)

Paper 6, Part IB Jun. 2003

(a) $\bar{y}(s) = \bar{d}_o(s) + \bar{d}_i(s)\,G(s) + \bar{e}(s)\,K(s)\,G(s)$

now

$$\bar{e}(s) = \bar{r}(s) - H(s)\,\bar{y}(s) \quad \text{so,}$$

$$\bar{y}(s) = \bar{d}_o(s) + \bar{d}_i(s)\,G(s) + (\bar{r}(s) - H(s)\,\bar{y}(s))\,K(s)\,G(s)$$

$$= \bar{d}_o(s) + \bar{d}_i(s)\,G(s) + \bar{r}(s)\,K(s)\,G(s) - \bar{y}(s)\,K(s)\,G(s)\,H(s)$$

$$\bar{y}(s)\,(1 + K(s)\,G(s)\,H(s)) = \bar{r}(s)\,K(s)\,G(s) + \bar{d}_i(s)\,G(s) + \bar{d}_o(s)$$

$$\therefore\ \bar{y}(s) = \frac{K(s)\,G(s)\,\bar{r}(s)}{1 + K(s)G(s)\,H(s)} + \frac{G(s)\,\bar{d}_i(s)}{1 + K(s)G(s)H(s)} + \frac{\bar{d}_o(s)}{1 + K(s)\,G(s)\,H(s)}$$

(b) (i) $r(t) = 0$, $d_i(t) = u(t)$ and $d_o(t) = 0$

$$\bar{y}(s) = \frac{G(s)}{1 + K(s)G(s)\,H(s)}\,\bar{u}(s)$$

Also,

$$\bar{e}(s) = -H(s)\,\bar{y}(s) \quad \text{so,}$$

$$\bar{e}(s) = \frac{-H(s)\,G(s)}{1 + K(s)G(s)H(s)}\,\bar{u}(s)$$

$$= \frac{\dfrac{-100}{s(s^2 + 20s + 100)}}{1 + K_p\,.\,\dfrac{100}{s(s^2 + 20s + 100)} \times 1} \cdot \left(\frac{1}{s}\right)$$

$$= \frac{\dfrac{-100}{s(s^2 + 20s + 100)}}{\dfrac{s(s^2 + 20s + 100) + 100K_p}{s(s^2 + 20s + 100)}} \cdot \left(\frac{1}{s}\right)$$

$$= \frac{-100}{s(s^2 + 20s + 100) + 100K_p} \cdot \frac{1}{s}$$

So, S.S. error $= \lim_{s \to 0} s\,\bar{e}(s)$

$$= \frac{-100}{100 K_p}$$

$$= \frac{-1}{K_p}$$

$\frac{3}{6}$

(ii) Now $d_o(t) = t \quad \therefore \bar{d}_o(s) = \frac{1}{s^2}$

$\bar{y}(s) = \frac{1}{1+K(s)G(s)H(s)} \cdot \frac{1}{s^2}$, $\quad \bar{e}(s) = -H(s)\bar{y}(s) \quad \therefore \bar{e}(s) = \frac{-1}{1+K(s)G(s)H(s)}\left(\frac{1}{s^2}\right)$

$$\bar{e}(s) = -\left(\frac{1}{s^2}\right)\cdot \frac{1}{1+K_p \frac{100}{s(s^2+20s+100)}} = \frac{-1}{s^2} \cdot \frac{s(s^2+20s+100)}{s(s^2+20s+100)+100K_p}$$

So,

$$\bar{e}(s) = -\frac{1}{s^2} \cdot \frac{s(s^2+20s+100)}{s(s^2+20s+100)+100K_p}$$

$$\bar{e}(s) = \frac{-1}{s} \quad \frac{(s^2+20s+100)}{s(s^2+20s+100)+100K_p}$$

$$\text{SS error} = \lim_{s\to 0} s\bar{e}(s)$$

$$= \lim_{s\to 0} \frac{-(s^2+20s+100)}{s(s^2+20s+100)+100K_p}$$

$$= \frac{-100}{100K_p} = \frac{-1}{K_p}$$

$\frac{3}{6}$ [6]

(c) Now $K(s) = K_p + \frac{K_I}{s}$

ie a P+I controller is used to eliminate SS error.

$\frac{1}{5}$

For (i), ie, $d_i(t) = u(t)$

$$\bar{e}(s) = \frac{\frac{-100}{s(s^2+20s+100)}}{1+\left(K_p+\frac{K_I}{s}\right)\frac{100}{s(s^2+20s+100)} \times 1} \times \left(\frac{1}{s}\right)$$

$$\bar{e}(s) = \frac{\frac{-100}{s(s^2+20s+100)}}{1+\frac{100K_p}{s(s^2+20s+100)}+\frac{100K_I}{s^2(s^2+20s+100)}} \cdot \left(\frac{1}{s}\right)$$

3)

$$\bar{e}(s) = \frac{\dfrac{-100}{s(s^2+20s+100)}}{\dfrac{s^2(s^2+20s+100)+100K_p s+100K_I}{s^2(s^2+20s+100)}} \cdot \left(\frac{1}{s}\right)$$

$$\bar{e}(s) = \frac{-100s}{s^2(s^2+20s+100)+100K_p s+100K_I} \cdot \left(\frac{1}{s}\right)$$

$$\bar{e}(s) = \frac{-100}{s^2(s^2+20s+100)+100K_p s+100K_I}$$

$$\text{ss error} = \lim_{s\to 0} s\bar{e}(s) = \lim_{s\to 0} \frac{-100s}{s^2(s^2+20s+100)+100K_p s+100K_I}$$

$$= \frac{0}{100K_I} = \underline{\underline{0}}$$

(2/5)

For (ii), $d_o(t) = t$

So,

$$\bar{e}(s) = \frac{-1}{1+\left(K_p+\frac{K_I}{s}\right)\left(\frac{100}{s(s^2+20s+100)}\right)} \cdot \left(\frac{1}{s^2}\right)$$

$$\bar{e}(s) = \frac{-1}{1+\dfrac{100K_p}{s(s^2+20s+100)}+\dfrac{100K_I}{s^2(s^2+20s+100)}} \cdot \left(\frac{1}{s^2}\right)$$

$$\bar{e}(s) = \frac{-s^2(s^2+20s+100)}{s^2(s^2+20s+100)+100K_p s+100K_I} \cdot \left(\frac{1}{s^2}\right)$$

$$\bar{e}(s) = \frac{-(s^2+20s+100)}{s^2(s^2+20s+100)+100K_p s+100K_I}$$

$$\text{ss error} = \lim_{s\to 0} s\bar{e}(s) = \lim_{s\to 0} \frac{-s(s^2+20s+100)}{s^2(s^2+20s+100)+100K_p s+100K_I}$$

$$= \frac{0}{100K_I} = \underline{\underline{0}}$$

(2/5)

[5]

(4)

(d) $$\bar{y}(s) = \frac{1}{1 + \frac{100K_p}{s(s^2+20s+100)}} \bar{d_o}(s)$$

$$= \frac{s(s^2+20s+100)}{s(s^2+20s+100) + 100K_p} \bar{d_o}(s)$$

$$= \frac{s^3 + 20s^2 + 100s}{s^3 + 20s^2 + 100s + 100k_p} \bar{d_o}(s)$$

We have $\bar{y}(s) = F(s)\, d_o(s)$ ∴

$$F(j\omega) = \frac{-j\omega^3 - 20\omega^2 + j\omega 100}{-j\omega^3 - 20\omega^2 + j\omega 100 + 100k_p}$$

$$F(j\omega) = \frac{-20\omega^2 + j(100\omega - \omega^3)}{100K_p - 20\omega^2 + j(100\omega - \omega^3)}$$

Now, $d_o(t) = \sin 10t$, ie, $\omega = 10$

$$\therefore F(j10) = \frac{-2000 + j(0)}{100K_p - 2000 + j(0)}$$

To keep $|F(j10)| \leq 2$

$$2 \leq \frac{-2000}{100K_p - 2000}$$

$\underline{K_p \leq 10}$

3/4

Also note soln at $k_p > 30$. However this will be unstable since,

$F(j10) \to \infty$ when $100K_p - 2000 = 0$

ie $K_p = \frac{2000}{100} = \underline{20}$

1/4

[4]

(5)

2.) (a)
$$\frac{d\phi(t)}{dt} = K_o\, v_c(t)$$

$$s\bar{\phi}(s) = K_o\, \bar{v}_c(s)$$

so,
$$\bar{\phi}(s) = \frac{K_o}{s}\, v_c(s)$$

Also,
$$e(t) = K\left[\theta(t) - \frac{\phi(t)}{N}\right]$$

$$\bar{e}(s) = K\left[\bar{\theta}(s) - \frac{\bar{\phi}(s)}{N}\right]$$

and
$$\bar{v}_c(s) = \bar{e}(s)\ F(s)$$

$$\bar{v}_c(s) = \bar{e}(s)\ .\ \left(\frac{1}{1+s\tau}\right)$$

So,
$$\bar{v}_c(s) = \left(K\bar{\theta}(s) - \frac{K}{N}\bar{\phi}(s)\right)\left(\frac{1}{1+s\tau}\right)$$

$$\bar{v}_c(s) = \left(K\bar{\theta}(s) - \frac{K}{N}\frac{K_o}{s}\, v_c(s)\right)\left(\frac{1}{1+s\tau}\right)$$

$$\bar{v}_c(s)\left(1 + \frac{KK_o}{Ns}\left(\frac{1}{1+s\tau}\right)\right) = \frac{K\bar{\theta}(s)}{1+s\tau}$$

$$\bar{v}_c(s) = \frac{Ns(1+s\tau) + KK_o}{Ns(1+s\tau)} = \frac{K\bar{\theta}(s)}{1+s\tau}$$

$$\bar{v}_c(s) = \frac{KNs(1+s\tau)}{(1+s\tau)(Ns(1+s\tau)+KK_o)}\ .\ \bar{\theta}(s)$$

$$\bar{v}_c(s) = \frac{KNs}{Ns(1+s\tau)+KK_o}\ .\ \bar{\theta}(s)$$

$$\bar{v}_c(s) = \frac{KNs}{N\tau s^2 + Ns + KK_o}\ \bar{\theta}(s)$$

$$\bar{v}_c(s) = \frac{\frac{KN}{KK_o}\, s}{\frac{N\tau}{KK_o}s^2 + \frac{N}{KK_o}s + 1}\ \bar{\theta}(s)$$

⑥

$$\bar{V_c}(s) = \frac{K_a s}{\frac{K_a \tau}{K} s^2 + \frac{K_a}{K} s + 1} \bar{\theta}(s)$$

[5/5]

(b) For a 2nd-order system

$$\omega_n = |p| \quad \text{and} \quad c = \frac{-\text{Real}(p)}{|p|}$$

$$\therefore -\text{Real}(p) = |p| c$$
$$-\text{Real}(p) = \omega_n c$$
$$-\text{Real}(p) = 2 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$
$$\text{Real}(p) = -\sqrt{2}.$$

So,
$$\sqrt{(-\sqrt{2})^2 + (\text{imag}(p))^2} = 2$$
$$\therefore \text{imag}(p) = \sqrt{2}$$
$$\therefore p = -\sqrt{2} \pm j\sqrt{2}$$

$$(s + \sqrt{2} - j\sqrt{2})(s + \sqrt{2} + j\sqrt{2}) = 0$$
$$s^2 + 2\sqrt{2}s + 4 = 0$$

Compare with the CE,

$$\frac{N\tau}{K_o K} s^2 + \frac{N}{K_o K} s + 1 = 0$$

$$s^2 + \frac{1}{\tau} s + \frac{K_o K}{N\tau} = 0$$

So,
$$\frac{1}{\tau} = 2\sqrt{2}$$
$$\therefore \tau = \sqrt{2}/4 \qquad 0.354$$

3/6

$$4 = \frac{K_o K}{N\tau}$$

$$\therefore K_o = \frac{4N\tau}{K}$$
$$K_o = \frac{4 \times 100 \times \sqrt{2}/4}{1}$$
$$K_o = 100\sqrt{2} \qquad 141$$

3/6

[6]

7

(c) Sub in values gives

$$\bar{v}(s) = \frac{\frac{100}{100\sqrt{2}}\, s}{\frac{100\sqrt{2}}{100\sqrt{2}\,4} s^2 + \frac{100}{100\sqrt{2}} s + 1} \quad \bar{\omega}(s)$$

$$\bar{v}(s) = \frac{2\sqrt{2}\, s}{s^2 + 2\sqrt{2}s + 4} \cdot \frac{\Delta\omega}{s^2}$$

where $\Delta\omega = \frac{1}{\sqrt{2}}$

so

$$\frac{2\sqrt{2}\,\Delta\omega}{s(s^2 + 2\sqrt{2}s + 4)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\sqrt{2}s + 4}$$

$$\frac{2}{s(s^2 + 2\sqrt{2}s + 4)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\sqrt{2}s + 4}$$

$$2 \equiv A(s^2 + 2\sqrt{2}s + 4) + (Bs + C)s$$
$$2 \equiv As^2 + 2\sqrt{2}As + 4A + Bs^2 + Cs$$

Equate coeff.

const: $2 = 4A \quad \therefore \quad A = 1/2$

s: $0 = \frac{2\sqrt{2}}{2} + C \quad \therefore \quad C = -\sqrt{2}$

s^2: $0 = \frac{1}{2} + B \quad \therefore \quad B = -1/2$

$\therefore$

$$\bar{v}(s) = \frac{0.5}{s} - \left(\frac{0.5s + \sqrt{2}}{s^2 + 2\sqrt{2}s + 4}\right)$$

$$\bar{v}(s) = \frac{1}{2}\left(\frac{1}{s} - \left(\frac{s + 2\sqrt{2}}{s^2 + 2\sqrt{2}s + 4}\right)\right)$$

$$\bar{v}(s) = \frac{1}{2}\left(\frac{1}{s} - \left(\frac{(s + \sqrt{2}) + \sqrt{2}}{(s + \sqrt{2})^2 + 2}\right)\right)$$

Laplace table (inverse) gives

$$v(t) = \frac{1}{2}\left(u(t) - e^{-\sqrt{2}t}(\cos\sqrt{2}t + \sin\sqrt{2}t)\right)$$
$$= \frac{1}{2}\left(u(t) - e^{-\sqrt{2}t}\sqrt{2}\cos(\sqrt{2}t - \pi/4)\right)$$

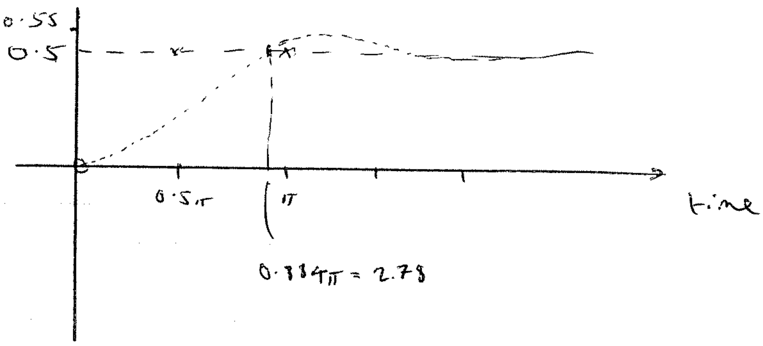
(4)

(8)

Response is actually that of a conventional 2nd-order system to a step.

(see also soln in Mechanical Data Book).

The only difference is is that the final value is 0.5 (not 1).



(4/8)

[8]

3.) (a) From graph
$\omega_n = 20 \text{ rad s}^{-1}$ (2/8)

$\frac{1}{T_1} = 1 \quad \therefore \quad T_1 = 1\text{s}$ (2/8)

$\frac{1}{T_2} = 300 \quad \therefore \quad T_2 = \frac{1}{300} = 3.333 \times 10^{-3}\text{s}$ (2/8)

Low frequency response = as (i.e. $\propto$ frequency)

Also see at $w=1$, Gain $= -3\text{dB}$ (0dB or unity for Bode approx)

Therefore $a = 1$. (2/8) [8]

(b) Input sinusoidal signals.
Measure gain and phase shift
Repeat at a number of frequencies over the desired frequency range. (2/5)

Problems,

- Unstable systems never reach steady state.
- Integral action - causes drift - system hits 'end-stops'.
- Noise, non-linearities, variation with time .... (3/5)

[5]

(c) GM is the extra gain factor which may be applied before closed-loop instability occurs. (1/7)

PM is the additional phase shift that may be applied before closed-loop instability occurs. (1/7) d)

With $K_p = 7$,

The GM is obtained from the Bode plot when the phase = $-180°$, ie at 75 rads^{-1}.

From the plot, the gain at this freq is

-23.3 dB $\equiv 0.0685$.

The open-loop gain is $7 \times 0.0685 = 0.479$.

$\therefore$ $GM = \dfrac{1}{0.479} = 2.087 = \underline{6.39\text{ dB}}$ (2/7)

PM is obtained when open-loop gain = 1, ie when the Bode-plot gain = $\dfrac{1}{7} = -16.9$ dB.

This occurs at $\omega = 55$ rads^{-1}

From plot gives phase = $-170°$

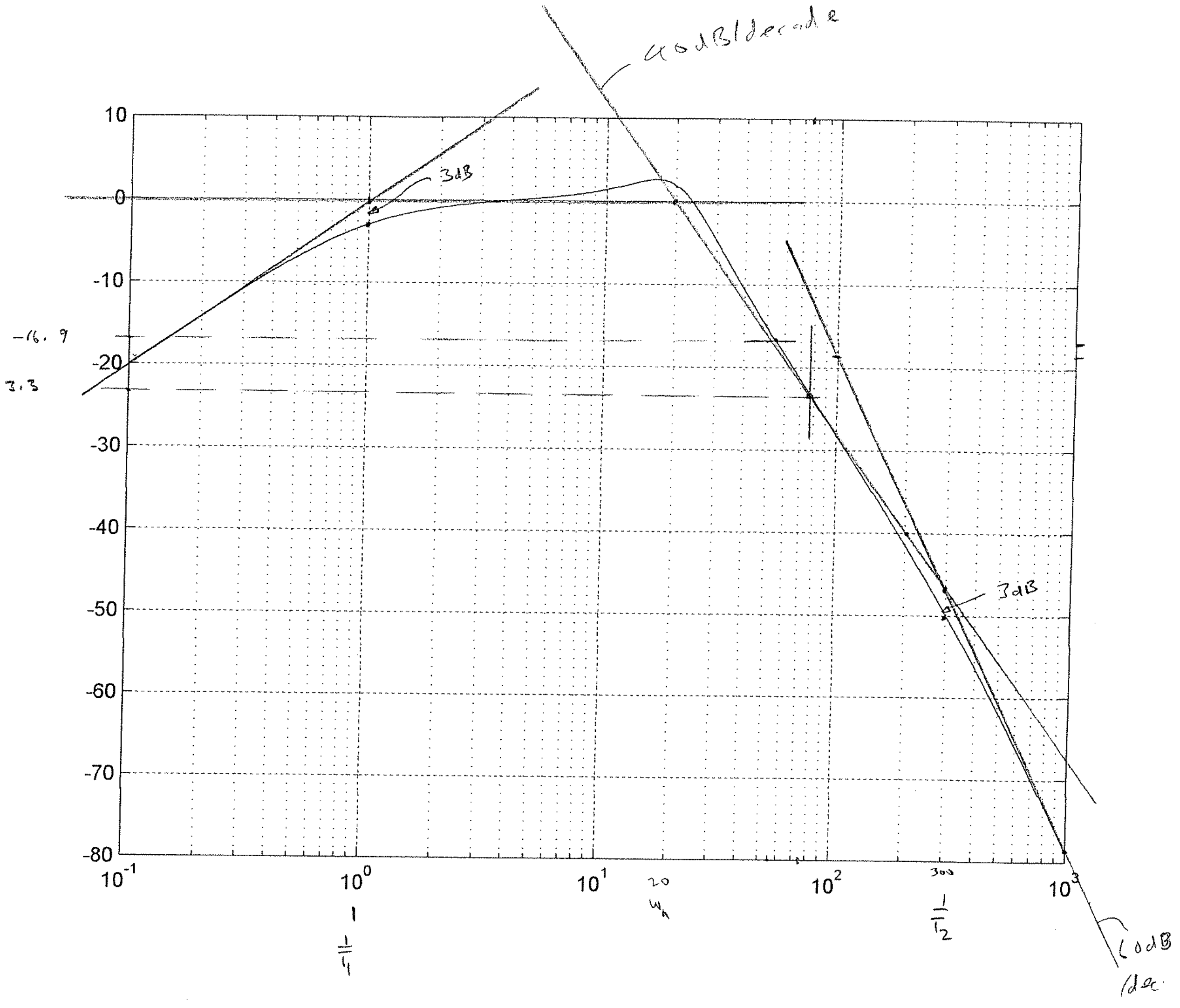
$\therefore$ $\underline{PM = 10°}$ (2/7)

Response will be very oscillatory - around $\underline{50\text{ rads}^{-1}}$ - since PM is very low.

(1/7)

[7]

11



$T_2 = \frac{1}{300} = 3.333 \times 10^{-3}$

$T_1 = \frac{1}{1}$

$a = 1$

$\omega_n = 20$

$c = 0.4$

$\frac{1}{\omega_n^2} = 2.5 \times 10^{-3}$

$\frac{2c}{\omega_n} = \frac{2 \times 0.4}{20} = 0.04$

Fig A

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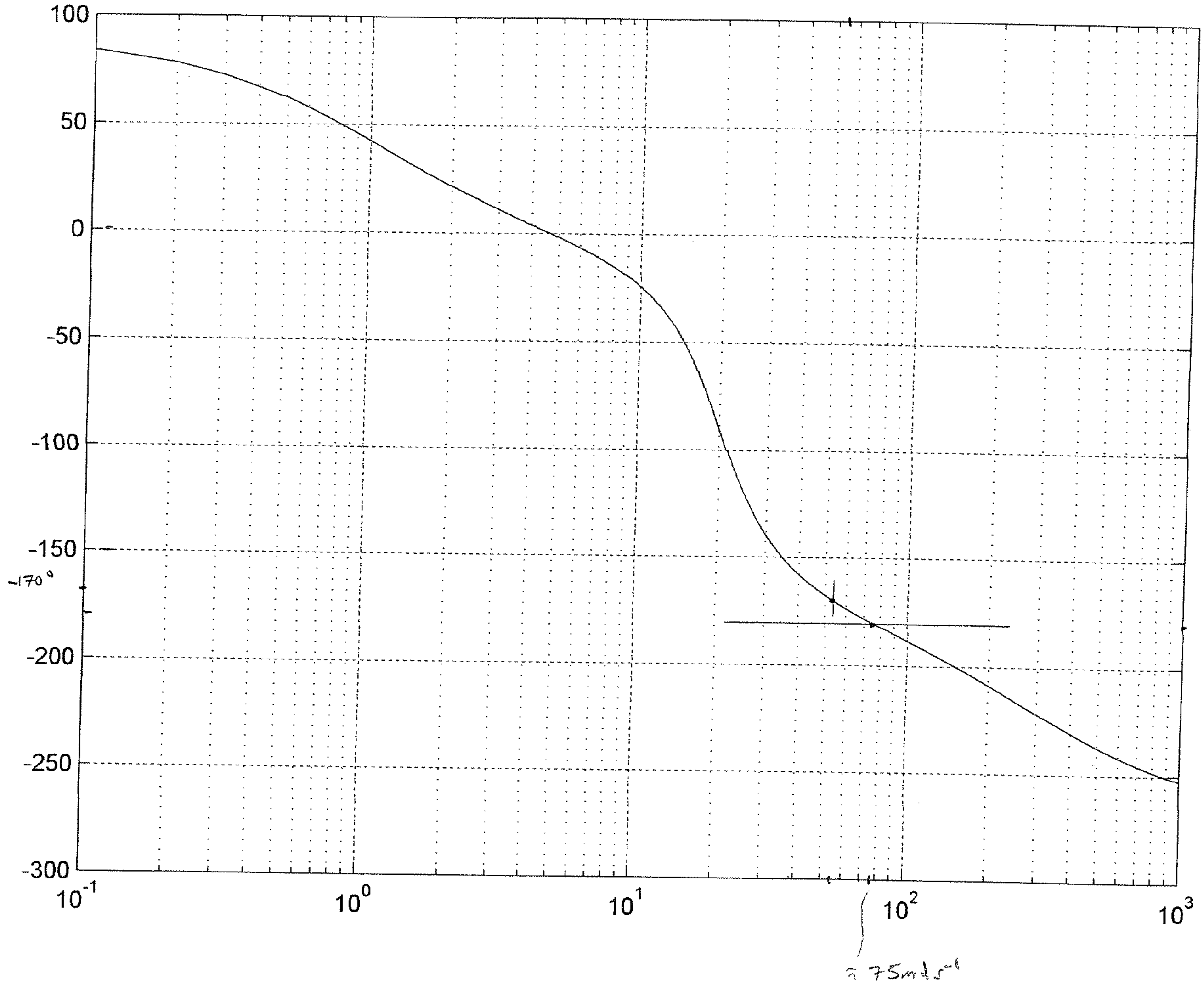


3

4.) (a) Stable if Nyquist diagram does not enclose the -1 point.
In this case we are talking about the complete Nyquist diagram of $K(s)\,G(s)$

[4]

(b) The limit of stability is at the -1 point

$\therefore$ Maximum positive value of K is, (from plot of $G(s)$)

$$K_{maxpos} = \frac{-1}{-0.2857} = 3.5$$

3/6

So with $K = 3.5$ the system is marginally stable. (occurs when $\omega = 1.2247$ rads^{-1}) 0.195 Hz.
This is the closed-loop frequency of oscillation.

3/6

[6]

(c) This time, max positive value of K is given by. (from plot of $G(s)\,K(s)$)

$$K_{maxpos} = \frac{-1}{-0.8} = 1.25$$

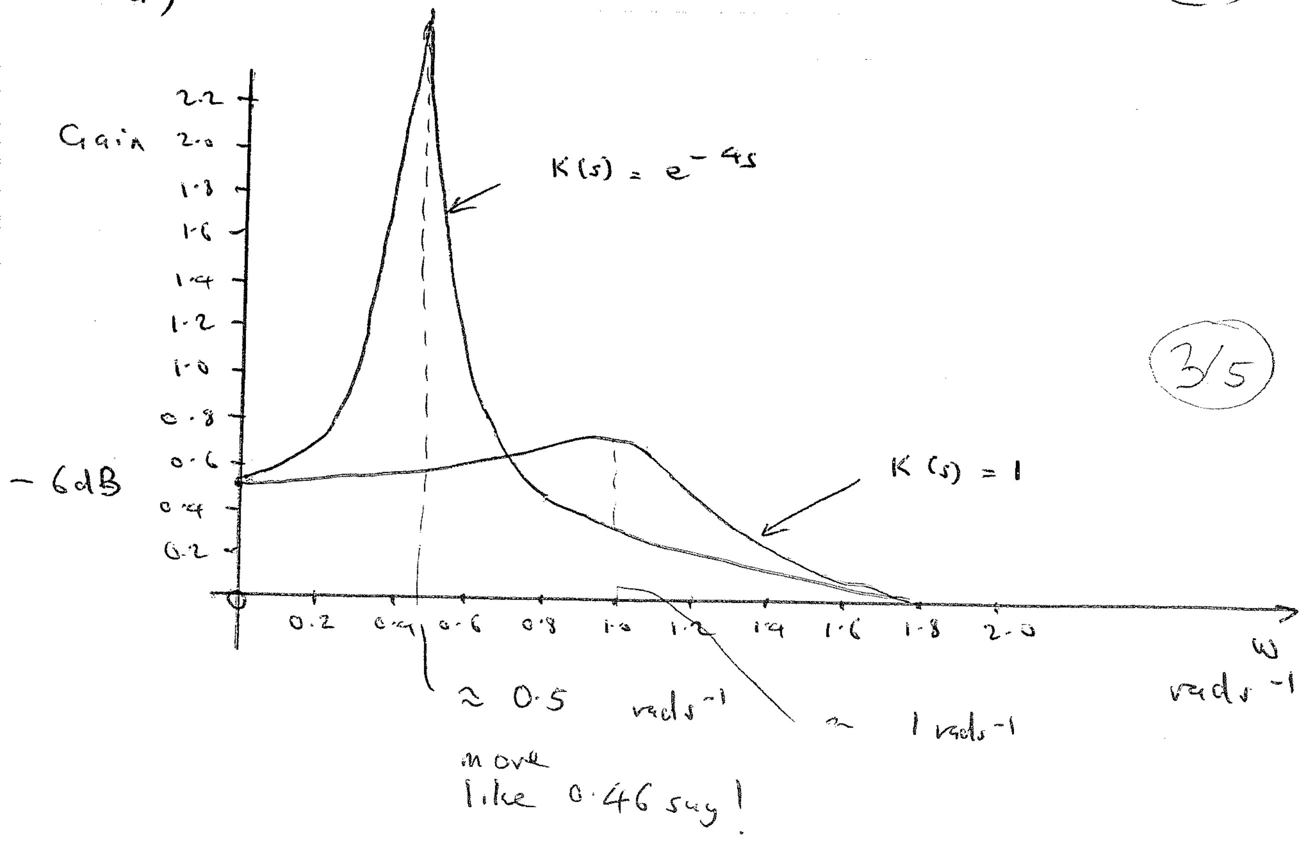
2/5

(See plot)

3/5

[5]

d)



3/5

)

With $K(s) = e^{-4s}$ response will be more oscillatory (at $w = 0.46$ rads^{-1}) in response to a step, ie will be very underdamped. Both will have final values of 0.5. (owing to low gain at d.c. - only 0.5).

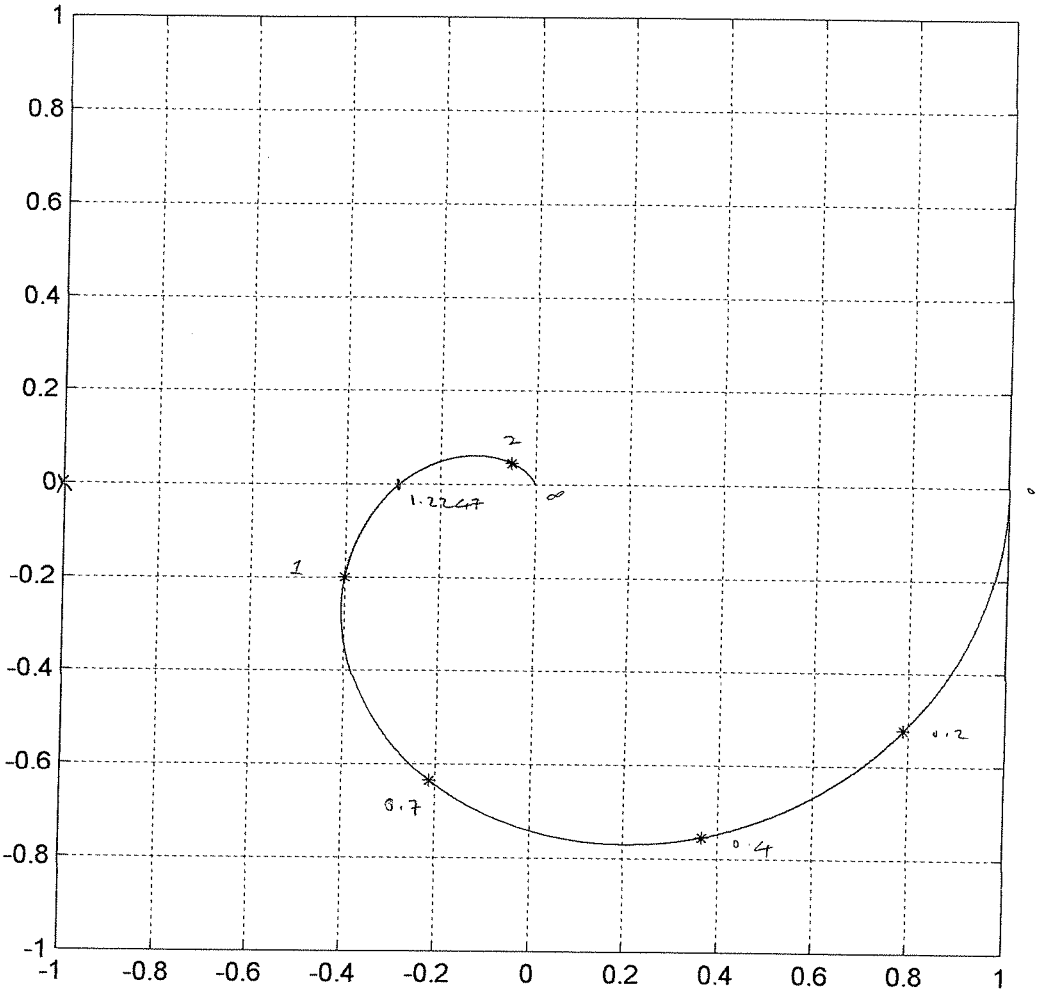
System with $K(s) = e^{-4s}$ will converge to final more quickly than the original system.

2/5

[5]

(15)

Closed loop freq response $= \frac{L}{1+L}$



ω	Gain
0	$\frac{62}{124} = 0.5$
0.2	$\frac{59}{116} = 0.51$
0.4	$\frac{52}{97} = 0.54$
0.7	$\frac{41}{63} = 0.65$
1.0	$\frac{28}{40} = 0.7$
1.225	$\frac{18}{44} = 0.41$
2	$\frac{4}{59} = 0.07$
∞	$0 = 0$
0.52	$\frac{47}{84} = 0.56$

$$G(s) = \frac{1}{(1+2s)(s^2+s+1)}$$

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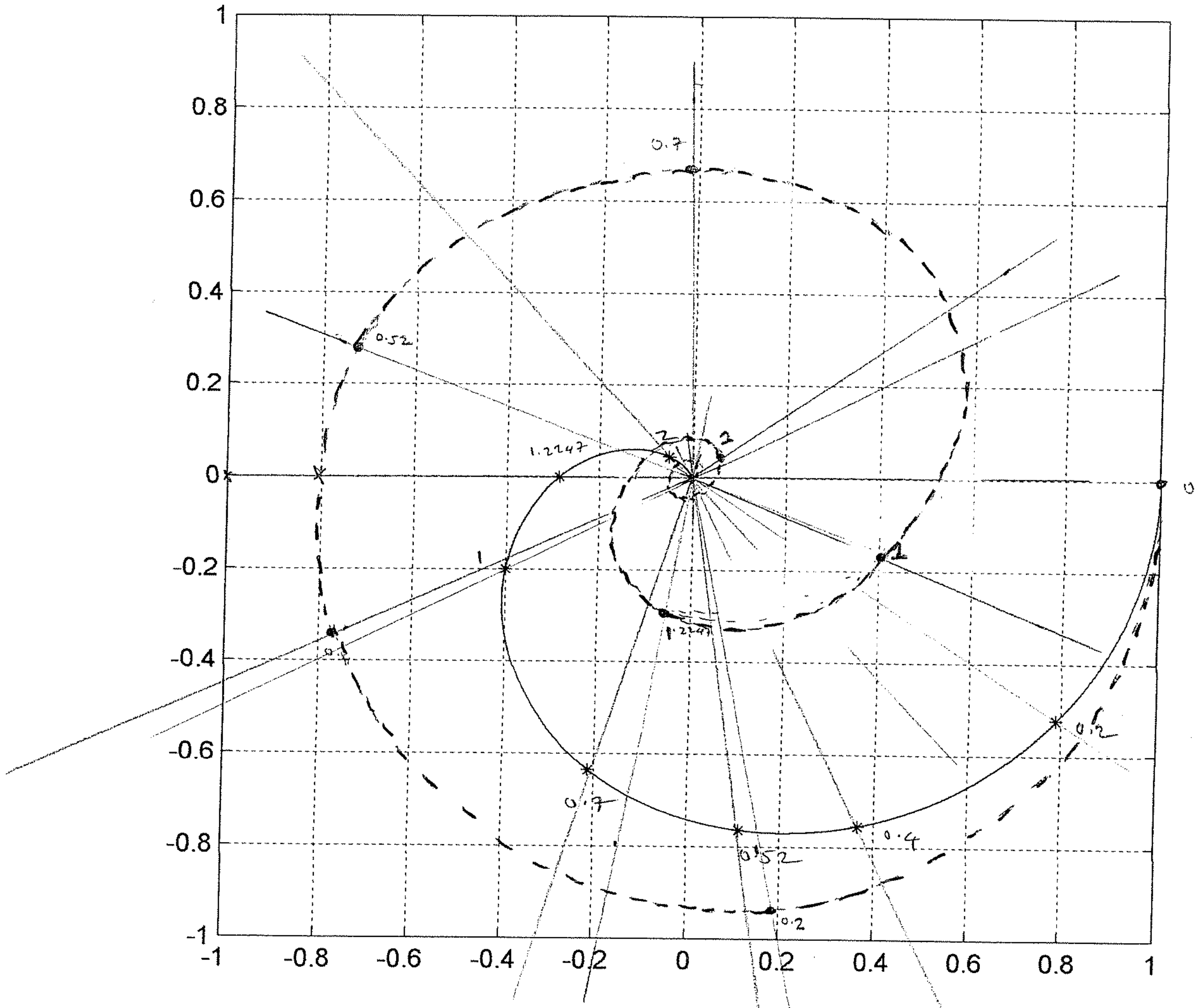

ω	$\angle e^{j4\omega}$	
0	0	0°
0.2	0.8	45.84°
0.4	1.6	91.67°
0.52	2.08	119.2°
0.7	2.8	160.4°
1	4	229.2°
1.2247	4.9	280.7
2	8	458.4° = 98.37°

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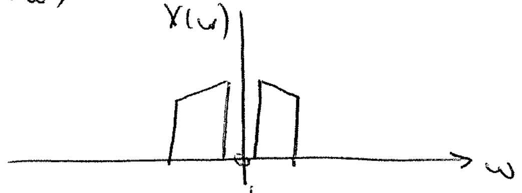
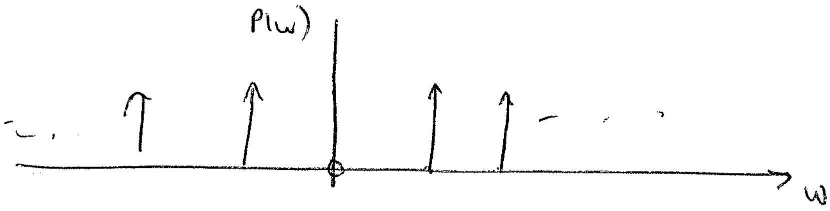
5.) (a) Aliasing distortion occurs if the sampling frequency f_s is too low for the input signal.

Consider the following example,

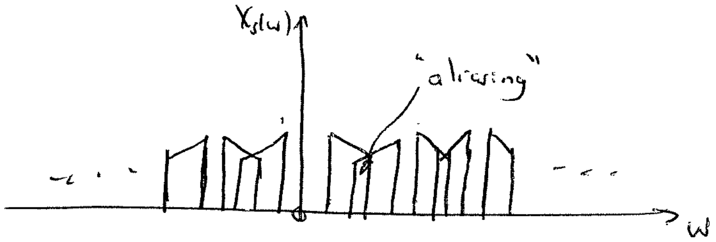
The spectrum of the signal to be sampled is $X(\omega)$



The power spectrum of the sampling signal is



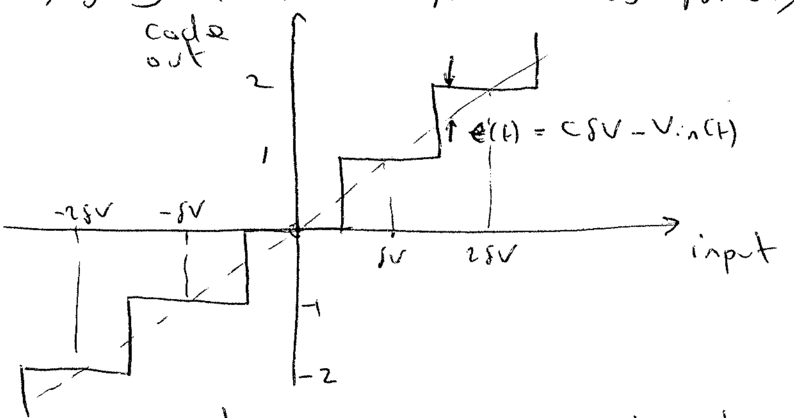
Giving a resulting spectrum $X_s(\omega)$ (after convolving $X(\omega)$ and $P(\omega)$)



To eliminate aliasing distortion, f_s must be at least twice the maximum frequency present in the input signal.

4/8

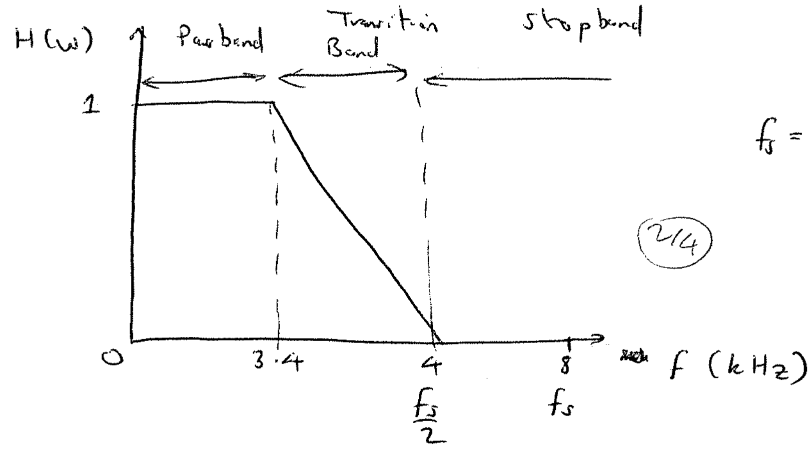
The ADC output is a finite precision binary number, giving a transfer function as follows,



See that the quantiser always rounds to the

nearest output value. Consequently the quantising error lies between $-\delta V/2$ and $+\delta V/2$.
Figure shows error voltage for a particular input V_{in}. (4/8)
[8]

(b)



$f_s = 4\,kHz$.

(2/4)

Transition band width = 4000 − 3400 = 600 Hz (1/4)

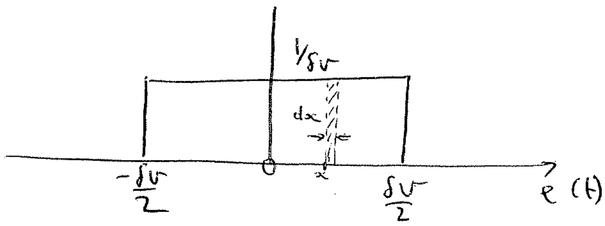
Percentage transition band width = $\frac{600}{3400} \times 100 = 17.65\%$ (1/4)

[4]

(c) Assumptions – quantisation has a uniform pdf from $-\delta V/2$ to $+\delta V/2$, ie, it is equally likely to lie anywhere in the range

– also quantising error is uncorrelated with the input signal. (1/4)

Probability of noise being between x and dx = $\frac{dx}{\delta V}$ if $|x| \leq \delta V/2$



The power (ie, volt^2) of a noise value of amplitude x is x^2

$\therefore$ Total error noise power = $\int_{-\delta V/2}^{\delta V/2} x^2 \frac{dx}{\delta V}$

i)

$$= \frac{1}{\delta v}\left[\frac{x^3}{3}\right]_{-\delta v/2}^{\delta v/2}$$

$$= \frac{1}{\delta v}\cdot\frac{1}{3}\left[\frac{\delta v^3}{8} + \frac{\delta v^3}{8}\right]$$

$$= \frac{1}{\delta v}\cdot\frac{\delta v^3}{12}$$

$$= \frac{\delta v^2}{12}$$

$\therefore$ rms noise voltage $= \underline{\underline{\dfrac{\delta v}{\sqrt{12}}}}$

3/4

[4]

d) Peak input signal range $= \pm A$

rms signal voltage $= A/\sqrt{3}$

rms noise voltage $= \delta V/\sqrt{12}$

where $\delta V = \dfrac{2A}{2^N}$

and N is the 'number of bits' in the quantiser.

$$SNR = 20\log_{10}\left(\frac{\text{rms signal voltage}}{\text{rms noise voltage}}\right)$$

$$= 20\log_{10}\left(\frac{A/\sqrt{3}}{2A/2^N\sqrt{12}}\right)$$

$$= 20\log_{10}\left(\frac{\sqrt{12}\,2^N}{2\sqrt{3}}\right) = 20\log_{10}(2^N)$$

$$= 20N\log_{10}2$$

$$= 6.02N$$

No. of bits (N)	SNR (dB)
5	30.1
6	36.1
7	42.1

So need at least $\underline{\underline{N = 6 \text{ bits}}}$ to achieve an SNR of 35dB.

3/4

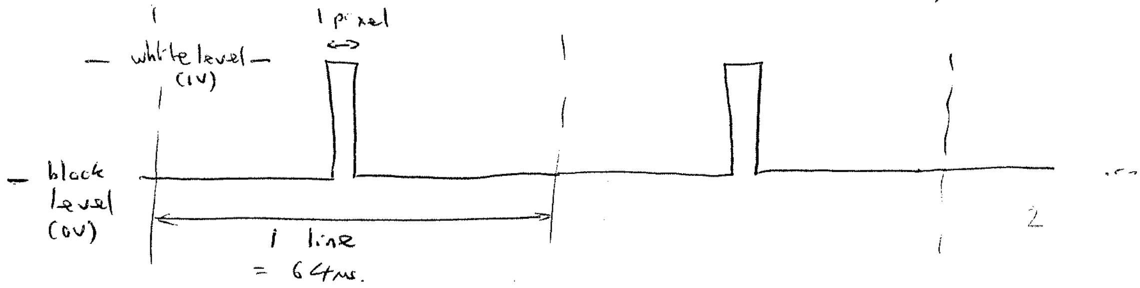
Non-linear quantisation, eg a logarithmic quantiser as employed in PCM will give a relatively constant SNR as a function of signal power. Also known as companding. Makes quantising steps small for small signals and larger for large signals $\therefore$ maintaining a constant SNR.

1/4

[4]

D

6.) (a) The transmitted signal has the form, eg.,



625 lines.

With equal horizontal and vertical resolution we have

$$625 \times \frac{4}{3} = 833.33 \text{ pixels per line.}$$ 1

We also have 625 lines every 1/25 s (with 2:1 interlacing)

So each line takes,

$$\frac{1}{25} \div 625 = 64\mu s.$$ 1

So the pixel duration is $\frac{64 \times 10^{-6}}{833.33} = 76.8\,ns$ 1

From the E and I Data book

$$y(t) = \frac{t_d}{T}\left[1 + 2\sum_{n=1}^{\infty} \frac{\sin(n\pi t_d/T)}{(n\pi t_d/T)} \cos(n\omega_0 t)\right]$$

where in this case,

$t_d = 76.8\,ns$ and $T = 64\mu s$. ($\therefore \frac{1}{T} = 15.625 \times 10^3$ Hz)

Also, $\omega_0 = 2\pi f_0 = \frac{2\pi}{T} = 2\pi \times 15.625 \times 10^3$, i.e., the cosinusoidal harmonics are at integer multiples of 15.625 kHz. 1

So,

$$y(t) = 1.2 \times 10^{-3}\left[1 + 2\sum_{n=1}^{\infty} \frac{\sin(1.2 \times 10^{-3}\pi n)}{1.2 \times 10^{-3}\pi n} \cos(2\pi \times 15.625 \times 10^3 n)\right]$$

2

[8]

(b) Amplitude of the fundamental component (n = 1) is

$$\frac{\sin 1.2 \times 10^{-3}\pi}{1.2 \times 10^{-3}\pi} \simeq 1$$ 1

The 3dB down point is given by the frequency component n which has an amplitude equal to 0.707 of that of the fundamental component, ie, 1)

$$\frac{\sin(1.2\times10^{-3} n\pi)}{1.2\times10^{-3} n\pi} = 0.707$$

ie, $\text{sinc}(1.2\times10^{-3} n\pi) = 0.707$

From the note on the question,

$$1.392 = 1.2\times10^{-3} n\pi$$

$$\therefore \quad n = \frac{1.392}{1.2\times10^{-3}\pi} = 369.24 \qquad 2$$

So, $n = 370$ is the first component which is $> 3\text{dB}$ down on the fundamental component. 1

$\therefore$ Transmitted bandwidth $= 370 \times 15.625\ \text{kHz}$
$= \underline{5.78\ \text{MHz}}$ 1

[5]

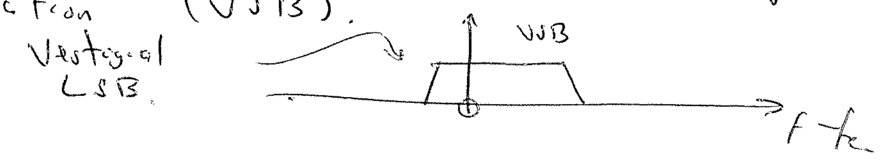
(c) (i) For AM, the transmitted bandwidth is 2× the baseband bandwidth, so,
Tx bandwidth $= 2\times 5.78\ \text{MHz} = \underline{11.56\ \text{MHz}}$, 2

(ii) For FM, the transmitted bandwidth using Carson's Rule is,

Tx bandwidth $= 2(M_F+1) f_m$
$= 2(3+1)\times 5.78\ \text{MHz}$
$= \underline{46.25\ \text{MHz}}$ 2

FM has a greater tolerance to noise than AM. Thus we are able to trade SNR for bandwidth in FM. Appropriate for a power limited system where a large bandwidth is available, eg, satellite TV. 2

In terrestrial TV we are bandwidth limited. The approach here is to eliminate virtually all of one of the sidebands. This is ok because a TV signal has most of its energy at low frequencies. In this way the bandwidth is $= \frac{1}{2}$ that of conventional AM. Known as Vestigial Sideband modulation (VSB). 1



[7]