before, $Q_{old} = 50xtan(cos^{-1}(0.8)) = 37.5$ MVAR

Therefore, difference = 37.5 MVAR – 24.216 MVAR = 13.28 MVAR

So, we want to reduce the amount of VARs in the system – we want to generate some => capacitors. If we star-connect three capacitors in parallel with the load, they generate

$Q_{difference} = 3V_{ph}^2/X = 13.28x10^6$

$=> X = 1312$

But, $X = 1/(2\pi fC)$, where f is 50 Hz, and C is the capacitance we are looking for.

Therefore, $C = 2.42\ \mu F$

If the capacitors were delta-connected instead, then you should use capacitors of value 809 nF (factor of 3 different)

(e) Given that $P = 3V_{ph}I_{ph}\cos\phi$, and V and $\cos\phi$ remain constant, there is no reason for P to vary with frequency. If P did change, so would V. Therefore, the Line current is independent of frequency as long as the power factor is maintained.

Very few people got this out, as most people tried to over-complicate the issue.

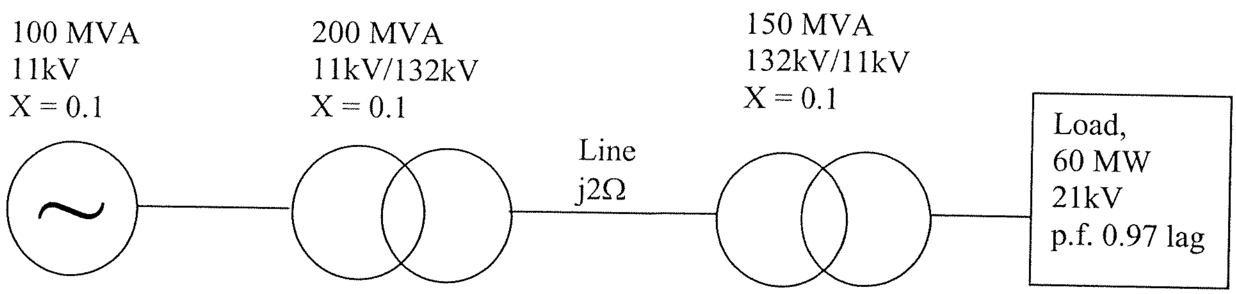
4.

(a) (i) All generator's excitations can be considered to be identical, and of strength 1 p.u.

(ii) Transformers can be replaced by inductors

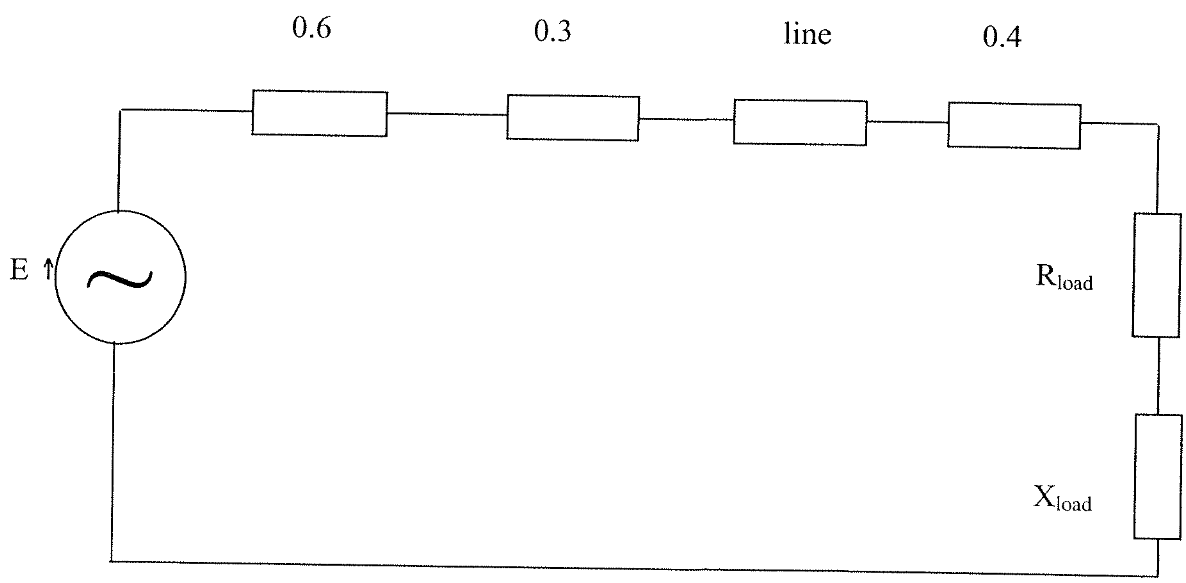
(iii) We end up with small, easy-to-manage numbers

(b)



Choose $MVA_b = 600MVA$

Then, circuit becomes:



The base impedance of the line is $Z_b = V_b^2/VA_b = 29\ \Omega$

Therefore, the line impedance = 2/29 = 0.07 p.u.

The current in the load, which is the same as the line current, is $P/(\surd 3 V_l \cos\phi) = 1700.6$ A

But, $I_{b,\,load} = VA_b/\surd 3 V_b = 15746$ A

=> $I_{load} = 0.108$ p.u.

This is the per-unit current flowing through the entire circuit (but it's actual value at any point will depend on the base-current at that point in the circuit). To find out the Generator current then, we must work out the base current there:

$I_{b,\,gen} = 600MVA/(\sqrt{3}x11kV) = 31492$ A

=> **Generator current = 3401 A**

How about the generator excitation? Now that we know the current everywhere, we need to know the impedance everywhere, then we can just apply Ohm's law to find E.

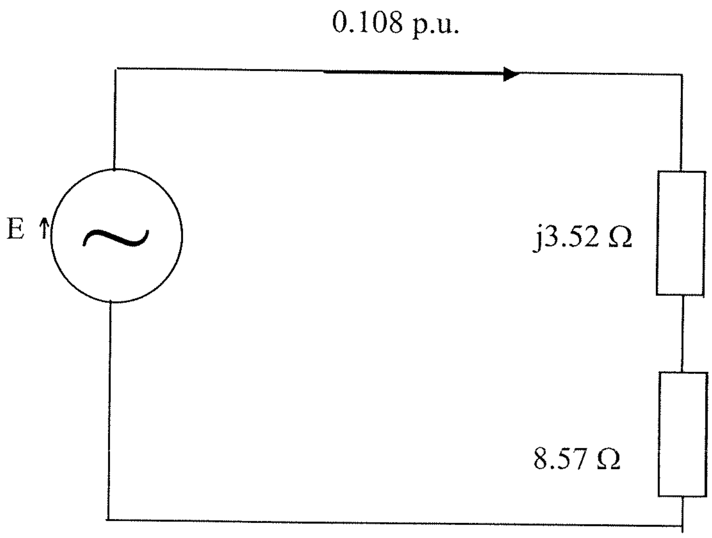
$V_{load} = 21$ kV/22 kV = 0.95 p.u.

$Z_{load} = V_{load}/I_{load} = 0.95/0.108 = 8.84$ p.u.

Then, $Z_{load} = Z_{load}\cos\phi + jZ_{load}\sin\phi$

$= 8.57 + 2.15j$

Then the circuit reduces down to:



Then, $E = IZ = 0.108\sqrt{3.52^2 + 8.57^2} = 1.006$ p.u.

=> **E = 11.006 kV**

(c) The fault current is worked out by simply replacing the load with a short circuit, and finding the net current through the circuit. The sum of the other impedances in the system is 1.37 p.u., so the fault current is 0.73 p.u. The base current after the last transformer is 15746 A, so the fault current is then **11494 A**.

The circuit breaker rating is given by $V_{ph}I_{fault} = 0.73$

Given that the $MVA_b = 600$ MVA, the circuit breaker should be rated to 600MVAx0.73 = **438 MVA**

To limit the fault to 170 MVA, we need the overall impedance to be 600/170 = 3.53, which means an additional reactance of 3.53-1.37 = 2.16 p.u.

But, $V_b = 22$ kV

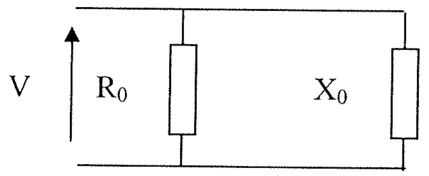
$\Rightarrow Z_b = V_b^2/VA_b = 0.806\ \Omega$

$\Rightarrow$ **$X = 0.806 \times 2.16 = 1.74\ \Omega$/phase**

5.

(a) Both the stator and rotor are wound with 3-phase windings. The current applied to the stator produces a rotating magnetic field, which induces currents in the rotor due to the changing flux linkage (similar to a transformer). The rotor currents then produce their own magnetic field. The rotor and stator magnetic fields interact to produce a torque. At synchronous speed, the rotor is spinning at the same frequency as the stator field, so the flux linkage remains constant, and there is no induced emf in the rotor coils, so there will be no rotor field, and no torque on the rotor.

(b) No-load test. The equivalent circuit becomes:



The power in, $P = 3V^2/R_0$ ....$V = 415/\sqrt{3}$

$\Rightarrow R_0 = 3V^2/P$

$\Rightarrow \mathbf{R_0 = 202.6\ \Omega}$

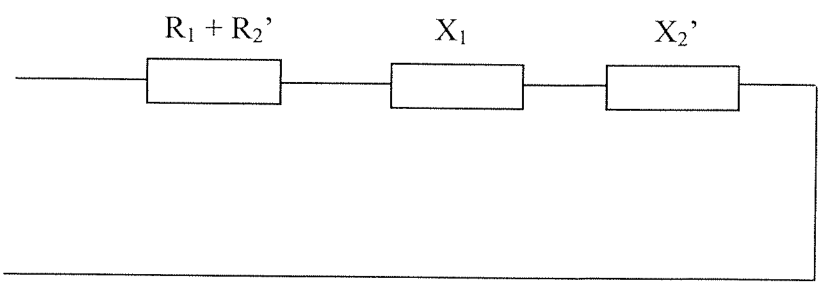
Also, $S^2 = P^2 + Q^2 = (3VI)^2$

$\Rightarrow Q^2 = (3VI)^2 - P^2$

$\Rightarrow Q = 2137.3$ VAR

$\Rightarrow \mathbf{X_0 = 80.6\ \Omega}$

(c) Locked-rotor test. The circuit reduces to:



Let $R_1 + R_2' = R$

$P_{in} = I^2R \Rightarrow R = 5000/3600 = 1.39$..............(***if you assume that the power given is the total power, R_2' turns out to be negative! Therefore, you should assume that the power given is per phase***).

Given that $R_1 = 0.7\ \Omega$, then $\mathbf{R_2' = 0.69\ \Omega}$

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Similarly, $Q^2 = S^2 - P^2$

$= (V_{ph}I_{ph})^2 - P^2$

$\Rightarrow Q = 4795.8 = I^2X$, where X is $X_1 + X_2'$

$\Rightarrow X = 4795.8/3600 = 1.33$

$\Rightarrow X_1 = 0.53\ \Omega$

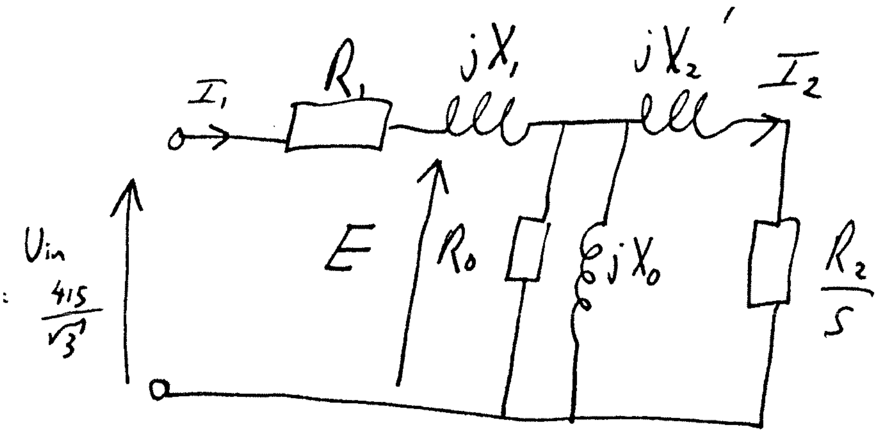
and $X_2 = 0.8\ \Omega$

(d) From the data book,

$$Torque = \frac{3I_2'^2 R_2'}{s\omega_s}$$

We want to find the current, I_2', through R_2'

To do that, we need to find the overall impedance of the circuit, from which we can find the input current. Then, we can work out the proportion of current passing through R_2'.
The circuit is below:



The net impedance of this is $Z_{in} = R_1 + X_1 + ((R_0 // X_0) // (X_2' + R_2'/s))$

This turns out to be $19.8 + j6.03$

Then the input current, $I_1 = V_{in}/Z_{in}$

$= 11.08 - j3.37$

Now, $E = V - I_1(R_1 + jX_1)$

$= 230.06 - j3.51$

The current through the magnetising branch, $I_m = E/Z_{mag}$, where Z_{mag} is $R_0 // jX_0$

=> $I_m = 1.09 - j2.87$

Therefore, $I_2 = I_1 - I_m = 9.95 - j0.5$

Using this in the equation for torque, we obtain a torque of **43.95 Nm**

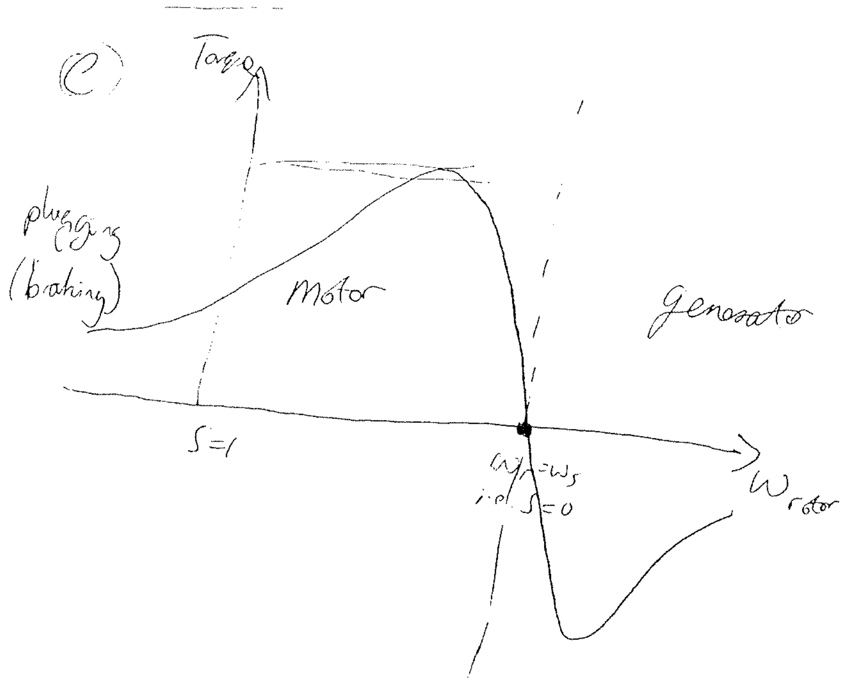
The efficiency is (power out)/(Power in)

= (Torque x Angular speed)/(3VIcosϕ)

The power factor is given as $\cos(\tan^{-1}(Im[I_1])/Re[I_1])$

That gives an efficiency of 97%

(e)



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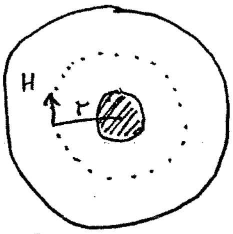
Part Ib Paper 5 Q6.

(a) Calculate the field strength H as a function of radius r.



Apply Ampere Law
$$\oint \underline{H}.\underline{dl} = \int_{\text{Surface}} \underline{J}.\underline{ds}$$

$$2\pi r H = I \quad \text{(total current } I\text{)}$$

Integrate from $r = b$ to a to find the total magnetic flux ψ per unit length

$$\psi = \int_b^a \mu_0 H \, dr = \int_b^a \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I}{2\pi} \ln(a/b)$$

By definition the inductance per unit length
$$L = \frac{\psi}{I} = \frac{\mu_0}{2\pi} \ln(a/b)$$
taking the relative permeability to be 1

(b) The characteristic impedance $Z_0 = \frac{V}{I}$ at any point in the coaxial line

$$Z_0 = \sqrt{\frac{L}{C}}$$

For this line $$Z_0 = \sqrt{\frac{\mu_0 \ln(a/b)}{2\pi} \cdot \frac{\ln(a/b)}{2\pi \varepsilon_0 \varepsilon_r}}$$
$$= \frac{\ln(a/b)}{2\pi} \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}$$

With $a = 2\,mm$

$b = 1\,mm$

$\varepsilon_0 = 2$

$$Z_0 = \frac{\ln 2}{2\pi} \sqrt{\frac{4\pi \times 10^{-7}}{8.9 \times 10^{-12} \times 2}} = \frac{\ln 2}{2\pi} \sqrt{\frac{2\pi \times 10^{5}}{8.9}}$$

$$= 29\,\Omega$$

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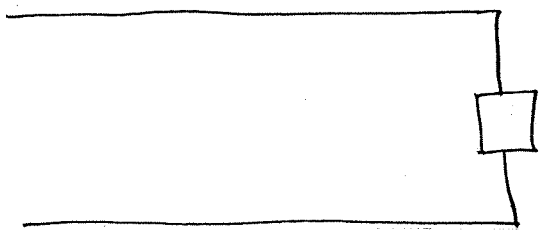
Part Ib Paper 5 Q6 (continued) Pfm 7.1.2003



$Z_\ell = 50\Omega$

At the load $V_F + V_B = V$

$$= Z_\ell I$$
$$= Z_\ell (I_F + I_B)$$
$$= Z_\ell \left(\frac{V_F}{Z_o} - \frac{V_B}{Z_o} \right)$$

$$\therefore Z_o V_F + Z_o V_B = Z_\ell V_F - Z_\ell V_B$$

Voltage reflection coefficient $\frac{V_B}{V_F} = \frac{Z_\ell - Z_o}{Z_\ell + Z_o}$

The power reflection coefficient is $\left| \frac{Z_\ell - Z_o}{Z_\ell + Z_o} \right|^2 = 0.08$

To reduce the reflected power to zero
The transmission line impedance must be increased to 50Ω
by decreasing the diameter of the inner conductor.
New diameter x.

require $\frac{\ln 2/x}{\ln 2} = \frac{50}{29.3}$

$$\ln(2/x) = 1.183$$
$$x = \frac{2}{e^{1.183}} = 0.61\,\text{mm}.$$

Reduce the diameter to 0.6 mm to match the load.

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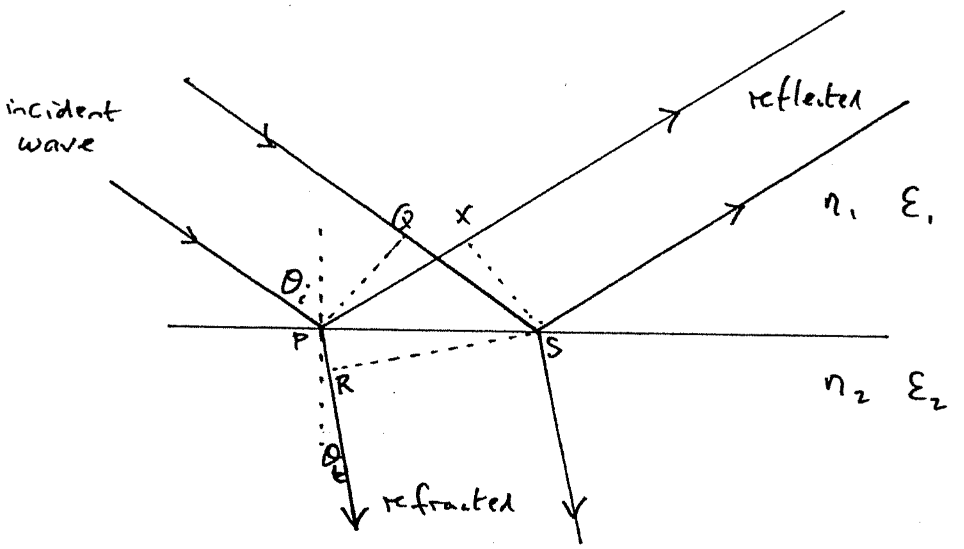
(a)



When the wave front travels from PQ to RS
the light has travelled a distance QS in medium 1
and PR in medium 2
the velocity is inversely proportional to η

Hence $\dfrac{n_1 \, t}{n_2 \, t} = \dfrac{PR}{QS} = \dfrac{PS \sin\theta_t}{PS \sin\theta_i}$

$\therefore \dfrac{n_1}{n_2} = \dfrac{\sin\theta_t}{\sin\theta_i}$

$\theta_t = \sin^{-1}\left(\dfrac{n_1}{n_2} \sin\theta_i\right)$ Snell Law of Refraction

Similarly for the reflective wave
when the wave front travels from PQ to SX
the light travels a distance QS in medium 1
and also PX in medium 1
Hence the angle of reflection must equal the angle of incidence.

Ib Paper 5 Q7 (b) ✱

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Boundary condition at the interface

Apply Gauss Law — since no charge at the interface

$$E_{\parallel}\ (\text{medium } 1) = E_{\parallel}\ (\text{medium } 2)$$

The incident power density is $\frac{(E_i)^2}{\eta_1} \cos\theta_i$ per unit area of interface.

No power is lost at the interface

Hence $$\frac{(E_i)^2}{\eta_1}\cos\theta_i = \frac{(E_r)^2}{\eta_1}\cos\theta_i + \frac{(E_t)^2}{\eta_2}\cos\theta_t$$

Same as angle of incidence

(c) apply boundary condition for parallel polarized wave

$$E_i \cos\theta_i - E_r \cos\theta_i = E_t \cos\theta_t .$$

by substitution

$$\frac{E_r}{E_i} = \frac{(\varepsilon_2/\varepsilon_1)\cos\theta_i - (\varepsilon_2/\varepsilon_1 - \sin^2\theta_i)^{1/2}}{(\varepsilon_2/\varepsilon_1)\cos\theta_i + (\varepsilon_2/\varepsilon_1 - \sin^2\theta_i)^{1/2}}$$

The minimum value of $\frac{E_r}{E_i}$ is zero corresponding to the Brewster angle (this effect used in polarized sun glasses)

As the ratio $\varepsilon_2/\varepsilon_1$ is increased then $\frac{E_r}{E_i}$ tends to 1

corresponding to perfect reflection.