ENGINEERING TRIPOS PART IB

Monday 2 June 2003 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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SECTION A

1 A uniform rod AB is of mass m and length $2a$. The end A is forced to move horizontally along a track at a steady velocity V while the rod rests on a support which is located a distance a below the track as shown in Fig. 1. At the instant shown, the rod makes an angle θ with the horizontal. The effects of friction can be neglected.

(a) Show that $\dot{\theta}$ and θ are related by the equation

$$\dot{\theta} = \frac{V}{a}\sin^2\theta \ .$$ [8]

(b) What will be the value of the angle θ when the rod just loses contact with the support? [8]

(c) What is then the magnitude of the horizontal force that is required at A? [4]

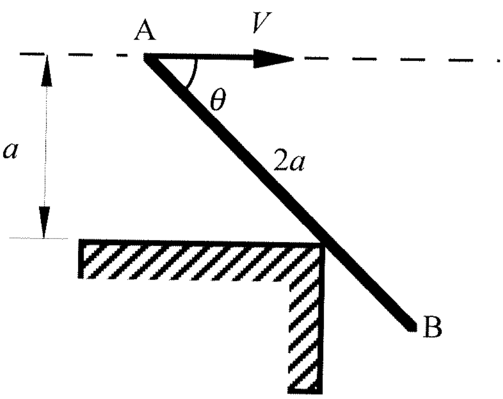

Fig. 1

2 A cylinder of radius R is fixed with its axis horizontal. A uniform, solid block of thickness $2h$ rests symmetrically on the cylinder as shown in Fig. 2.

(a) If friction is sufficient to prevent any slipping at the interface show that this equilibrium position is stable provided that

$$R \geq h \ .$$ [8]

(b) The radius of gyration of the block about a perpendicular axis through its centre of mass is k. If the block is displaced slightly from the equilibrium position, determine the frequency of the subsequent oscillatory motion in terms of R, h, k and g. [12]

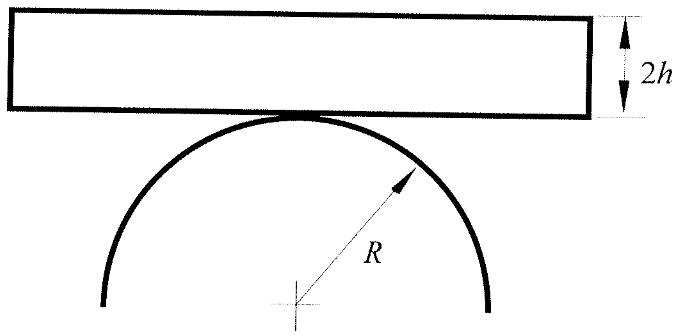

Fig. 2

3 Figure 3 shows a shaft which carries three thin discs each of mass 1 kg and diameter 100 mm at axial spacings of 100 mm. The shaft is supported in two bearings at the positions shown which are midway between the adjacent discs.

Disc C is mounted with its centre of mass on the axis of rotation but discs A and B both have their centres of mass displaced from the axis of the shaft by 0.05 mm. The angular displacement between the offsets of their centres of mass is 90°.

(a) The shaft runs at a speed of 10,000 rpm: calculate the magnitude of the forces supplied by the bearings. [8]

(b) The system is to be statically and dynamically balanced by adding masses to the rims of discs A and C. What are the magnitudes of the balancing masses and their angular positions relative to the displacement of the centre of mass of disc A? [12]

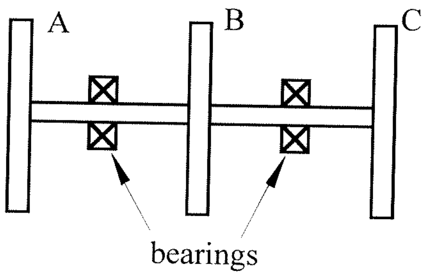

Fig. 3

SECTION B

4 Show that the polar moment of inertia of a square lamina of mass m and side $2a$ about a perpendicular axis through its centre is equal to $\frac{2}{3}ma^2$. [6]

A prismatic bar of mass m whose cross-section is a square of side $2a$ is rolling down a rough inclined plane which makes 45° with the horizontal, as shown in Fig. 4. Friction between the block and the plane is sufficient to prevent any sliding.

At the instant shown, the bar is rotating about corner A with an angular velocity of magnitude ω and corner D is just on the point of making contact with the plane. With what magnitude of angular velocity does the bar begin to rotate immediately after impact? What proportion of the kinetic energy of the bar is lost during the impact? [6]

If there is no average acceleration of the block down the plane find an expression for ω in terms of a and g. [8]

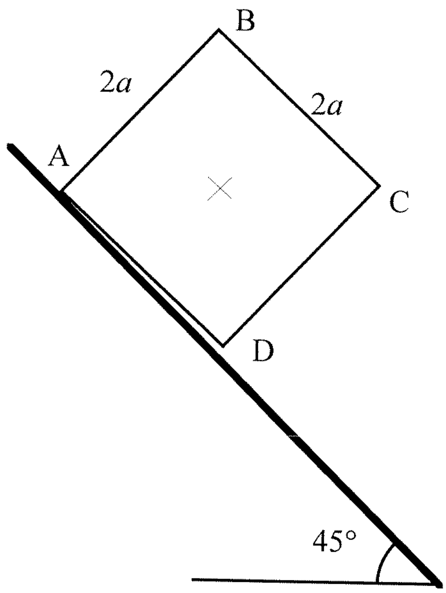

Fig. 4

5 Figure 5 shows a rigid, uniform bar OAB of total mass $2m$. OA = AB = a, the angle at A is 90° and the bar lies in a horizontal plane. The bar is pivoted about a vertical axis at O and, at the instant shown, is moving with angular velocity ω and angular acceleration $\dot{\omega}$ in the sense shown.

Obtain an expression for the moment of inertia of OAB about the point O and, hence or otherwise, show that the torque required to provide this motion is equal to

$$\frac{5}{3} ma^2\dot{\omega} .$$ [5]

Find an expression for the component of acceleration normal to the bar at a point in AB which is at a distance x from A. Hence obtain expressions for the way in which the shear force and bending moment are distributed as x varies from zero to a. The effect of gravity can be neglected. [9]

If the numerical values of ω and $\dot{\omega}$ are 8 rad s^{-1} and 100 rad s^{-2} respectively, find the value of x at which the bending moment in AB reaches a maximum. [6]

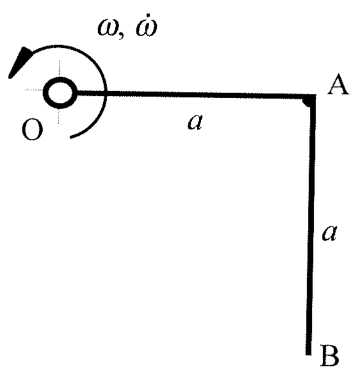

Fig. 5

6 Figure 6 shows a plane mechanism in which the crank OP, which is of length a, rotates at a steady angular speed ω. The crank is freely pivoted to the slider at P and the bar QR which is of length $2a$ is similarly freely pivoted at the point Q. The angles θ and ϕ measure respectively the inclination of the bars OP and QR to the horizontal. At the instant shown the angle θ is equal to 60°.

(a) What are then the magnitudes of $\dot{\phi}$ the angular velocity and $\ddot{\phi}$ the angular acceleration of the bar QR? Solutions by either drawing or calculation are equally acceptable. [14]

(b) If the two bars are each of mass m per unit length and the mass of the slider and any friction at P are negligible, what is the instantaneous torque required at the point O? The effect of gravity can be neglected. [6]

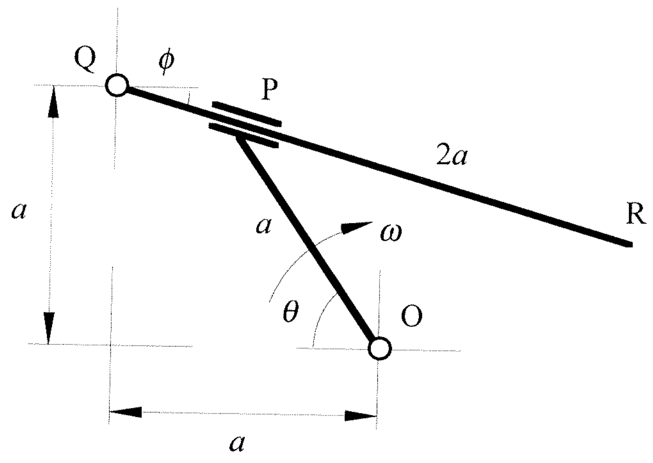

Fig. 6

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